

# Documentation of class `ScaleSpaceStudies`

## Methods

- `main`:

The main method allows to set

- the parameters of the phantom, which determine the type and the size of the phantom. There are five possible types of phantom (see Figure 1):
  - \* custom phantom of geometric shapes
  - \* Shepp-Logan phantom
  - \* uniform circle phantom
  - \* ring phantom
  - \* "dartboard" phantom with nine rings
- the parameters of the Gaussian / Laplacian of Gaussian, which determine the kernel size and the values for the Gaussian width  $\sigma$
- whether the Gaussian, Laplacian or the Laplacian of Gaussian is used. Moreover, it is possible to compare the Laplacian of Gaussian referred to the method `laplacianOfGaussian` (see below) with the successive application of filtering an image with the Gaussian and then with the Laplacian.

At first, the phantom is created according to the set parameters. By default, the Shepp-Logan phantom is taken. A fan-beam projector creates a projection image, i. e. a sinogram, from the phantom. This sinogram is convolved with either the Laplacian or the Gaussian respectively the Laplacian of Gaussian for the different values of  $\sigma$ . By default, the convolution with the Laplacian of Gaussian is executed. The resulting sinogram is corrected using a Ram-Lak filter and a cosine filter for every row of the sinogram, before the fan beam backprojector creates the back-projected image out of it. At last, an image displaying the difference between the phantom and the reconstruction is computed in case of the Gaussian. For the comparison of the Laplacian of Gaussian with the successive application of a gaussian and a laplacian filter, the difference between both reconstructions is additionally shown in an image.

- `ScaleSpaceStudies`:

The constructor creates the custom phantom. It consists of

- a big circle slightly shifted from the center to the lower right with the intensity 1.0
- a small circle slightly shifted from the center to the upper left with the intensity 1.5
- an ellipse in the upper left with the intensity 0.9
- a rectangle in the lower left with the intensity 1.5

The intensity in the area, where the big circle and the rectangle intersect, is the sum of both intensities of the circle and the rectangle, i. e. 2.5 (see Figure 1(a)).

- `phantomDartboard`:

This method of type `Grid2D` creates a phantom that looks like a dartboard with nine rings by adding up circles with different radii. The given radius  $r$  (input of type `double`) defines the radius of the outermost circle. The radii of the other circles depend irregularly on  $r$ , so that some rings are wider or smaller than others (see Figure 1(e)).

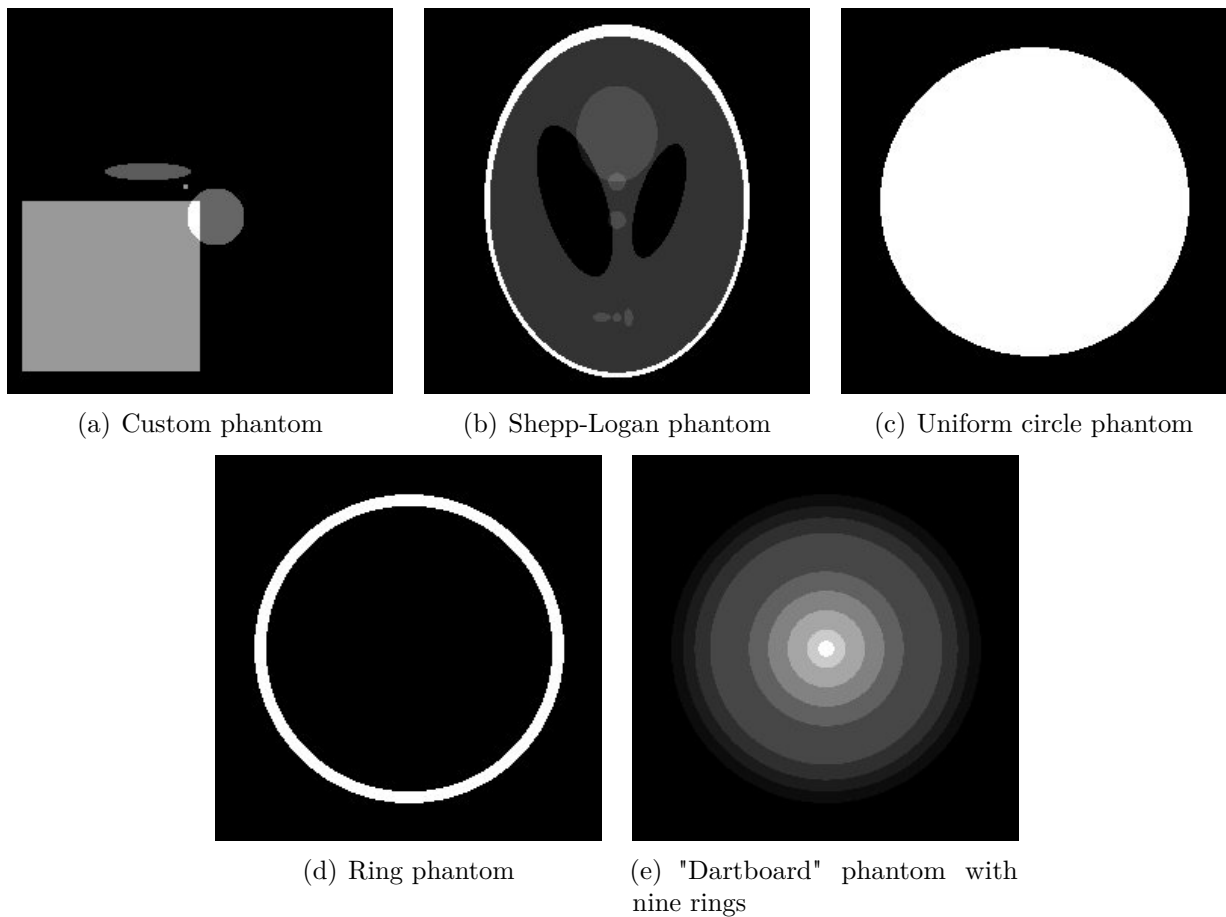


Figure 1: Possible types of phantom

- **gaussianSinogram:**

This method returns a sinogram of type `Grid2D` which results out of the convolution of a given sinogram (input of type `Grid2D`) with the Gaussian function for a given  $\sigma$  (input of type `double`). Therefore, the sinogram is splitted into single rows which are transformed in the Fourier space to multiply them by the Fourier transform of the Gaussian distribution  $e^{-\frac{1}{2}\sigma^2\omega^2}$ . Finally, the resulted rows are transformed back using an inverse Fourier transform and copied back into the output sinogram.

- **laplacianOfGaussianSinogram:**

Using this method of type `Grid2D`, the convolution of a given sinogram (input of type `Grid2D`) with the Laplacian of Gaussian for a given  $\sigma$  (input of type `double`) is computed. Therefore, the input sinogram is splitted into single rows which are convolved with the Laplacian of Gaussian using the method `convolutionLoG` (see below) which additionally needs the kernel size (type `int`) as input. Finally, the computed rows are copied back into the output sinogram.

- **convolutionLoG:**

This method of type `Grid1D` computes the convolution of a signal (input of type `Grid1D`) with the Laplacian of Gaussian for a given  $\sigma$  (input of type `double`) and a given kernel size (input of type `int`). The result has the same size as the input signal. Successively, for each element  $n$  of the input signal a kernel window with the lower bound  $k_{\min}$  and the upper bound  $k_{\max}$  is determined. These bounds ensure that only values within the signal are taken into account, i. e. they prevent an out-of-bound-exception. For better results, the signal is converted to a higher sampling rate  $v$  (upsampling).

The value at position  $n$  of the convolved result is gained by the successive addition of all values given by the multiplication of the interpolated signal values and the values from the Laplacian of Gaussian kernel. Corresponding values are given for  $k \in [k_{\min}; k_{\max}]$  for the signal at  $(1 - u) \cdot \text{signal}(k) + u \cdot \text{signal}(k + 1)$  and the kernel at  $n - v$ , where  $v = k + u$  and  $u$  is the sampling step.

- **laplacianOfGaussian:**

Use this method of type `double` to calculate the second derivative of the Gaussian distribution at a given position  $x$  (input index of type `int`) and for a given  $\sigma$  (input of type `double`) according to

$$\text{LoG}(x) = -\frac{1}{\sigma^3\sqrt{2\pi}} \cdot \left(1 - \frac{x^2}{\sigma^2}\right) \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right).$$

## Results

Figure 2 shows the custom phantom of geometric shapes (left hand side) and the corresponding sinogram (right hand side). Figure 3 to Figure 6 show the sinograms of the own created phantom achieved from the convolution with the Laplacian of Gaussian and the resulted back-projected images for increasing values of  $\sigma \in \{0.1, 1.0, 3.0, 5.0\}$ . Figure 7 shows the "dartboard" phantom (left hand side) and the corresponding sinogram (right hand side). Figure 8 shows the sinogram and the resulted back-projected image for the convolution with the Laplacian filter. Figure 9 to Figure 12 show the sinograms of the "dartboard" phantom achieved from the convolution with the Laplacian of Gaussian and the resulted back-projected images for increasing values of  $\sigma \in \{0.1, 1.0, 3.0, 5.0\}$ . Comparing Figure 8 with Figure 9, it can be seen, that the Laplacian and the Laplacian of Gaussian are almost the same for small values of  $\sigma$ . Nevertheless, Figure 9 looks a bit more uniform due to the smoothing effect of  $\sigma$  in the Laplacian of Gaussian. In general, the Laplacian of Gaussian highlights the edges with a bright and a dark line along the edges. Small values of  $\sigma$  lead to images with sharp edges and therefore to a representation of the edge structure (see Figure 3 and Figure 9). With increasing values of  $\sigma$ , the edges expand. Small objects, like the circle in the upper left in the custom phantom, vanish (see Figure 6). Especially Figure 11 shows, that large values result in the illustration of the transitions. Bigger regions with intensity modification are pointed out.

Figure 13 to Figure 16 show the comparison for increasing values of  $\sigma \in \{0.1, 1.0, 3.0, 5.0\}$  of the Laplacian of Gaussian referred to the method `laplacianOfGaussian` (left hand side) with the successive application of filtering the sinogram with the Gaussian and then with the Laplacian before the back-projection (right hand side). Underneath, a plot shows the gray values of a straight line through the middle of the difference image of both reconstructions. The dominant peaks appear at the edges. The higher the value of  $\sigma$  is, the smoother the difference image is and the smaller the peaks are.

Figure 17 to Figure 20 show the sinograms achieved from the convolution with the Gaussian, the resulted back-projected images and the differences between the phantom and the back-projected images for increasing values of  $\sigma \in \{0.1, 1.0, 3.0, 5.0\}$ . The Gaussian blurs the edges and with increasing values of  $\sigma$ , the different objects with different intensities merge into one single object with one intensity. The convolution with a Gaussian containing a very small  $\sigma$  subtracted from the phantom results in an image with very distinct edges (see Figure 17, rightmost image).

The experiments show, that filtering the projections (sinograms) with the Laplacian of Gaussian provides a result which is similar to what is expected when applying filtering of the reconstructions. An advantage of the former method is that computation of the scale-space features is performed in 1D.

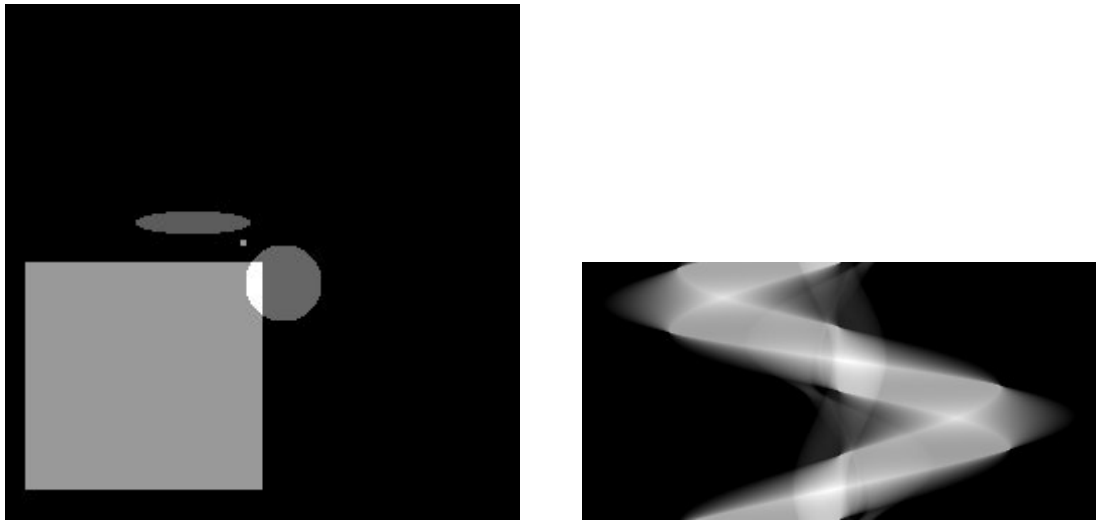


Figure 2: Custom phantom and resulting sinogram

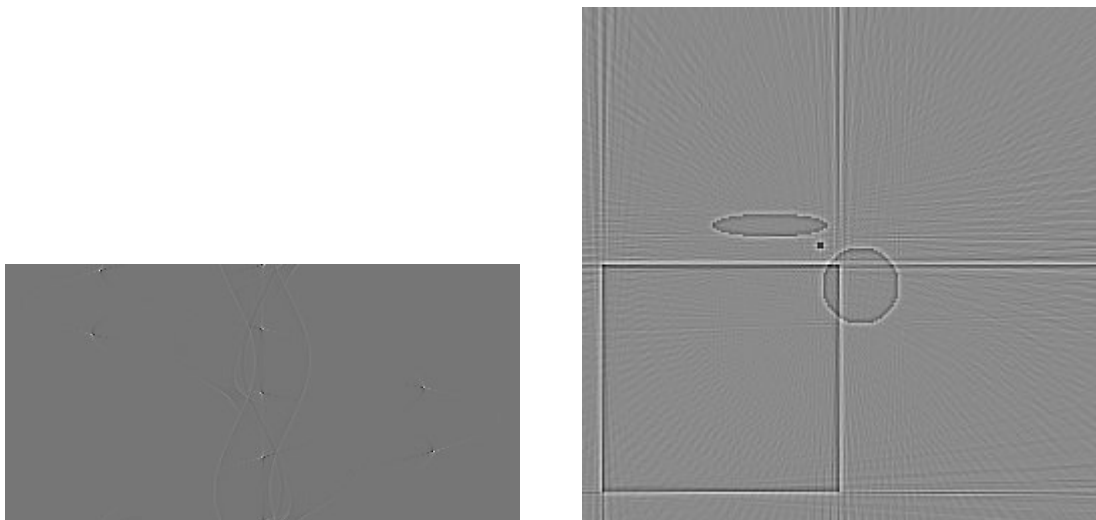


Figure 3: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 0.1$

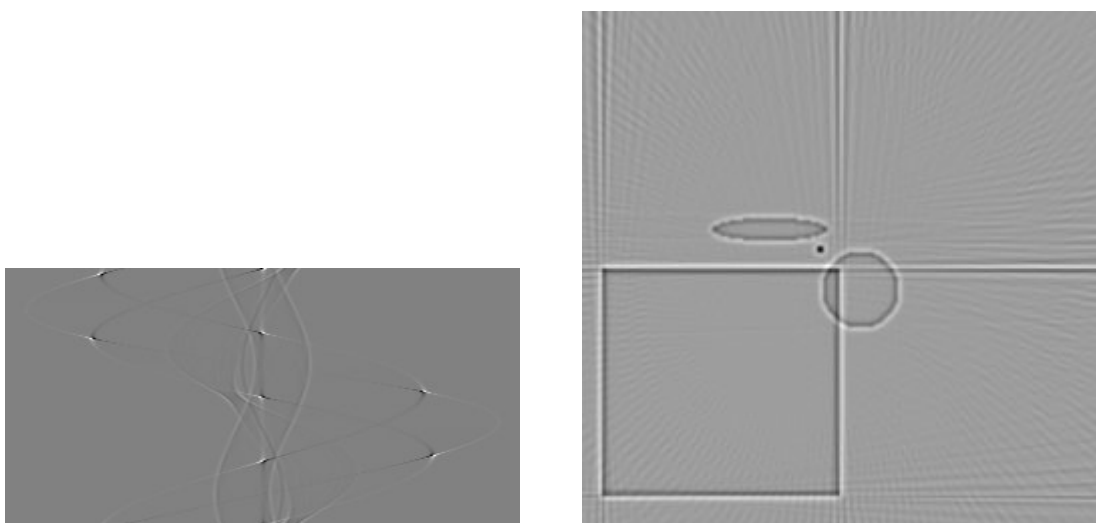


Figure 4: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 1.0$

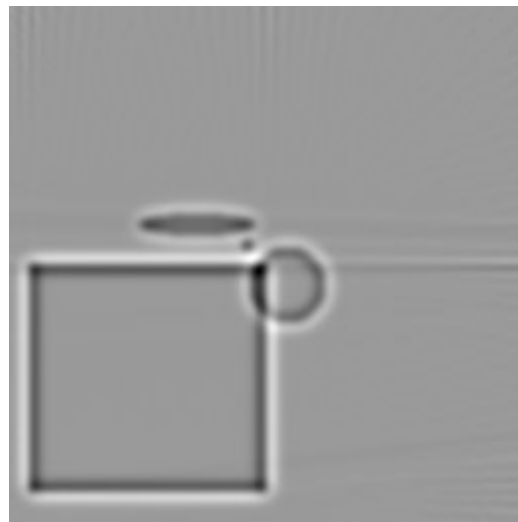
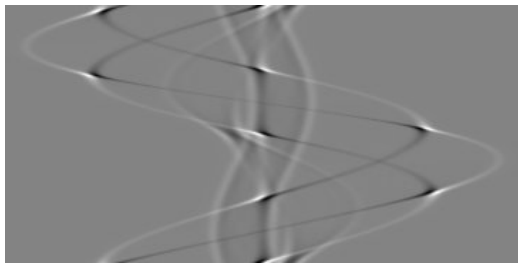


Figure 5: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 3.0$

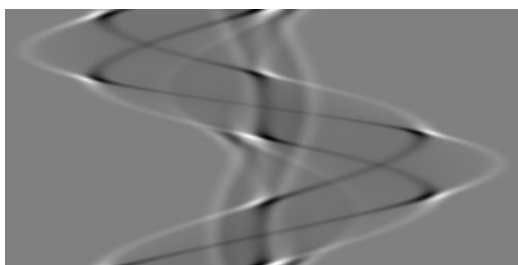


Figure 6: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 5.0$

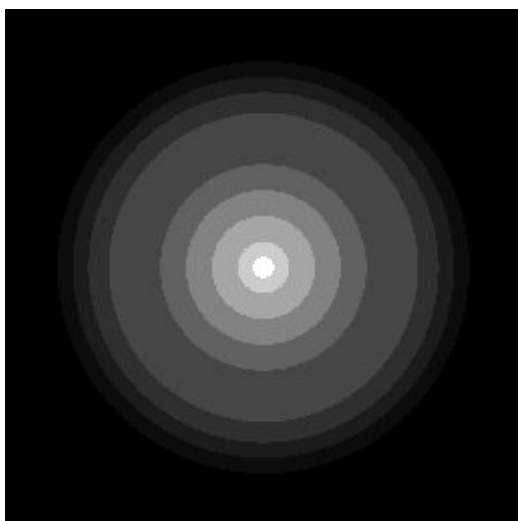


Figure 7: "Dartboard" phantom and resulting sinogram

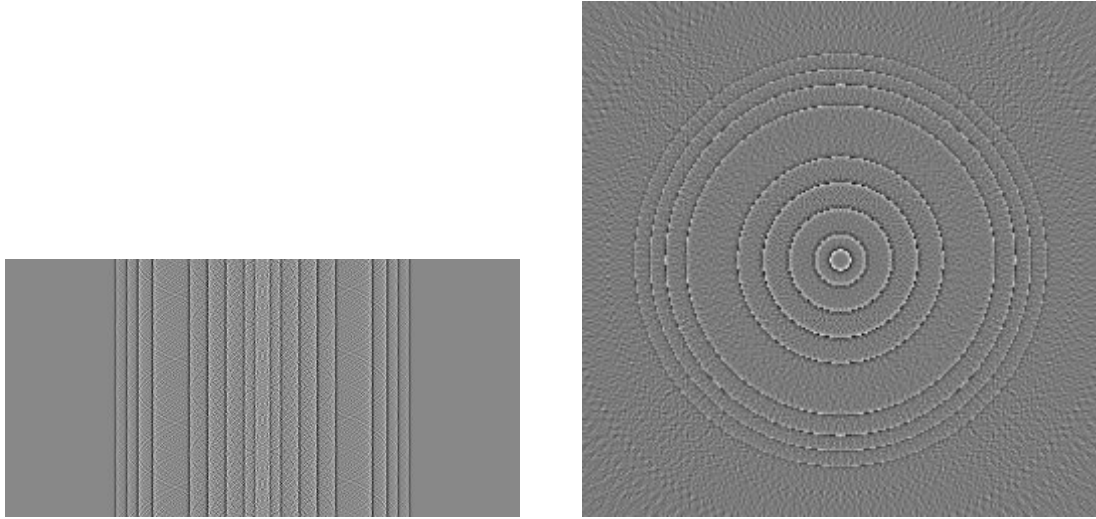


Figure 8: Sinogram and back-projected image for Laplacian

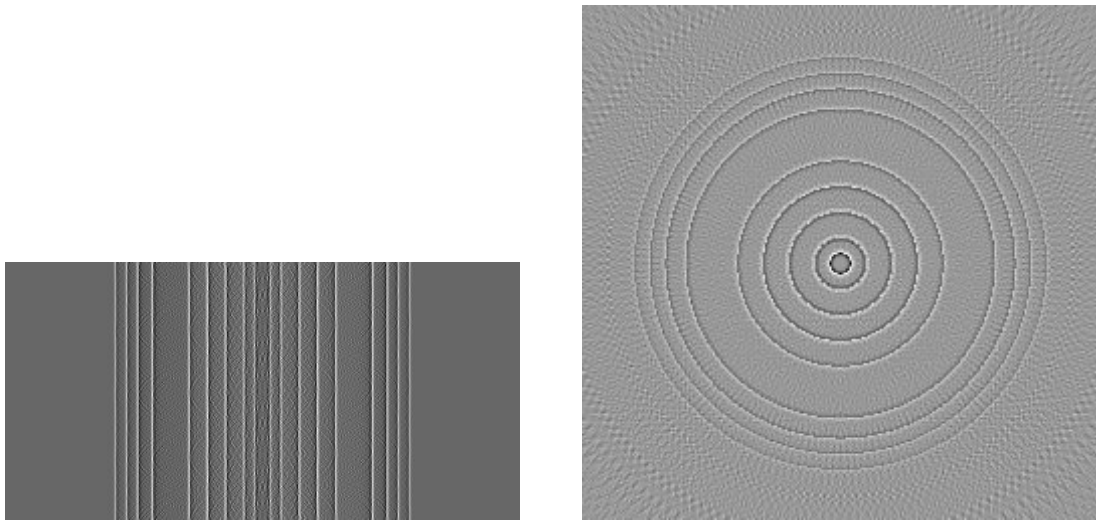


Figure 9: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 0.1$

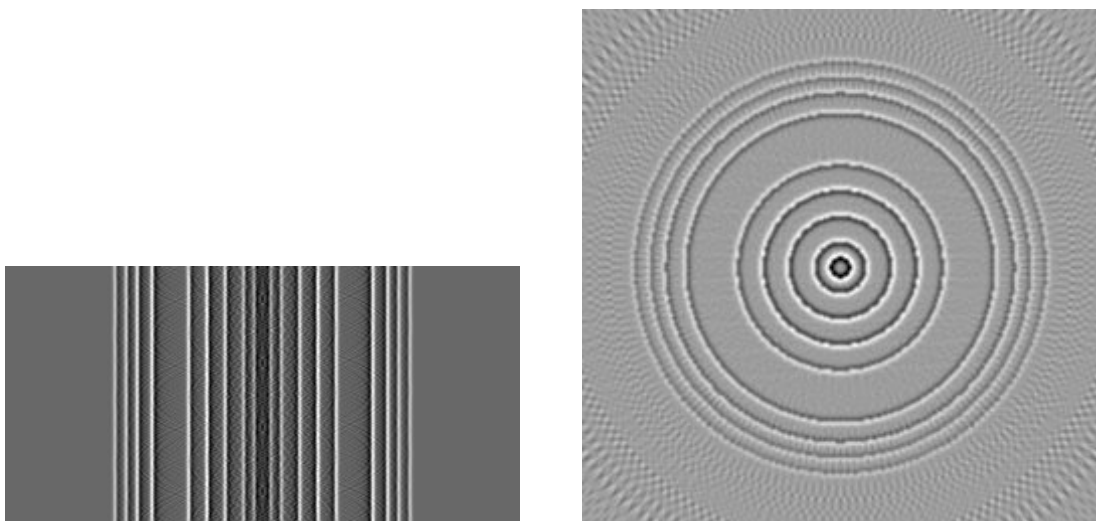


Figure 10: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 1.0$



Figure 11: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 3.0$



Figure 12: Sinogram and back-projected image for Laplacian of Gaussian with  $\sigma = 5.0$

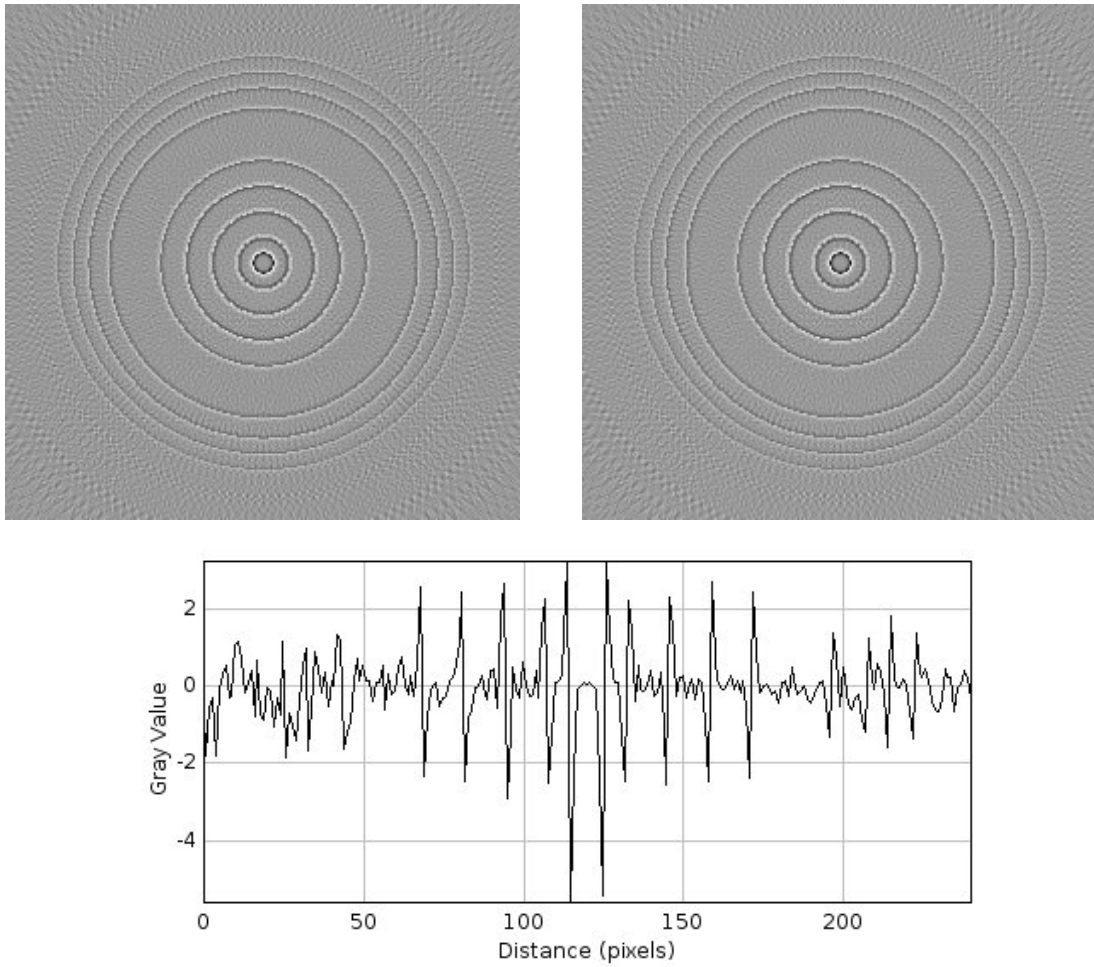


Figure 13: back-projected image for Laplacian of Gaussian (left) and for successive filtering with Gaussian and Laplacian (right) for  $\sigma = 0.1$  and resulting plot of the centric line of the difference image



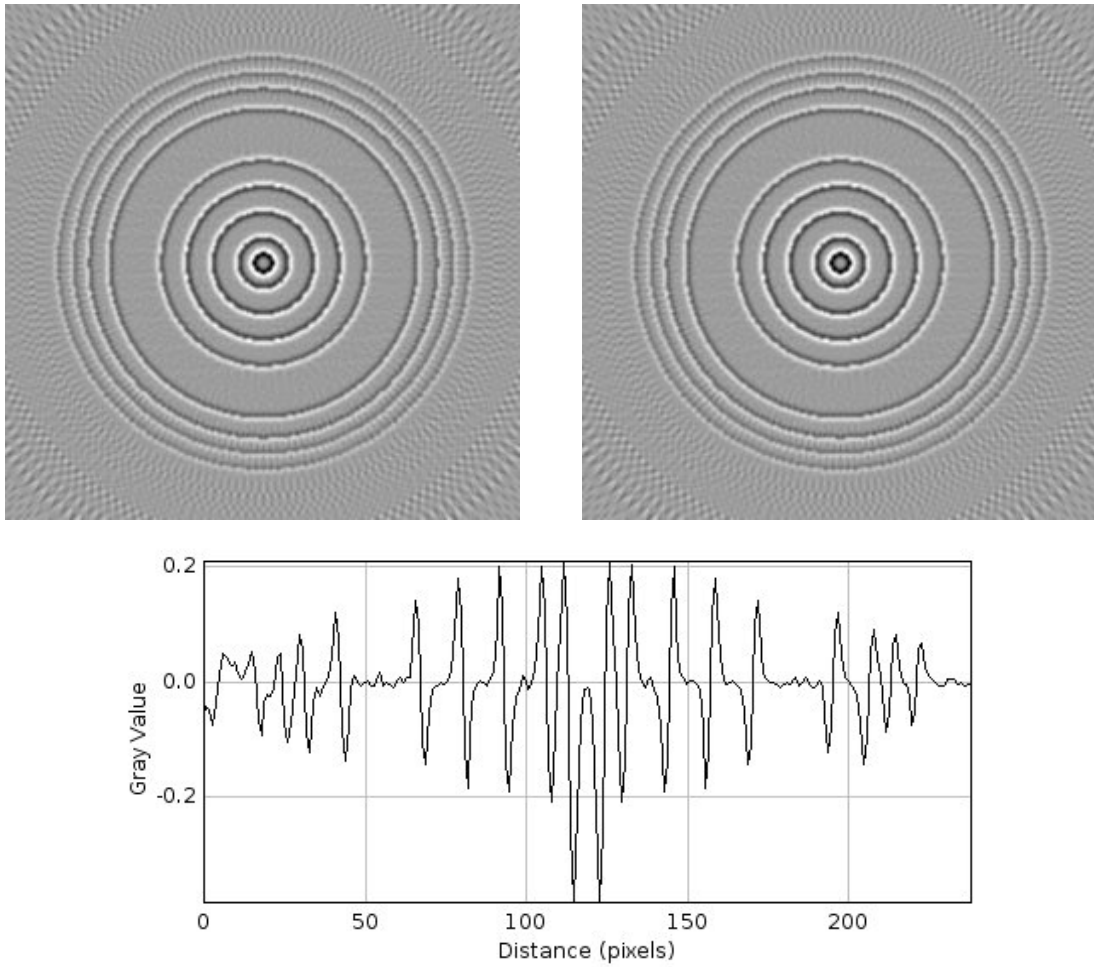


Figure 14: back-projected image for Laplacian of Gaussian (left) and for successive filtering with Gaussian and Laplacian (right) for  $\sigma = 1.0$  and resulting plot of the centric line of the difference image

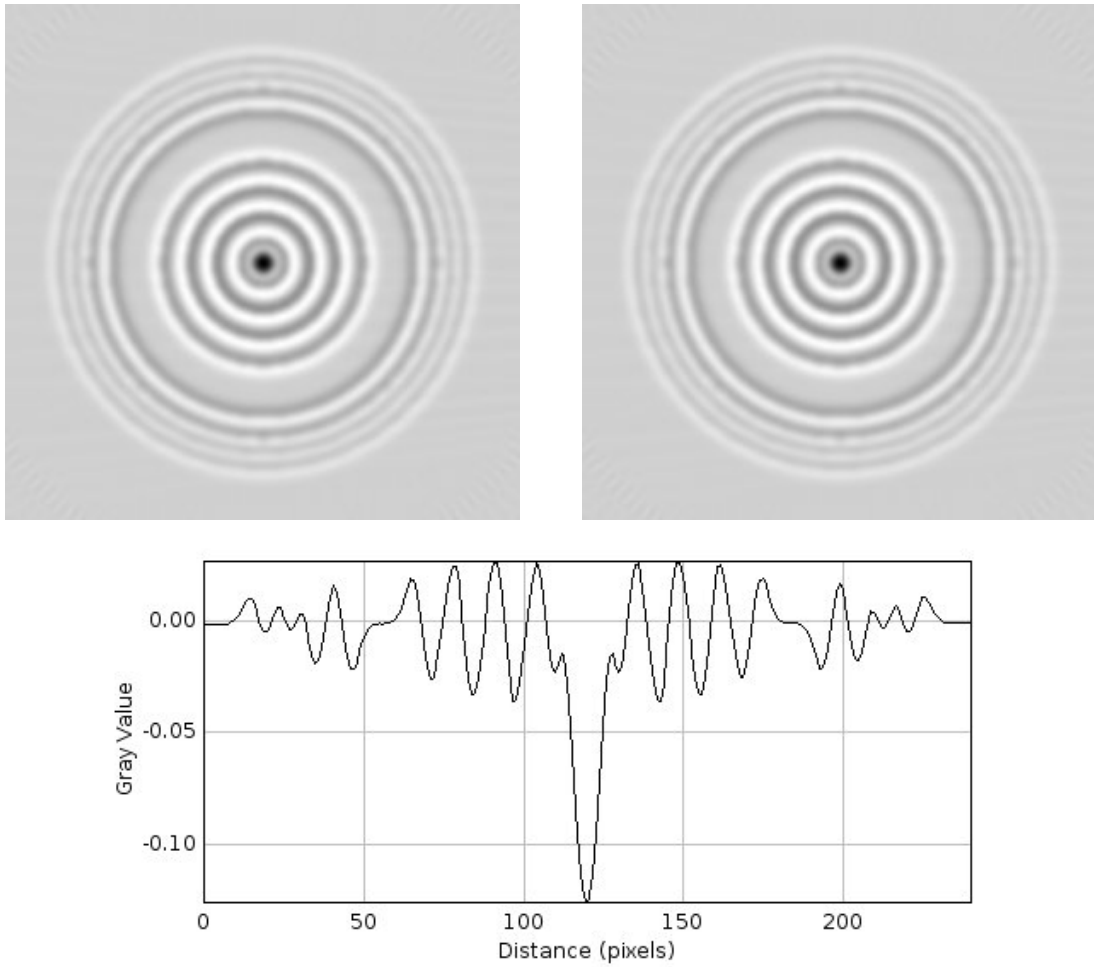


Figure 15: back-projected image for Laplacian of Gaussian (left) and for successive filtering with Gaussian and Laplacian (right) for  $\sigma = 3.0$  and resulting plot of the centric line of the difference image

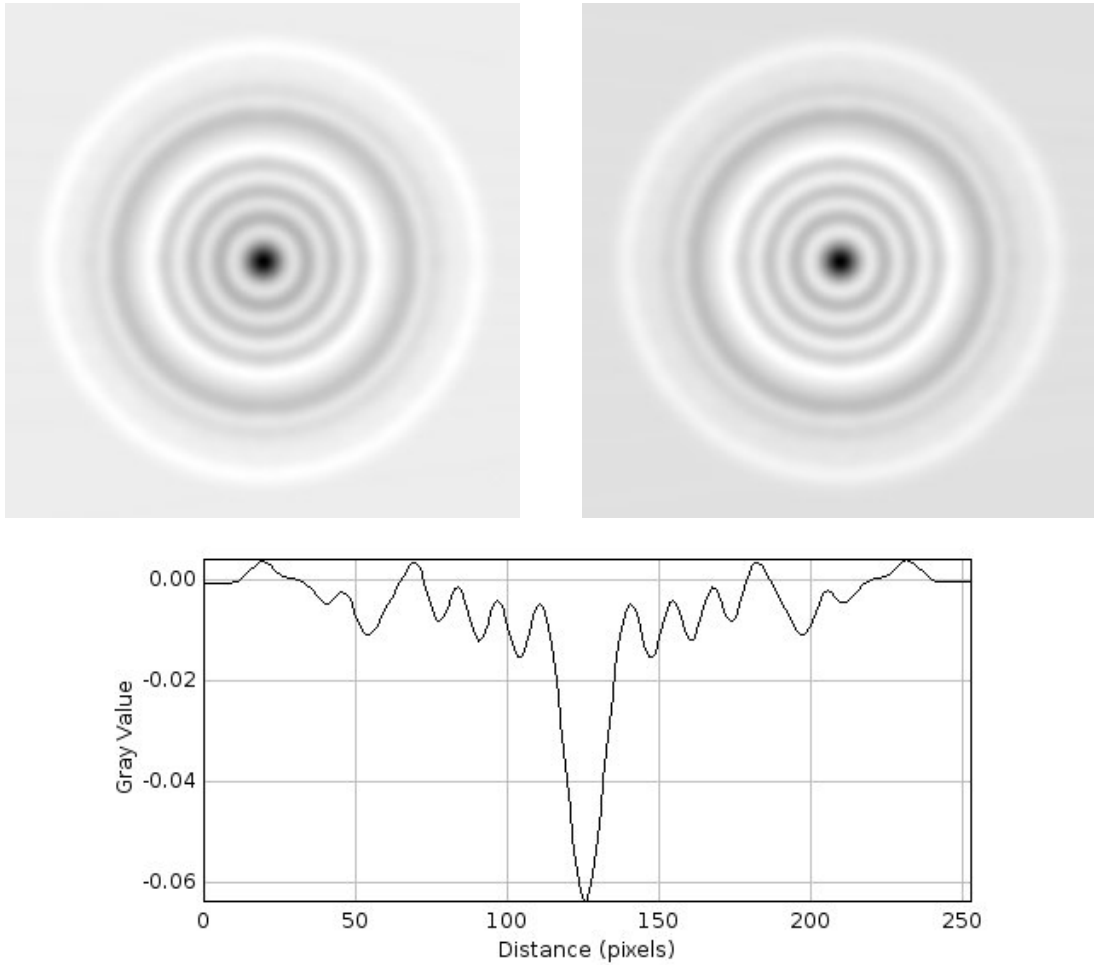


Figure 16: back-projected image for Laplacian of Gaussian (left) and for successive filtering with Gaussian and Laplacian (right) for  $\sigma = 5.0$  and resulting plot of the centric line of the difference image

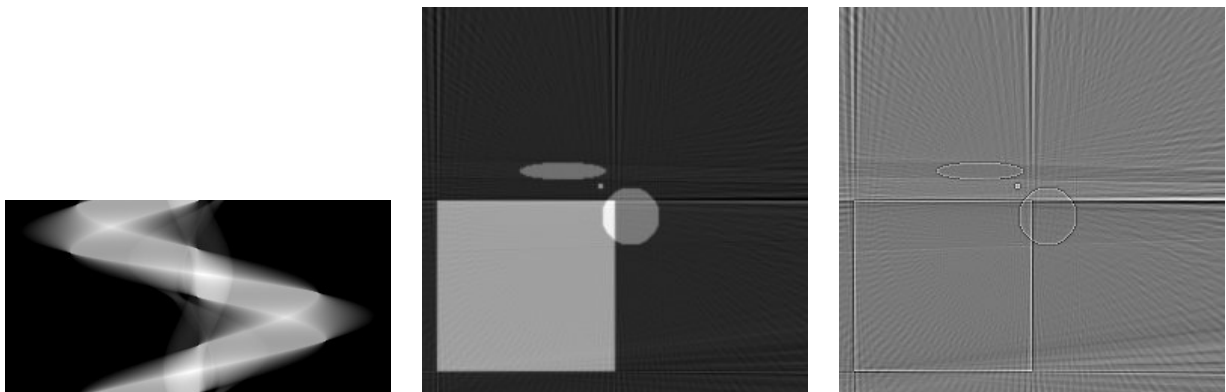


Figure 17: Sinogram, back-projected image and difference between the phantom and the back-projected image for Gaussian with  $\sigma = 0.1$

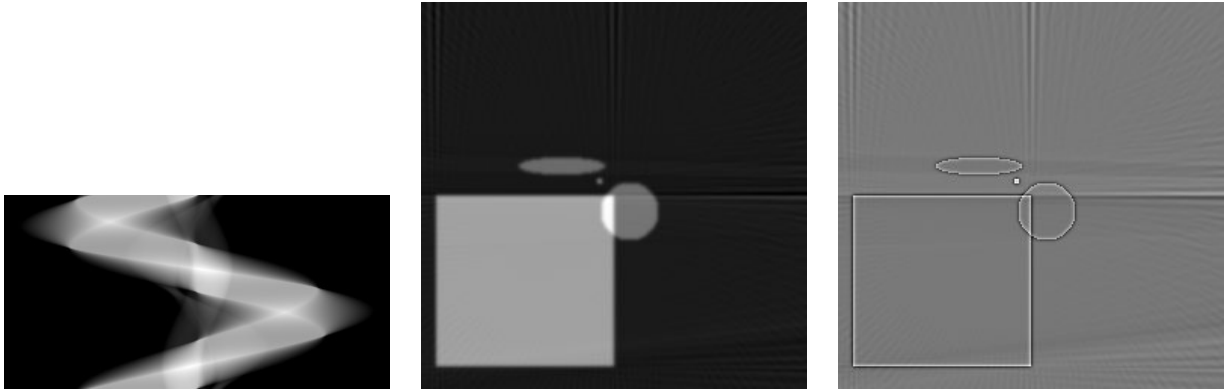


Figure 18: Sinogram, back-projected image and difference between the phantom and the back-projected image for Gaussian with  $\sigma = 1.0$

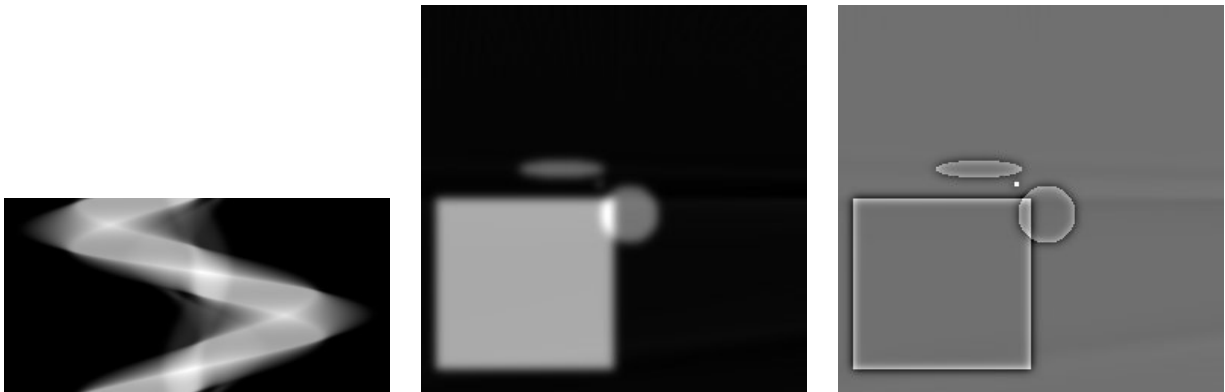


Figure 19: Sinogram, back-projected image and difference between the phantom and the back-projected image for Gaussian with  $\sigma = 3.0$

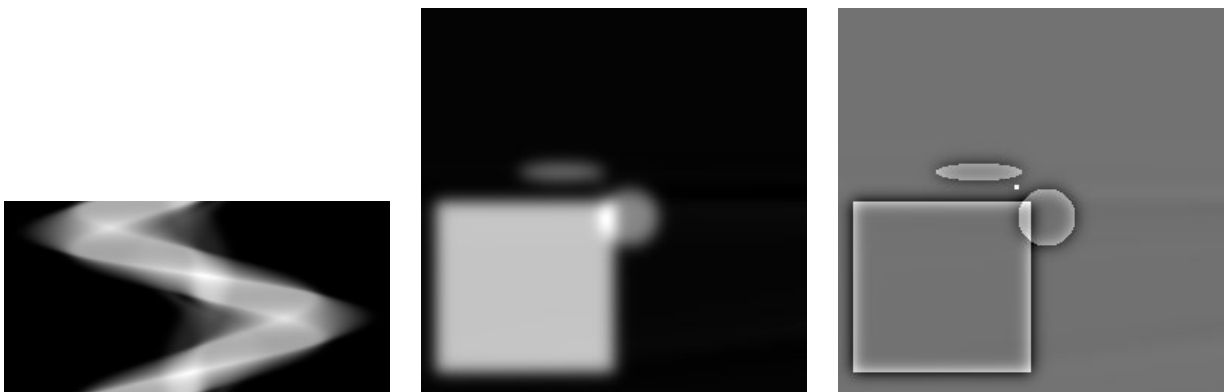


Figure 20: Sinogram, back-projected image and difference between the phantom and the back-projected image for Gaussian with  $\sigma = 5.0$