Tortuosity measurements

Attila Budai

Chair of Pattern Recognition (Computer Science 5)

Friedrich-Alexander-University Erlangen-Nuremberg

Outline

- Introduction
  - Tortuosity measurements
- measurements
  - Arc Length Over Chord Length Ratio
  - tc/tsc over Arch length/Chord Length
  - Mean direction Angle Change
  - Inflection Count Metric
  - Proposed method of the article
- Summary

Introduction

- Based on article:
  - A Novel Method for the Automatic Grading of Retinal Vessel Tortuosity
  - Enrico Grisan, Marco Foracchia, and Alfredo Ruggeri
  - IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 27, NO. 3, MARCH 2008
- Requirements:
  - For an automatic measurement a segmentation and thinning algorithm is needed

Arc Length Over Chord Length Ratio (implemented)

- Chord Length:
  - Distance of end points (L_x)
- Arc Length:
  - The real length of the curve (L_c)
- Measurement:
  - The tortuosity is the ratio of the 2 lengths
- Problems:
  - A curved, but not tortuous vessel can have a high value too
  - Highly depends on amplitude, and low dependency on frequency

Measures Involving Curvature (implemented)

- Definitions:
  - s(l) is the curved line
  - tc is absolute curvature
  - tsc is squared curvature
- Measurements:
  - The ratio of tc/tsc and the arc/chord length

$$s(l) = [x(l), y(l)] : D \subset \mathbb{R} \to \mathbb{R}^2$$

$$\kappa(l) = \frac{\frac{dx}{dl}\frac{d^2y}{dl^2} - \frac{d^2x}{dl^2}\frac{dy}{dl}}{\left(\left(\frac{dx}{dl}\right)^2 + \left(\frac{dy}{dl}\right)^2\right)^{3/2}}.$$

$$tc = \int_{\min(D)}^{\max(D)} |\kappa(l)|\, dl$$

$$tsc = \int_{\min(D)}^{\max(D)} |\kappa(l)|^2\, dl.$$

Derivative of Curvature (implemented)

- Measurement:
  - Integral of the squared derivative of curvature

$$\mathrm{DCI} = \frac{1}{L_c} \int_{\min(D)}^{\max(D)} \left| \frac{d\kappa(l)}{dl} \right|^2 dl.$$

Mean direction Angle Change (not implemented yet)

- Definitions:
  - d_i: coordinates of the ith samplepoint
  - v_{i+n} (V_{i-n}): A vector from the ith sample point to the „i +n"th („i-n"th) sample point

$$\boldsymbol{v}_{i+n} = \boldsymbol{d}_{\boldsymbol{i+n}} - \boldsymbol{d}_{\boldsymbol{i}}$$
$$\boldsymbol{v}_{i-n} = \boldsymbol{d}_{\boldsymbol{i-n}} - \boldsymbol{d}_{\boldsymbol{i}}$$

$$\theta(i) = \arccos\left(\boldsymbol{v}_{\boldsymbol{i+n}} \cdot \boldsymbol{v}_{\boldsymbol{i-n}}\right)$$

$$MAC = \frac{1}{N - 2 \cdot n} \sum_{i=1}^{n} \theta(i)$$

Inflection Count Metric (implemented)

- Measurement:
  - Arc length over chord length value multiplied by the „number of inflection points +1"

Proposed method of the article (not implemented yet)

- Measurement:
  - They segment the curved line into n segments with constant curveture (using an unknown algorithm) and summarize them using the equationto the right.

$$\tau(s) = \frac{n-1}{n}\frac{1}{L_c}\sum_{i=1}^{n}\left[\frac{L_{c_{s_i}}}{L_{\chi_{s_i}}} - 1\right].$$

Summary

- No ground-truth for tests
- Raster images generate not existing curvature changes
- No solution for bifurcations and vessel crossings