

Embedded trees for 1D-DWT

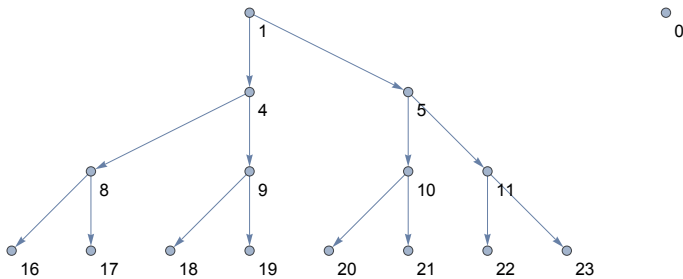
Constructing and displaying embedded trees

```
In[10]:= extracttrees[length_, levels_] := Module[{short, vertices, edges, a},  
  If[! Divisible[length, 2^levels],  
    Throw["length not feasible"]];  
  a = length/2^levels;  
  vertices = Range[0, length - 1];  
  edges =  
    Join[Table[2 j + 1 → a + 2 j, {j, 0, Floor[a/2] - 1}],  
        Table[2 j + 1 → a + 2 j + 1, {j, 0, Floor[a/2] - 1}]];  
  For[j = a, j < length/2, j++,  
    edges = Join[edges, {j → 2 j, j → 2 j + 1}];  
  {vertices, edges}  
]
```

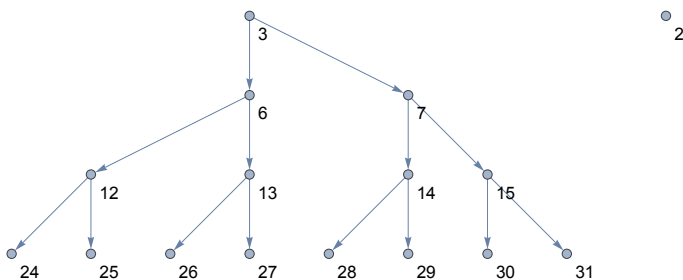
```
In[11]:= extracttrees[32, 3]
```

```
Out[11]= {{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
  16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31},  
  {1 → 4, 3 → 6, 1 → 5, 3 → 7, 4 → 8, 4 → 9, 5 → 10, 5 → 11, 6 → 12, 6 → 13,  
  7 → 14, 7 → 15, 8 → 16, 8 → 17, 9 → 18, 9 → 19, 10 → 20, 10 → 21, 11 → 22,  
  11 → 23, 12 → 24, 12 → 25, 13 → 26, 13 → 27, 14 → 28, 14 → 29, 15 → 30, 15 → 31}}
```

```
In[12]:= Graph[%[[1]], %[[2]],  
  VertexLabels → Table[x → Placed[x, {After, Below}], {x, 0, 31}]]
```



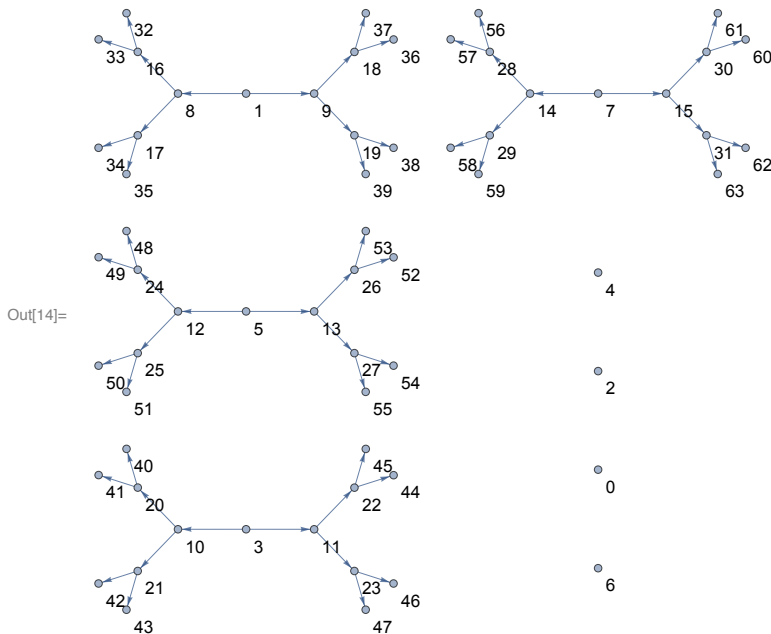
```
Out[12]=
```



```
In[13]:= extracttrees[64, 3]
```

```
Out[13]= {{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23,
24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43,
44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63},
{1 ↔ 8, 3 ↔ 10, 5 ↔ 12, 7 ↔ 14, 1 ↔ 9, 3 ↔ 11, 5 ↔ 13, 7 ↔ 15, 8 ↔ 16,
8 ↔ 17, 9 ↔ 18, 9 ↔ 19, 10 ↔ 20, 10 ↔ 21, 11 ↔ 22, 11 ↔ 23, 12 ↔ 24,
12 ↔ 25, 13 ↔ 26, 13 ↔ 27, 14 ↔ 28, 14 ↔ 29, 15 ↔ 30, 15 ↔ 31, 16 ↔ 32,
16 ↔ 33, 17 ↔ 34, 17 ↔ 35, 18 ↔ 36, 18 ↔ 37, 19 ↔ 38, 19 ↔ 39, 20 ↔ 40,
20 ↔ 41, 21 ↔ 42, 21 ↔ 43, 22 ↔ 44, 22 ↔ 45, 23 ↔ 46, 23 ↔ 47, 24 ↔ 48,
24 ↔ 49, 25 ↔ 50, 25 ↔ 51, 26 ↔ 52, 26 ↔ 53, 27 ↔ 54, 27 ↔ 55, 28 ↔ 56,
28 ↔ 57, 29 ↔ 58, 29 ↔ 59, 30 ↔ 60, 30 ↔ 61, 31 ↔ 62, 31 ↔ 63}}
```

```
In[14]:= Graph[%[[1]], %[[2]],
VertexLabels → Table[x → Placed[x, {After, Below}], {x, 0, 63}]]
```



```
In[15]:= showtrees[dwddata_, level_, root_] :=
Module[{length, vertices, edges, dwdlist, wcc, comp},
length = Length[dwddata];
{vertices, edges} = extracttrees[length, level];
G = Graph[vertices, edges];
wcc = WeaklyConnectedComponents[G];
comp = Select[wcc, MemberQ[#, root] &];
(*Print[Subgraph[G, comp,
VertexLabels → Table[x → Placed[x, {After, Below}], {x, 0, length - 1}]]];*)
Subgraph[G, comp, VertexLabels → Table[
x → Placed[x → dwddata[[x + 1]], {After, Below}], {x, 0, length - 1}]]
]
```

The data

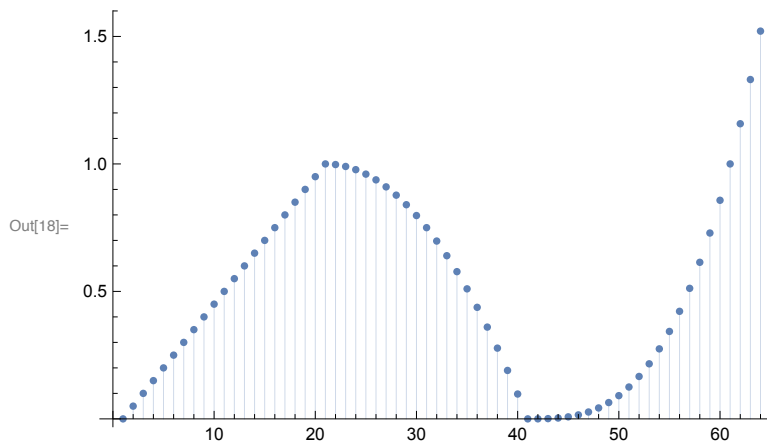
```
In[16]:= data =
  Join[
    Table[x/20, {x, 0, 19}],
    Table[2 (x/20) - (x/20)^2, {x, 20, 39}],
    Table[(x/20 - 2)^3, {x, 40, 63}]
  ]
```

```
Out[16]= {0,  $\frac{1}{20}$ ,  $\frac{1}{10}$ ,  $\frac{3}{20}$ ,  $\frac{1}{5}$ ,  $\frac{1}{4}$ ,  $\frac{3}{10}$ ,  $\frac{7}{20}$ ,  $\frac{2}{5}$ ,  $\frac{9}{20}$ ,  $\frac{1}{2}$ ,  $\frac{11}{20}$ ,  $\frac{3}{5}$ ,  $\frac{13}{20}$ ,  $\frac{7}{10}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{17}{20}$ ,  $\frac{9}{10}$ ,  $\frac{19}{20}$ , 1,
 $\frac{399}{400}$ ,  $\frac{99}{100}$ ,  $\frac{391}{400}$ ,  $\frac{24}{25}$ ,  $\frac{15}{16}$ ,  $\frac{91}{100}$ ,  $\frac{351}{400}$ ,  $\frac{21}{25}$ ,  $\frac{319}{400}$ ,  $\frac{3}{4}$ ,  $\frac{279}{400}$ ,  $\frac{16}{25}$ ,  $\frac{231}{400}$ ,  $\frac{51}{100}$ ,  $\frac{7}{16}$ ,  $\frac{9}{25}$ ,
 $\frac{111}{400}$ ,  $\frac{19}{100}$ ,  $\frac{39}{400}$ , 0,  $\frac{1}{8000}$ ,  $\frac{1}{1000}$ ,  $\frac{27}{8000}$ ,  $\frac{1}{125}$ ,  $\frac{1}{64}$ ,  $\frac{1}{1000}$ ,  $\frac{343}{8000}$ ,  $\frac{8}{125}$ ,  $\frac{729}{8000}$ ,  $\frac{1}{8}$ ,
 $\frac{1331}{8000}$ ,  $\frac{27}{125}$ ,  $\frac{2197}{8000}$ ,  $\frac{343}{1000}$ ,  $\frac{27}{64}$ ,  $\frac{4913}{8000}$ ,  $\frac{729}{1000}$ ,  $\frac{6859}{8000}$ , 1,  $\frac{9261}{8000}$ ,  $\frac{1331}{1000}$ ,  $\frac{12167}{8000}$ }
```

```
In[17]:= Length[data]
```

```
Out[17]= 64
```

```
In[18]:= ListPlot[data, Filling -> Axis]
```



A slightly modified DWT scheme

```
In[19]:= myDWT[data_, wv_, lev_] :=  
  Module[{dwd, low, high, flen},  
    dwd = DiscreteWaveletTransform[data, wv, 1];  
    low = dwd[[1, 1]];  
    high = dwd[[1, 2]];  
    flen = Length[dwd[[7]]];  
    If[flen > 2,  
      low = Drop[low, (flen - 2) / 2];  
      high = Drop[high, (flen - 2) / 2];  
    If[lev == 1,  
      Return[Join[low, high]],  
      Return[Join[myDWT[low, wv, lev - 1], high]]]  
  ]
```

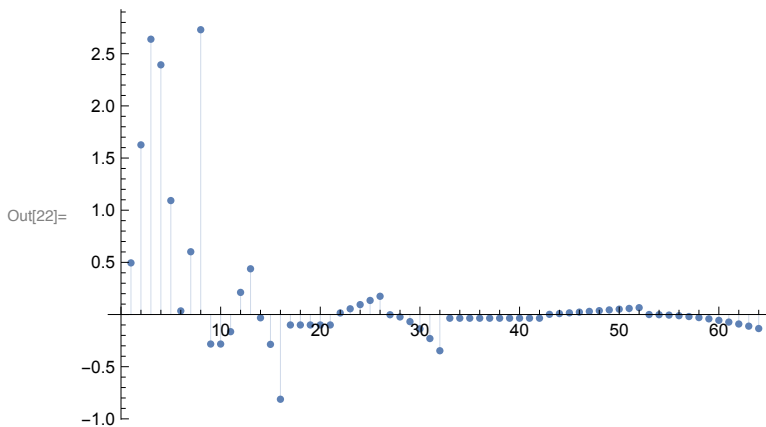
Using the Haar wavelet

```
In[20]:= dwdhaar = myDWT[data, HaarWavelet[], 3];
```

```
In[21]:= dwdhaar
```

```
Out[21]= {0.494975, 1.62635, 2.63928, 2.39356, 1.09248, 0.0346482, 0.601748, 2.73014,
  -0.282843, -0.282843, -0.164402, 0.212132, 0.438406, -0.0314663, -0.286025,
  -0.812112, -0.1, -0.1, -0.1, -0.1, -0.1, 0.015, 0.055, 0.095, 0.135, 0.175,
  -0.002125, -0.023125, -0.068125, -0.137125, -0.230125, -0.347125, -0.0353553,
  -0.0353553, -0.0353553, -0.0353553, -0.0353553, -0.0353553, -0.0353553,
  -0.0353553, -0.0353553, -0.0353553, 0.00176777, 0.00883883, 0.0159099,
  0.022981, 0.030052, 0.0371231, 0.0441942, 0.0512652, 0.0583363, 0.0654074,
  -0.0000883883, -0.00167938, -0.00539169, -0.0112253, -0.0191803, -0.0292565,
  -0.0414541, -0.055773, -0.0722133, -0.0907748, -0.111458, -0.134262}
```

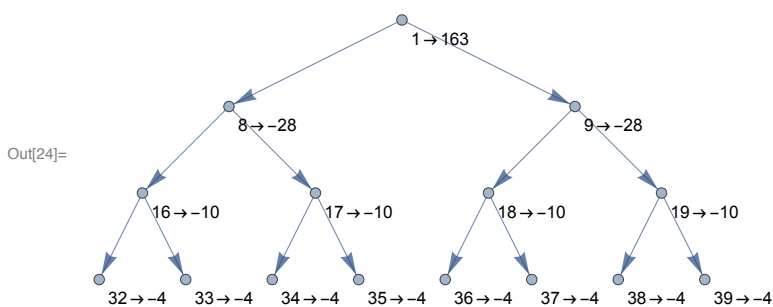
```
In[22]:= ListPlot[dwdhaar, PlotRange -> All, Filling -> Axis]
```



```
In[23]:= dwdhaarmodif = Map[Round[# * 10^2] &, dwdhaar]
```

```
Out[23]= {49, 163, 264, 239, 109, 3, 60, 273, -28, -28, -16, 21, 44,
  -3, -29, -81, -10, -10, -10, -10, -10, 2, 5, 9, 14, 18, 0, -2, -7,
  -14, -23, -35, -4, -4, -4, -4, -4, -4, -4, -4, -4, 0, 1, 2, 2,
  3, 4, 4, 5, 6, 7, 0, 0, -1, -1, -2, -3, -4, -6, -7, -9, -11, -13}
```

```
In[24]:= showtrees[dwdhaarmodif, 3, 1]
```

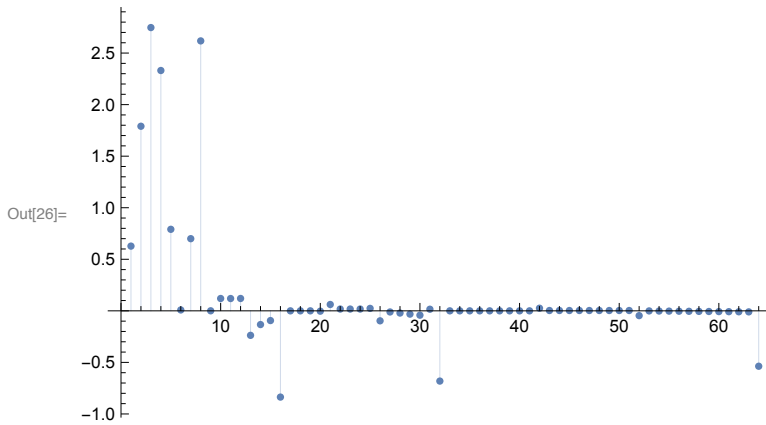


Using the D4 wavelet

In[25]:= `dwdd4 = myDWT[data, DaubechiesWavelet[2], 3]`

Out[25]= {0.627717, 1.79026, 2.74728, 2.33087, 0.790665, 0.00854885, 0.699824, 2.618, 0.000424625, 0.119688, 0.119359, 0.119942, -0.237658, -0.133105, -0.0943269, -0.836475, 5.55112×10^{-17} , 0., 1.11022×10^{-16} , -0.00328125, 0.0615869, 0.0173205, 0.0173205, 0.0173205, 0.0237346, -0.096841, -0.0109413, -0.0213336, -0.031726, -0.0421183, 0.0156056, -0.681205, 1.38778×10^{-17} , 2.77556×10^{-17} , 0., 5.55112×10^{-17} , 0., 0., -1.11022×10^{-16} , 1.11022×10^{-16} , 1.11022×10^{-16} , 0.0253556, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, 0.00306186, -0.0465022, -0.000821502, -0.00174006, -0.00265862, -0.00357718, -0.00449574, -0.00541429, -0.00633285, -0.00725141, -0.00816997, -0.00908853, -0.0100071, -0.537287}

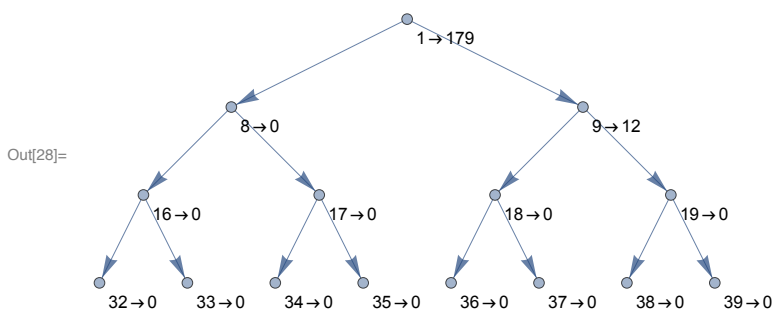
In[26]:= `ListPlot[dwdd4, PlotRange -> All, Filling -> Axis]`



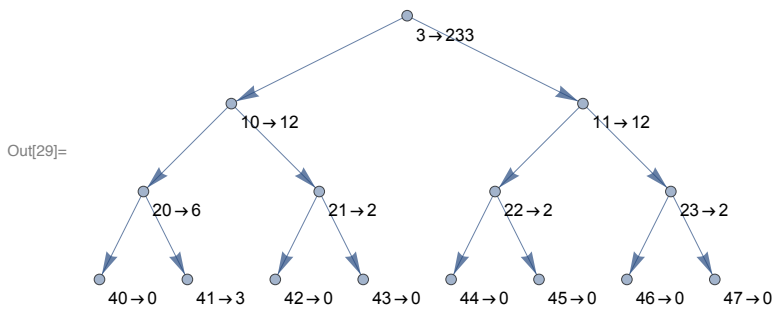
In[27]:= `dwdd4modif = Map[Round[# * 10^2] &, dwdd4]`

Out[27]= {63, 179, 275, 233, 79, 1, 70, 262, 0, 12, 12, 12, -24, -13, -9, -84, 0, 0, 0, 0, 6, 2, 2, 2, 2, -10, -1, -2, -3, -4, 2, -68, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, -5, 0, 0, 0, 0, 0, -1, -1, -1, -1, -1, -1, -54}

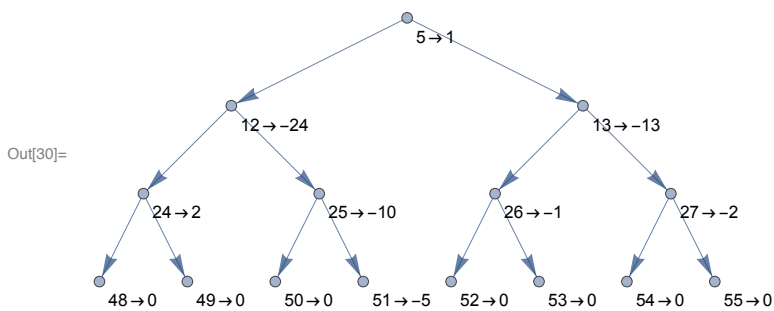
In[28]:= `showtrees[dwdd4modif, 3, 1]`



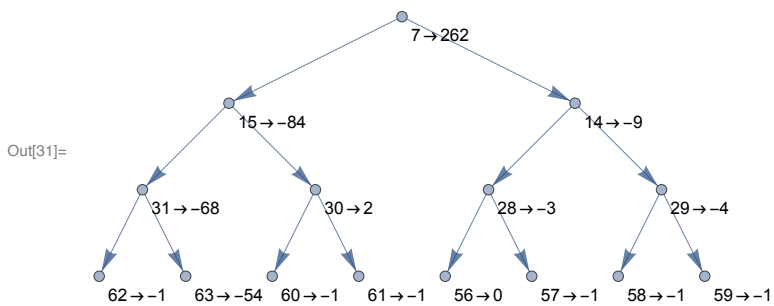
In[29]:= **showtrees[dwdd4modif, 3, 3]**



In[30]:= **showtrees[dwdd4modif, 3, 5]**



In[31]:= **showtrees[dwdd4modif, 3, 7]**

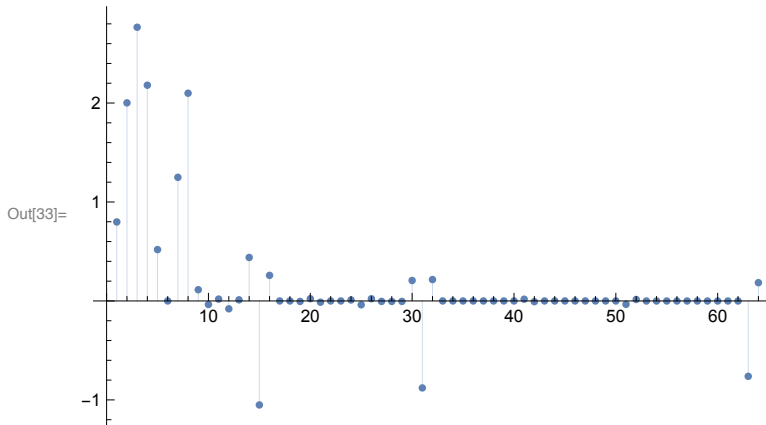


Using the D6 wavelet

```
In[32]:= dwdd6 = myDWT[data, DaubechiesWavelet[3], 3]
```

```
Out[32]:= {0.798358, 2.00169, 2.76576, 2.18008, 0.519046, -0.0000634177, 1.24923,
2.09906, 0.11335, -0.0358876, 0.0186975, -0.0796583, 0.0111948, 0.43945,
-1.05099, 0.258536, 5.55112 × 10-17, 2.22045 × 10-16, -0.00495538, 0.0225296,
-0.012564, 0., 1.11022 × 10-16, 0.00970488, -0.0391055, 0.0223419,
-0.00474342, -0.00474342, -0.00474342, 0.206649, -0.878949, 0.215915,
5.55112 × 10-17, 8.32667 × 10-17, 1.11022 × 10-16, 1.11022 × 10-16, 5.55112 × 10-17,
1.11022 × 10-16, 2.22045 × 10-16, 2.22045 × 10-16, 0.0174652, -0.00722883, 0.,
3.33067 × 10-16, 1.11022 × 10-16, 1.11022 × 10-16, 5.55112 × 10-17, 5.55112 × 10-17,
5.55112 × 10-17, 1.38778 × 10-17, -0.0341403, 0.0146502, -0.000419263,
-0.000419263, -0.000419263, -0.000419263, -0.000419263, -0.000419263,
-0.000419263, -0.000419263, -0.000419263, -0.000419263, -0.761614, 0.184626}
```

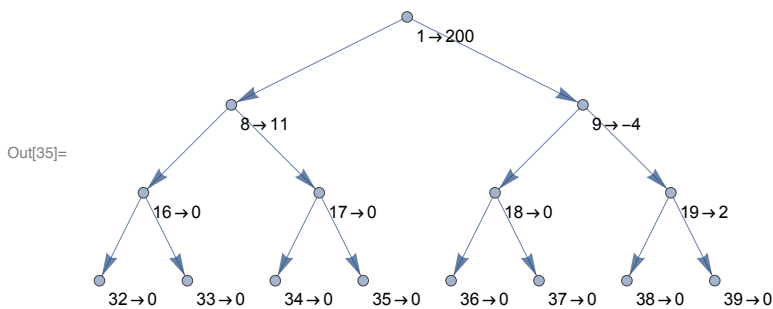
```
In[33]:= ListPlot[dwdd6, PlotRange → All, Filling → Axis]
```



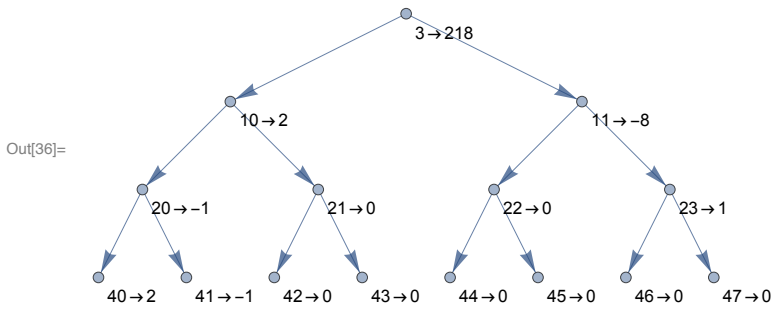
```
In[34]:= dwdd6modif = Map[Round[# * 10^2] &, dwdd6]
```

```
Out[34]:= {80, 200, 277, 218, 52, 0, 125, 210, 11, -4, 2, -8, 1, 44, -105, 26, 0,
0, 0, 2, -1, 0, 0, 1, -4, 2, 0, 0, 0, 21, -88, 22, 0, 0, 0, 0, 0, 0, 0, 0,
2, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -76, 18}
```

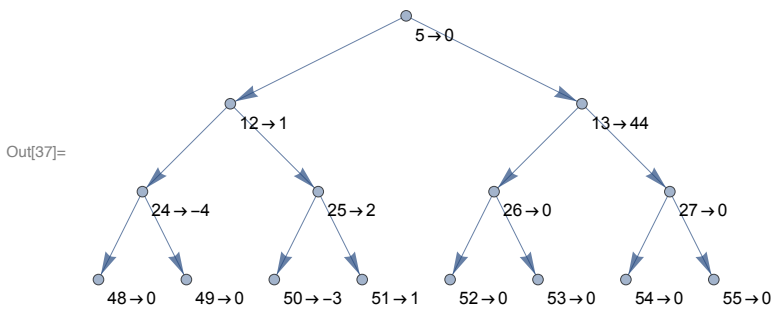
```
In[35]:= showtrees[dwdd6modif, 3, 1]
```



In[36]:= **showtrees[dwdd6modif, 3, 3]**



In[37]:= **showtrees[dwdd6modif, 3, 5]**



In[38]:= **showtrees[dwdd6modif, 3, 7]**

