

# DMIP – Exercise

## *Sinograms and Filtered Backprojection (FBP) for Fan Beam*

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# Fan-Beam Reconstruction

- Start with parallel FBP equation:

$$f(x, y) = \frac{1}{2} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \Theta) h(x \cos(\Theta) + y \sin(\Theta) - s) ds d\Theta$$

- Use following identities:

$$x = r \cos(\varphi)$$

$$y = r \sin(\varphi)$$

$$r(\cos(\varphi)\cos(\Theta) + \sin(\varphi)\sin(\Theta)) = r \cos(\Theta - \varphi)$$

- This gets us the polar-coordinate representation:

$$f(r, \varphi) = \frac{1}{2} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \Theta) h(r \cos(\Theta - \varphi) - s) ds d\Theta$$

# Fan-Beam Reconstruction

- Change of variables ( We want to get rid of  $\Theta$  and  $s$ )

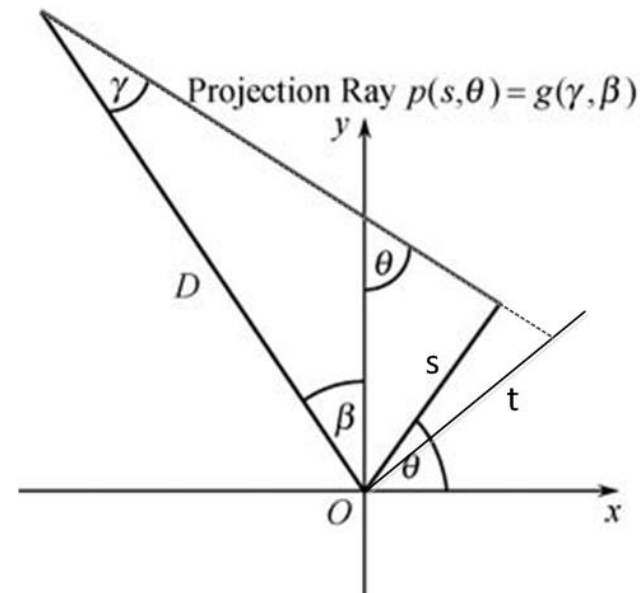
$$f(r, \varphi) = \frac{1}{2} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \Theta) h(r \cos(\Theta - \varphi) - s) ds d\Theta$$

- Use following identities:

$$g(t, \beta) = g(\gamma, \beta) = p(s, \Theta)$$

$$s = \frac{D}{\sqrt{D^2 + t^2}} t = \cos(\gamma) t$$

$$\Theta = \beta + \tan^{-1}(t/D) = \beta + \gamma$$





# Fan-Beam Reconstruction

- Change of variables: After some magical math [Zeng09]

$$f(r, \varphi) = \frac{1}{2} \int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2 + t^2}} g(t, \beta) h(\hat{t} - t) dt d\beta$$

- With cosine weight:

$$c(t, \beta) = \frac{D}{\sqrt{D^2 + t^2}} = \cos(\gamma)$$

- With distance weight:

$$U = \frac{D + r \sin(\beta - \phi)}{D}$$



# Fan-Beam Backprojection

- Similar approach as in parallel beam, however:
  - Projection of pixels onto detector not parallel
  - Intersection of rays need to be computed for each pixel
  - We have to perform cosine weighting before backprojection
  - During the backprojection we need to apply distance weighting
  - The distance weight depends on the projection angle and pixel position
  - → Distance weight needs to be calculated for each pixel



# Fan-Beam Backprojection

- Intersection of two lines (Hesse normal form)

$$\text{X-ray line: } \vec{x}^T \vec{n}_r - d_r = 0$$

$$\text{Detector line: } \vec{x}^T \vec{n}_s - d_s = 0$$

- Reshape to matrix form

$$\begin{pmatrix} \vec{n}_{r_1} & \vec{n}_{r_2} \\ \vec{n}_{s_1} & \vec{n}_{s_2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d_r \\ d_s \end{pmatrix}$$

- Solve by calculating the inverse

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \vec{n}_{r_1} & \vec{n}_{r_2} \\ \vec{n}_{s_1} & \vec{n}_{s_2} \end{pmatrix}^{-1} \begin{pmatrix} d_r \\ d_s \end{pmatrix}$$