

Correct solution for exercise 1c.

The i -th Lagrangian multiplier is given by:

$$\lambda_i = \left(\sum_{j=1}^K (q \cdot d(\mathbf{x}_i, \boldsymbol{\mu}_j))^{\frac{1}{1-q}} \right)^{1-q}$$

Substituting λ leads to:

$$\begin{aligned} c_{mn} &= \left(\frac{\lambda_m}{q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_n)} \right)^{\frac{1}{q-1}} \\ &= \frac{\left(\left(\sum_{j=1}^K (q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_j))^{\frac{1}{1-q}} \right)^{1-q} \right)^{\frac{1}{q-1}}}{(q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_n))^{\frac{1}{q-1}}} \\ &= \frac{\left(\sum_{j=1}^K (q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_j))^{\frac{1}{1-q}} \right)^{-1}}{\left((q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_n))^{\frac{1}{1-q}} \right)^{-1}} \\ &= \frac{(q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_n))^{\frac{1}{1-q}}}{\sum_{j=1}^K (q \cdot d(\mathbf{x}_m, \boldsymbol{\mu}_j))^{\frac{1}{1-q}}} \\ &= \frac{\|\mathbf{x}_m - \boldsymbol{\mu}_n\|_2^{\frac{2}{1-q}}}{\sum_{j=1}^K \|\mathbf{x}_m - \boldsymbol{\mu}_j\|_2^{\frac{2}{1-q}}} \end{aligned}$$