Precision Learning: Reconstruction Filter Kernel Discretization

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September 28, 2018

Reconstruction Pipeline as a Neural Network

Already proposed

- Filtered back-projection (FBP) algorithm as Neural Network[1]
- Compensation weights to reduce limited angle artifacts[2]

Benefits

- data-driven knowledge-enhancing abilities[3]
- allows to exchange heuristically method

Question

$\rightarrow$ Can we learn the reconstruction filter ?

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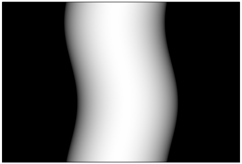
[1] **Tobias W\"urfl, Florin Cristian Ghesu, Vincent Christlein, and Andreas Maier, "Deep Learning Computed Tomography", in MICCAI 2016: 19th International Conference, Proceedings, Part III, 2016, vol. 3, pp. 432-440.**.

[2] **Kerstin Hammernik, Tobias W\"urfl, Thomas Pock, and Andreas Maier, "A deep learning architecture for limited-angle computed tomography reconstruction", in BVM 2017 Heidelberg, 2017, pp. 92-97, Springer Berlin Heidelberg**.

[3] **Ge Wang, "A perspective on deep imaging", IEEE Access, vol.4, pp. 8914-8924, 2016.**

Recap: CT Reconstruction

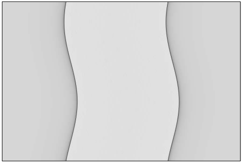
Sinogram



$p(s, \theta)$

$\xrightarrow{\quad}$ $h(s) * p(s, \theta)$

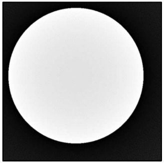
Filtered sinogram



$q(s, \theta)$

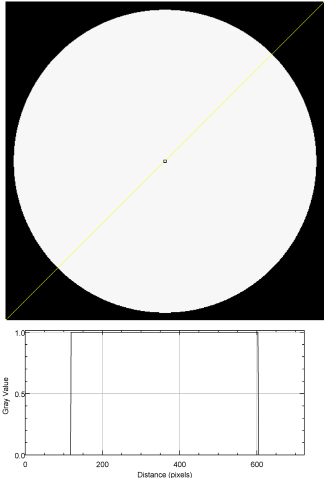
$\xrightarrow{\quad}$ $f(x, y) = \int q(s, \theta) d\theta$

Reconstruction

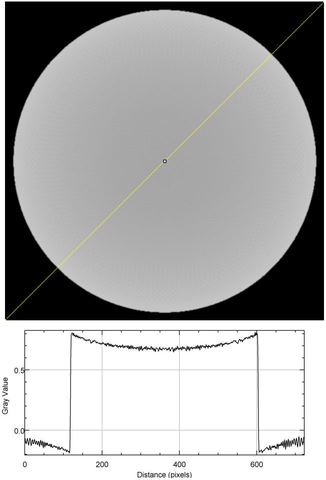


where $h(s)$ is the Ramp-Filter

Cupping Artifacts



Line profile through phantom.



Line profile through Ramp-Reco

Deriving the Network Topology

Discrete reconstruction problem:

$$\mathbf{A}\mathbf{x} = \mathbf{p}$$

$$\mathbf{x} = \underbrace{\mathbf{A}^\top}_{\text{Back-projection}} \underbrace{\left(\mathbf{A}\mathbf{A}^\top\right)^{-1}}_{\text{Filter}} \mathbf{p}$$

substituting the inverse:

$$\mathbf{x} = \mathbf{A}^\top \mathbf{F}^\mathbf{H} \mathbf{K} \mathbf{F} \mathbf{p}$$

where

- $\mathbf{A}$ is the system matrix
- $\mathbf{x}$ is the object
- $\mathbf{p}$ is the sinogram
- $\mathbf{F}, \mathbf{F}^\mathbf{H}$ is the Fourier and inverse Fourier-transform
- $\mathbf{K}$ is the filter in Fourier domain

Deriving the Network Topology

Objective function:

$$f(\mathbf{K}) = \frac{1}{2}\|\mathbf{A}^{\top}\mathbf{F}^{\mathbf{H}}\mathbf{K}\mathbf{F}\mathbf{p} - \mathbf{x}\|_2^2$$

Derivative:

$$\frac{\partial f(\mathbf{K})}{\partial \mathbf{K}} = \mathbf{F}\mathbf{A}(\mathbf{A}^{\mathbf{T}}\mathbf{F}^{\mathbf{H}}\mathbf{K}\mathbf{F}\mathbf{p} - \mathbf{x})(\mathbf{F}\mathbf{p})^{\top}$$

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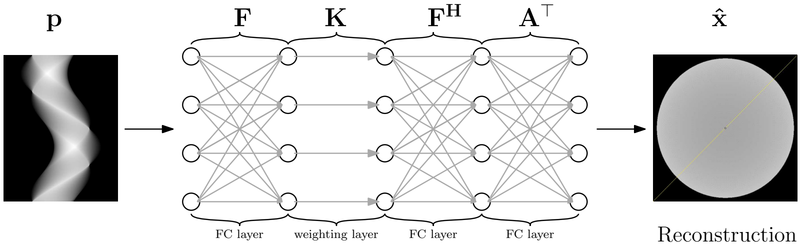
Deriving the Network Topology

Objective function:

$$f(\mathbf{K}) = \frac{1}{2}\left\|\mathbf{A}^\top \mathbf{F}^\mathbf{H}\mathbf{K}\mathbf{F}\mathbf{p} - \mathbf{x}\right\|_2^2$$

Derivative:

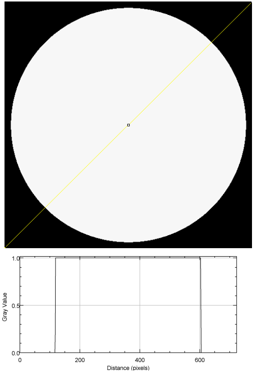
$$\frac{\partial f(\mathbf{K})}{\partial \mathbf{K}} = \underbrace{\mathbf{F}\mathbf{A}\underbrace{\left(\mathbf{A}^\mathbf{T}\mathbf{F}^\mathbf{H}\mathbf{K}\mathbf{F}\mathbf{p} - \mathbf{x}\right)}_{\text{Error}}}_{\text{Back-propagation}}\Big(\underbrace{\mathbf{F}\mathbf{p}}_{l-1}\Big)^\top$$



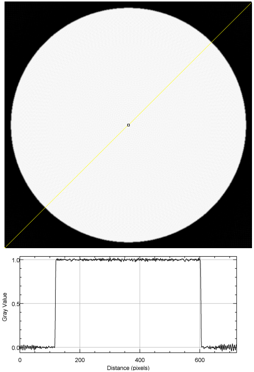
Experimental Setup

- **K** is initialized with the Ramp
- For training 10 numerical disc phantoms (increasing radii)
- Evaluation on real CT-dataset

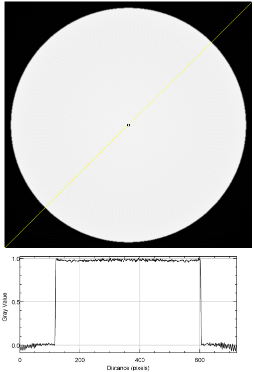
Results: Phantoms



Line profile through GT.

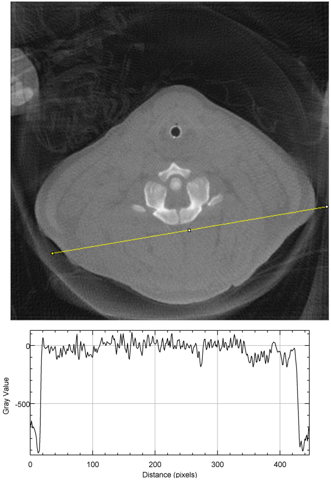


Line profile through Ram-Lak-reco.

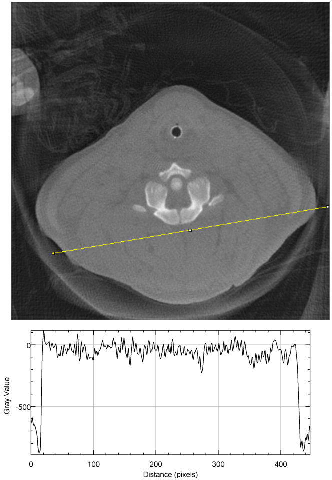


Line profile through Learned-reco.

Results: CT data



Reconstruction with Ram-Lak filter.



Reconstruction with learned filter.

Results: Quantitative Evaluation

Phantom data (absolute difference):

	mean	std. dev.	min	max
Ramp-reco	0.235	0.07	0.001	0.596
Ram-Lak-reco	0.01	0.031	0	0.41
Learned-reco	0.023	0.03	6.76E-09	0.409

CT data:

	mean	std. dev.	min	max
Ram-Lak-reco	66.99	61.401	6.10E-5	1634.82
Learned-reco	83.53	68.06	8.39E-5	1685.70

Conclusion

Outlook:

- Apply noise models to the training data
- Setup a complete CT Reconstruction pipeline

Take home message:

- Derive the network topology from the continuous analytical problem description
- Neural network intrinsically compensate for discretization errors
- Interesting link between neural network techniques and signal processing

Thanks for listening.

Any questions?