

Dyadic approximation of the scaling and wavelet functions

```
In[695]= fa = Filling → Axis; pa = PlotRange → All; ta → TableAlignments → ".";
```

Solving the eigenvalue problem

```
In[786]= eigen[h_] := Module[{A, vec, sum, hlen, len},
  hlen = Length[h];
  len = hlen - 2;
  A = Table[
    If[1 ≤ 2 i + 1 - j ≤ hlen, h[[2 i + 1 - j]], 0], {i, 1, len},
    {j, 1, len}];
  Print["The matrix for the eigenvalue problem"];
  Print[MatrixForm[A]];
  vec = Eigenvectors[A, 1][[1]];
  sum = Total[vec];
  Print["The eigenvector for the eigenvalue 1"];
  Print[MatrixForm[vec / sum, ta]];
  vec / sum
]

In[697]= eigen::usage = "Solves the eigenvalue problem belonging to the
  scaling equation given by the filter h.
  Needed for starting the dyadic interpolation
  for the true values of the scaling function
  belonging to h";
```

Interpolation by iterating the scaling equation

```

In[789]= dphi[h_, j_] := Module[{hlen, hvec, vec, tmp, vlen},
  hlen = Length[h];
  hvec = eigen[h];
  vec[0] = Join[{0}, hvec, {0}];
  Do[
    vlen = Length[vec[s - 1]];
    tmp = Table[
      Sum[h[[n]] (vec[s - 1]) [[
        Min[
          Max[2 k - 2^(s - 1) (n - 1), 1], vlen]]],
      {n, 1, hlen}],
    {k, 1, vlen - 1}];
    vec[s] = Riffle[vec[s - 1], tmp], {s, 1, j}];
  Table[vec[i], {i, 1, j}]
]

In[793]= dyadicphi[h_, j_] := Module[{hlen, data},
  hlen = Length[h];
  data = Last[dphi[h, j]];
  Table[{k/2^j, data[[k + 1]]}, {k, 0, 2^(j) (hlen - 1)}]
]

dyadicphi::usage =
  "level-j dyadic approximation of the scaling function belonging to the filter h";

dyadicphiall[h_, n_] := Module[{hlen, data},
  hlen = Length[h];
  data = dphi[h, n];
  Table[Table[{k/2^j, data[[j, k + 1]]}, {k, 0, 2^(j) (hlen - 1)}], {j, 1, n}]
]

In[762]= phianimate[h_, n_] := Module[{data},
  data = dyadicphiall[h, n];
  ListAnimate[Table[ListPlot[
    data[[t]],
    Filling -> Axis,
    PlotRange -> All], {t, 1, n}],
  AnimationRunning -> False]
]

```

```

In[769]= dyadicpsi[h_, j_] := Module[{hlen, vec, vlen},
  If[j == 0, Return[{0}]];
  hlen = Length[h];
  vec = Last[dphi[h, j - 1]];
  vlen = Length[vec];
  Table[{k/2^j,
    Sum[(-1)^n h[[hlen - n]] vec[[
      Min[
        Max[k + 1 - 2^(j - 1) n, 1], vlen]]],
      {n, 0, hlen - 1}]],
    {k, 0, 2^(j) (hlen - 1)}]
]

In[770]= dyadicpsiall[h_, n_] := Module[{hlen, vec, vlen},
  If[n == 0, Return[{0}]];
  hlen = Length[h];
  vec = dphi[h, n - 1];
  Table[
  vlen = Length[vec];
  Table[{k/2^j,
    Sum[(-1)^m h[[hlen - m]] ×
      vec[[j,
        Min[
          Max[k + 1 - 2^(j - 1) m, 1], Length[vec[[j]]]]]],
      {m, 0, hlen - 1}]], {k, 0, 2^(j) (hlen - 1)}], {j, 1, n - 1}
  ]

In[771]= psianimate[h_, n_] := Module[{data},
  data = dyadicpsiall[h, n];
  ListAnimate[Table[ListPlot[
    data[[t]],
    Filling -> Axis,
    PlotRange -> All], {t, 1, n - 1}],
  AnimationRunning -> False]
]

In[772]= dyadicpsiall[db4, 2]
The matrix for the eigenvalue problem
( 1.18301  0.683013
 -0.183013 0.316987 )
The eigenvector for the eigenvalue 1
( 1.36603
 -0.366025 )

Out[772]= {{0, 0}, {1/2, -0.170753}, {1, -0.545753}, {3/2, 0.670753}, {2, 1.04575},
  {5/2, -0.829247}, {3, -0.454247}}

In[702]= dyadicpsi::usage =
  "level-j dyadic approximation of the wavelet function belonging to the filter h";

```

Example Daubechies-4

```

In[703]= db4 = 2 Map[#[[2]] &, WaveletFilterCoefficients[DaubechiesWavelet[2]]];
TableForm[db4, ta]
      0.683013
      1.18301
Out[704]/TableForm= 0.316987
                    -0.183013

In[705]= eigen[db4]
The matrix for the eigenvalue problem
( 1.18301  0.683013 )
(-0.183013  0.316987)
The eigenvector for the eigenvalue 1
( 1.36603 )
(-0.366025)

In[749]= dyadicphi[db4, 2]

The matrix for the eigenvalue problem
( 1.18301  0.683013 )
(-0.183013  0.316987)
The eigenvector for the eigenvalue 1
( 1.36603 )
(-0.366025)

Out[749]= { {0, 0}, {1/4, 0.63726}, {1/2, 0.933013}, {3/4, 1.10377}, {1, 1.36603}, {5/4, 0.341506},
            {3/2, 1.66533 × 10-16}, {7/4, -0.0915064}, {2, -0.366025}, {9/4, 0.0212341}, {5/2, 0.0669873},
            {11/4, -0.0122595}, {3, 0} }

```

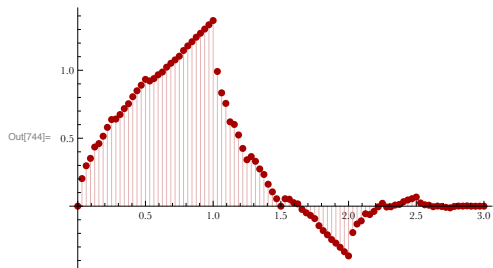
```
In[744]= ListPlot[dyadicphi[db4, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.18301 & 0.683013 \\ -0.183013 & 0.316987 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.36603 \\ -0.366025 \end{pmatrix}$$



```
In[763]= phianimate[db4, 5]
```

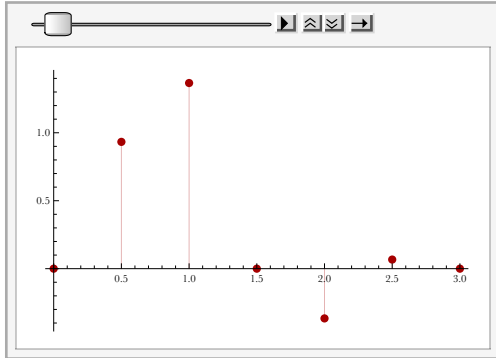
The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.18301 & 0.683013 \\ -0.183013 & 0.316987 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.36603 \\ -0.366025 \end{pmatrix}$$

Out[763]=



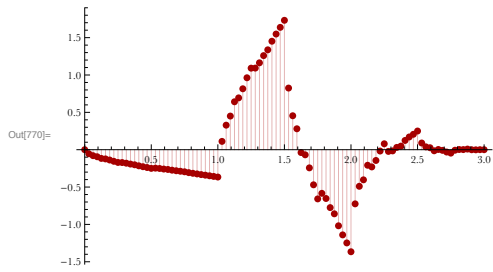
```
In[770]:= ListPlot[dyadicpsi[db4, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.18301 & 0.683013 \\ -0.183013 & 0.316987 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.36603 \\ -0.366025 \end{pmatrix}$$



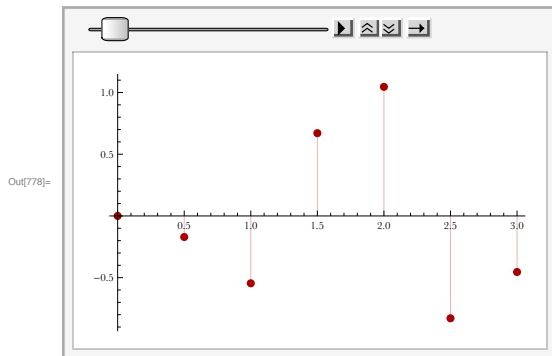
```
In[778]= psianimate[db4, 5]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.18301 & 0.683013 \\ -0.183013 & 0.316987 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.36603 \\ -0.366025 \end{pmatrix}$$



Example Daubechies-6

```
In[710]= db6 = 2 Map[#[[2]] &, WaveletFilterCoefficients[DaubechiesWavelet[3]]];
```

```
TableForm[db6, ta]
```

```
0.470467
1.14112
0.650365
-0.190934
-0.120832
0.0498175
```

```
Out[711]/TableForm=
```

```
In[712]= eigen[db6]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$

```
Out[712]= {1.28634, -0.385837, 0.0952675, 0.00423435}
```

```
In[723]= ListPlot[dyadicphi[db6, 5], Filling -> Axis, PlotRange -> All]
```

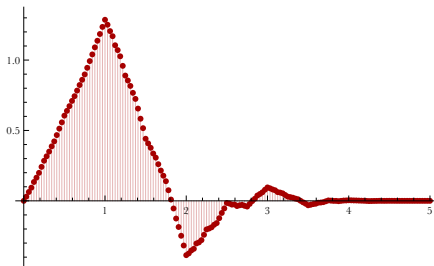
The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$

```
Out[723]=
```



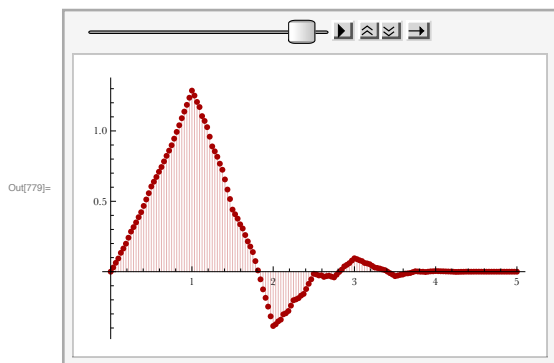
```
In[779]= phianimate[db6, 5]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$



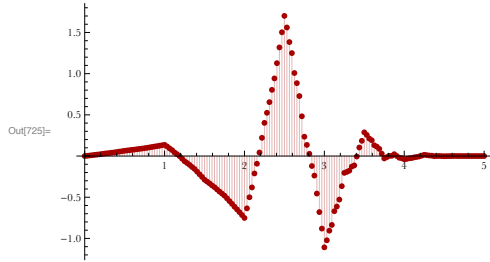
```
In[725]= ListPlot[dyadicpsi[db6, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$



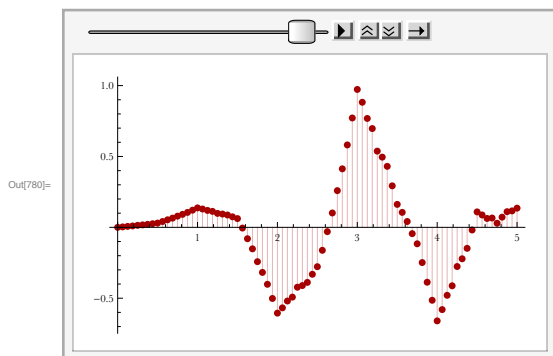
```
In[780]= psianimate[db6, 5]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$



Example Daubechies-8

```

In[791]= db8 = 2 Map[#[[2]] &, WaveletFilterCoefficients[DaubechiesWavelet[4]]];
TableForm[db8, ta]
      0.325803
      1.01095
      0.8922
      -0.039575
Out[792]/TableForm=
      -0.264507
      0.0436163
      0.0465036
      -0.014987

In[793]= eigen[db8]
The matrix for the eigenvalue problem
      (
      1.01095  0.325803  0  0  0  0
      -0.039575  0.8922  1.01095  0.325803  0  0
      0.0436163 -0.264507 -0.039575  0.8922  1.01095  0.325803
      -0.014987  0.0465036  0.0436163 -0.264507 -0.039575  0.8922
      0  0 -0.014987  0.0465036  0.0436163 -0.264507
      0  0  0  0 -0.014987  0.0465036
      )
The eigenvector for the eigenvalue 1
      (
      1.00717
      -0.033837
      0.0396105
      -0.0117644
      -0.00119796
      0.0000188294
      )
Out[793]= {1.00717, -0.033837, 0.0396105, -0.0117644, -0.00119796, 0.0000188294}

```

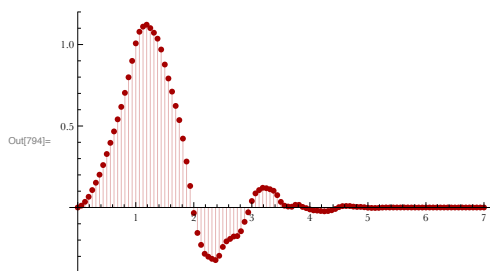
```
In[784]= ListPlot[dyadicphi[db8, 4], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.01095 & 0.325803 & 0 & 0 & 0 & 0 \\ -0.039575 & 0.8922 & 1.01095 & 0.325803 & 0 & 0 \\ 0.0436163 & -0.264507 & -0.039575 & 0.8922 & 1.01095 & 0.325803 \\ -0.014987 & 0.0465036 & 0.0436163 & -0.264507 & -0.039575 & 0.8922 \\ 0 & 0 & -0.014987 & 0.0465036 & 0.0436163 & -0.264507 \\ 0 & 0 & 0 & 0 & -0.014987 & 0.0465036 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.00717 \\ -0.033837 \\ 0.0396105 \\ -0.0117644 \\ -0.00119796 \\ 0.0000188294 \end{pmatrix}$$



```
In[795]= phianimate[db6, 5]
```

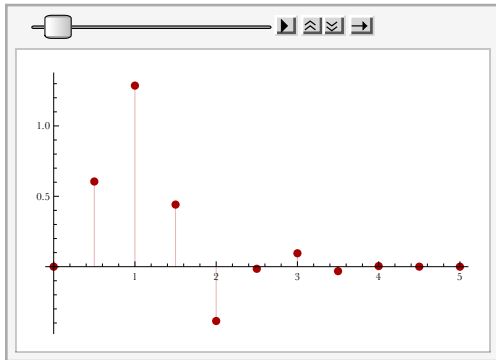
The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.14112 & 0.470467 & 0 & 0 \\ -0.190934 & 0.650365 & 1.14112 & 0.470467 \\ 0.0498175 & -0.120832 & -0.190934 & 0.650365 \\ 0 & 0 & 0.0498175 & -0.120832 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.28634 \\ -0.385837 \\ 0.0952675 \\ 0.00423435 \end{pmatrix}$$

Out[795]=



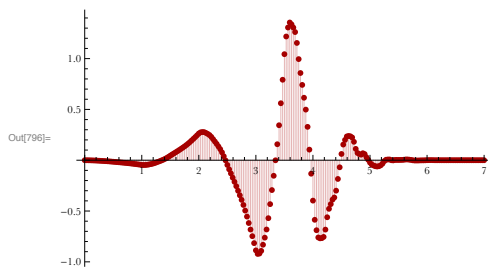
```
In[796]= ListPlot[dyadicpsi[db8, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.01095 & 0.325803 & 0 & 0 & 0 & 0 \\ -0.039575 & 0.8922 & 1.01095 & 0.325803 & 0 & 0 \\ 0.0436163 & -0.264507 & -0.039575 & 0.8922 & 1.01095 & 0.325803 \\ -0.014987 & 0.0465036 & 0.0436163 & -0.264507 & -0.039575 & 0.8922 \\ 0 & 0 & -0.014987 & 0.0465036 & 0.0436163 & -0.264507 \\ 0 & 0 & 0 & 0 & -0.014987 & 0.0465036 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.00717 \\ -0.033837 \\ 0.0396105 \\ -0.0117644 \\ -0.00119796 \\ 0.0000188294 \end{pmatrix}$$



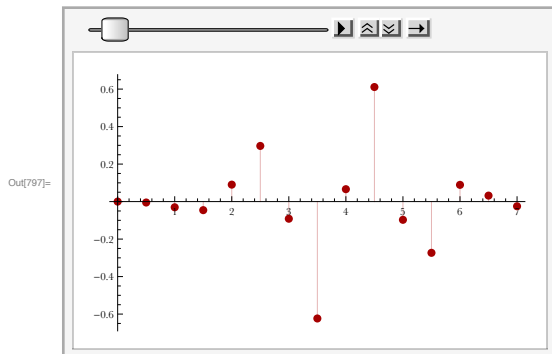

```
In[797]:= psianimate[db8, 5]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 1.01095 & 0.325803 & 0 & 0 & 0 & 0 \\ -0.039575 & 0.8922 & 1.01095 & 0.325803 & 0 & 0 \\ 0.0436163 & -0.264507 & -0.039575 & 0.8922 & 1.01095 & 0.325803 \\ -0.014987 & 0.0465036 & 0.0436163 & -0.264507 & -0.039575 & 0.8922 \\ 0 & 0 & -0.014987 & 0.0465036 & 0.0436163 & -0.264507 \\ 0 & 0 & 0 & 0 & -0.014987 & 0.0465036 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 1.00717 \\ -0.033837 \\ 0.0396105 \\ -0.0117644 \\ -0.00119796 \\ 0.0000188294 \end{pmatrix}$$



Example Daubechies-12

```
db12 = 2 Map[#[[2]] &, WaveletFilterCoefficients[ DaubechiesWavelet[6]]];
```

```
TableForm[db12, ta]
```

```
0.157742
0.699504
1.06226
0.445831
-0.319987
-0.183518
0.137888
0.0389232
-0.0446637
0.000783251
0.00675606
-0.00152353
```

Out[799]/TableForm=

```
In[800]= eigen[db12]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 0.699504 & 0.157742 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 & 0 & 0 \\ -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 \\ 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 \\ 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 \\ -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 \\ 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 \\ 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.436912 \\ 0.832309 \\ -0.384754 \\ 0.142798 \\ -0.0255074 \\ -0.00353052 \\ 0.00175977 \\ 0.0000155912 \\ -2.57792 \times 10^{-6} \\ 3.95427 \times 10^{-9} \end{pmatrix}$$

```
Out[800]= {0.436912, 0.832309, -0.384754, 0.142798, -0.0255074, -0.00353052, 0.00175977,
0.0000155912, -2.57792 × 10-6, 3.95427 × 10-9}
```

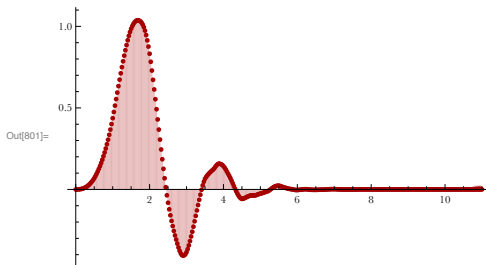
```
In[801]:= ListPlot[dyadicphi[db12, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 0.699504 & 0.157742 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 & 0 & 0 \\ -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 \\ 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 \\ 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 \\ -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 \\ 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 \\ 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.436912 \\ 0.832309 \\ -0.384754 \\ 0.142798 \\ -0.0255074 \\ -0.00353052 \\ 0.00175977 \\ 0.0000155912 \\ -2.57792 \times 10^{-6} \\ 3.95427 \times 10^{-9} \end{pmatrix}$$



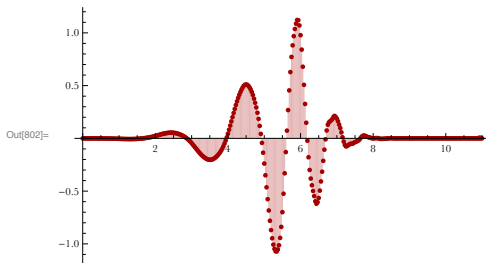
```
In[802]:= ListPlot[dyadicpsi[db12, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} 0.699504 & 0.157742 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 & 0 & 0 \\ -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 & 0 & 0 \\ 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 & 0.699504 & 0.157742 \\ 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 & 0.445831 & 1.06226 \\ -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 & -0.183518 & -0.319987 \\ 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 & 0.0389232 & 0.137888 \\ 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 & 0.000783251 & -0.0446637 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00152353 & 0.00675606 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.436912 \\ 0.832309 \\ -0.384754 \\ 0.142798 \\ -0.0255074 \\ -0.00353052 \\ 0.00175977 \\ 0.0000155912 \\ -2.57792 \times 10^{-6} \\ 3.95427 \times 10^{-9} \end{pmatrix}$$



Example Coiflet-12

```
coif12 = 2 Map[#[[2]] &,
  WaveletFilterCoefficients[CoifletWavelet[2]]];
TableForm[coif12, ta]
```

```
0.0231752
-0.0586403
-0.0952792
0.546042
1.14936
0.589734
-0.108171
-0.084053
0.0334888
0.00793577
-0.00257841
-0.00101901
```

Out[804]/TableForm=

```
In[805]= eigen[coif12]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 \\ 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 \\ -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 \\ 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 \\ -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 \\ 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 \\ 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.000750387 + 0. i \\ 0.0342776 + 0. i \\ -0.147015 + 0. i \\ 1.23031 + 0. i \\ -0.162912 + 0. i \\ 0.0470858 + 0. i \\ -0.00252724 + 0. i \\ 0.0000251784 + 0. i \\ 2.53036 \times 10^{-6} + 0. i \\ -2.57183 \times 10^{-9} + 0. i \end{pmatrix}$$

```
Out[805]= {0.000750387 + 0. i, 0.0342776 + 0. i, -0.147015 + 0. i, 1.23031 + 0. i, -0.162912 + 0. i,
0.0470858 + 0. i, -0.00252724 + 0. i, 0.0000251784 + 0. i, 2.53036 × 10-6 + 0. i,
-2.57183 × 10-9 + 0. i}
```

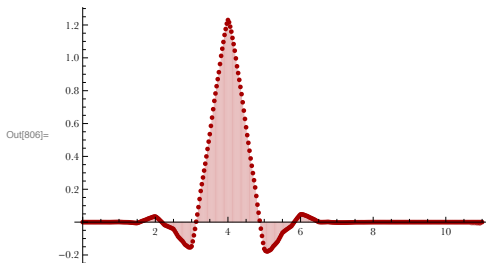
```
In[806]= ListPlot[dyadicphi[coif12, 5], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 \\ 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 \\ -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 \\ 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 \\ -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 \\ 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 \\ 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.000750387 + 0. i \\ 0.0342776 + 0. i \\ -0.147015 + 0. i \\ 1.23031 + 0. i \\ -0.162912 + 0. i \\ 0.0470858 + 0. i \\ -0.00252724 + 0. i \\ 0.0000251784 + 0. i \\ 2.53036 \times 10^{-6} + 0. i \\ -2.57183 \times 10^{-9} + 0. i \end{pmatrix}$$



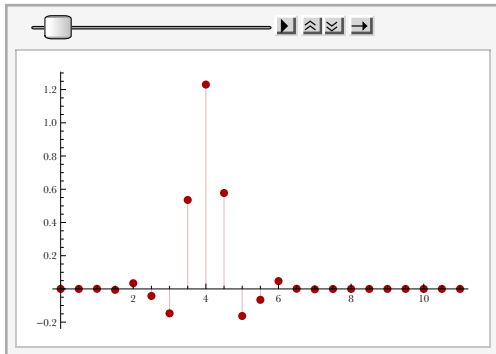
In[808]= `phianimate[coif12, 5]`

The matrix for the eigenvalue problem

$$\begin{pmatrix} -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 \\ 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 \\ -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 \\ 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 \\ -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 \\ 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 \\ 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

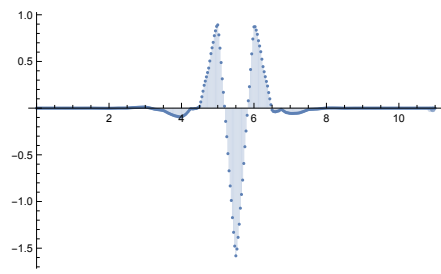
The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.000750387 + 0. i \\ 0.0342776 + 0. i \\ -0.147015 + 0. i \\ 1.23031 + 0. i \\ -0.162912 + 0. i \\ 0.0470858 + 0. i \\ -0.00252724 + 0. i \\ 0.0000251784 + 0. i \\ 2.53036 \times 10^{-6} + 0. i \\ -2.57183 \times 10^{-9} + 0. i \end{pmatrix}$$



Out[808]=

```
ListPlot[dyadicpsi[coif12, 5], Filling -> Axis, PlotRange -> All]
```



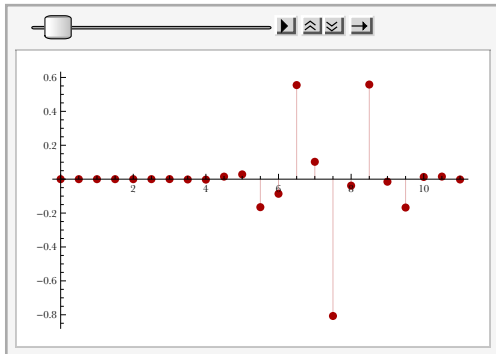

```
In[809]= psianimate[coif12, 5]
```

The matrix for the eigenvalue problem

$$\begin{pmatrix} -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 & 0 & 0 \\ 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 & 0 & 0 \\ -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 & -0.0586403 & 0.0231752 \\ 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 & 0.546042 & -0.0952792 \\ -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 & 0.589734 & 1.14936 \\ 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 & -0.084053 & -0.108171 \\ 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 & 0.00793577 & 0.0334888 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.00101901 & -0.00257841 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvector for the eigenvalue 1

$$\begin{pmatrix} 0.000750387 + 0. i \\ 0.0342776 + 0. i \\ -0.147015 + 0. i \\ 1.23031 + 0. i \\ -0.162912 + 0. i \\ 0.0470858 + 0. i \\ -0.00252724 + 0. i \\ 0.0000251784 + 0. i \\ 2.53036 \times 10^{-6} + 0. i \\ -2.57183 \times 10^{-9} + 0. i \end{pmatrix}$$



Example Coiflet-24

```
In[810]= coif24 = 2 Map[#[[2]] &, WaveletFilterCoefficients[CoifletWavelet[4]]];  
TableForm[coif24, ta]  
0.00126192  
-0.00230445  
-0.0103891  
0.0227249  
0.0377345  
-0.114928  
-0.0793053  
0.587335  
1.10625  
0.614315  
-0.0942255  
-0.136076  
Out[811]/TableForm=  
0.0556273  
0.0354717  
-0.0215126  
-0.00800202  
0.00530533  
0.00179119  
-0.000833  
-0.000367659  
0.0000881605  
0.0000441657  
-4.60984 × 10-6  
-2.52436 × 10-6
```

In[812]= eigen[coif24]

The matrix for the eigenvalue problem

-0.00230445	0.00126192	0	0	0	0	0
0.0227249	-0.0103891	-0.00230445	0.00126192	0	0	0
-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192	0
0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445
0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345	0.0227249
-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053	-0.114928
0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625	0.587335
-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255	0.614315
0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076
-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126	0.000367659
0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533	-0.0000441657
-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119
0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659
0	0	0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657
0	0	0	0	0	0	-2.52436
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

The eigenvector for the eigenvalue 1

$1.33974 \times 10^{-9} + 0. i$
$1.06411 \times 10^{-6} + 0. i$
$0.000067639 + 0. i$
$0.000975503 + 0. i$
$-0.0105753 + 0. i$
$0.0410692 + 0. i$
$-0.0880722 + 0. i$
$1.11781 + 0. i$
$-0.102635 + 0. i$
$0.0585417 + 0. i$
$-0.0214361 + 0. i$
$0.00490709 + 0. i$
$-0.000724169 + 0. i$
$0.0000752093 + 0. i$
$-1.38855 \times 10^{-6} + 0. i$
$-3.25612 \times 10^{-7} + 0. i$
$6.94007 \times 10^{-9} + 0. i$
$1.38763 \times 10^{-9} + 0. i$
$5.43309 \times 10^{-12} + 0. i$
$-2.36781 \times 10^{-14} + 0. i$
$-1.36065 \times 10^{-17} + 0. i$
$3.43476 \times 10^{-23} + 0. i$

Out[812]= { $1.33974 \times 10^{-9} + 0. i$, $1.06411 \times 10^{-6} + 0. i$, $0.000067639 + 0. i$, $0.000975503 + 0. i$,
 $-0.0105753 + 0. i$, $0.0410692 + 0. i$, $-0.0880722 + 0. i$, $1.11781 + 0. i$, $-0.102635 + 0. i$,
 $0.0585417 + 0. i$, $-0.0214361 + 0. i$, $0.00490709 + 0. i$, $-0.000724169 + 0. i$,
 $0.0000752093 + 0. i$, $-1.38855 \times 10^{-6} + 0. i$, $-3.25612 \times 10^{-7} + 0. i$, $6.94007 \times 10^{-9} + 0. i$,
 $1.38763 \times 10^{-9} + 0. i$, $5.43309 \times 10^{-12} + 0. i$, $-2.36781 \times 10^{-14} + 0. i$, $-1.36065 \times 10^{-17} + 0. i$,
 $3.43476 \times 10^{-23} + 0. i$ }

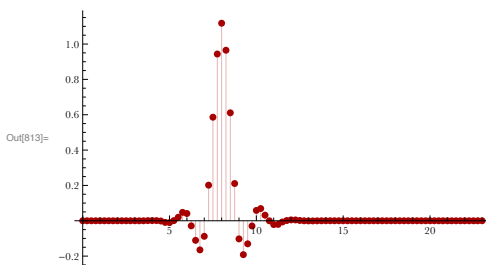
```
In[813]= ListPlot[dyadicphi[coif24, 2], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

-0.00230445	0.00126192	0	0	0	0	0
0.0227249	-0.0103891	-0.00230445	0.00126192	0	0	0
-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192	0
0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445
0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345	0.0227249
-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053	-0.114928
0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625	0.587335
-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255	0.614315
0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076
-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717
0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533	-0.000833
-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119
0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659
0	0	0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657
0	0	0	0	0	0	-2.52436
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

The eigenvector for the eigenvalue 1

$1.33974 \times 10^{-9} + 0. i$
$1.06411 \times 10^{-6} + 0. i$
$0.000067639 + 0. i$
$0.000975503 + 0. i$
$-0.0105753 + 0. i$
$0.0410692 + 0. i$
$-0.0880722 + 0. i$
$1.11781 + 0. i$
$-0.102635 + 0. i$
$0.0585417 + 0. i$
$-0.0214361 + 0. i$
$0.00490709 + 0. i$
$-0.000724169 + 0. i$
$0.0000752093 + 0. i$
$-1.38855 \times 10^{-6} + 0. i$
$-3.25612 \times 10^{-7} + 0. i$
$6.94007 \times 10^{-9} + 0. i$
$1.38763 \times 10^{-9} + 0. i$
$5.43309 \times 10^{-12} + 0. i$
$-2.36781 \times 10^{-14} + 0. i$
$-1.36065 \times 10^{-17} + 0. i$
$3.43476 \times 10^{-23} + 0. i$



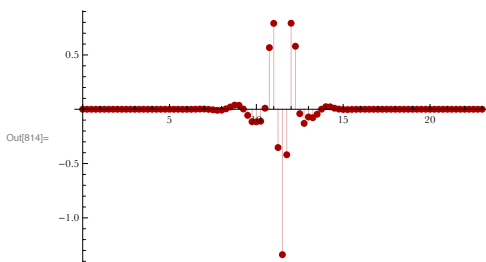
```
In[814]= ListPlot[dyadicpsi[coif24, 2], Filling -> Axis, PlotRange -> All]
```

The matrix for the eigenvalue problem

-0.00230445	0.00126192	0	0	0	0	0
0.0227249	-0.0103891	-0.00230445	0.00126192	0	0	0
-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192	0
0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445
0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345	0.0227249
-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053	-0.114928
0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625	0.587335
-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255	0.614315
0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076
-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126	0.000367659
0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533	-0.0000441657
-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119
0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659
0	0	0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657
0	0	0	0	0	0	-2.52436
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

The eigenvector for the eigenvalue 1

$1.33974 \times 10^{-9} + 0. i$
$1.06411 \times 10^{-6} + 0. i$
$0.000067639 + 0. i$
$0.000975503 + 0. i$
$-0.0105753 + 0. i$
$0.0410692 + 0. i$
$-0.0880722 + 0. i$
$1.11781 + 0. i$
$-0.102635 + 0. i$
$0.0585417 + 0. i$
$-0.0214361 + 0. i$
$0.00490709 + 0. i$
$-0.000724169 + 0. i$
$0.0000752093 + 0. i$
$-1.38855 \times 10^{-6} + 0. i$
$-3.25612 \times 10^{-7} + 0. i$
$6.94007 \times 10^{-9} + 0. i$
$1.38763 \times 10^{-9} + 0. i$
$5.43309 \times 10^{-12} + 0. i$
$-2.36781 \times 10^{-14} + 0. i$
$-1.36065 \times 10^{-17} + 0. i$
$3.43476 \times 10^{-23} + 0. i$



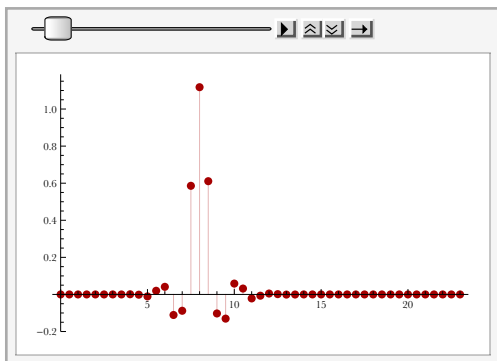
In[815]= phianimate[coif24, 5]

The matrix for the eigenvalue problem

-0.00230445	0.00126192	0	0	0	0	0
0.0227249	-0.0103891	-0.00230445	0.00126192	0	0	0
-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192	0
0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445
0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345	0.0227249
-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053	-0.114928
0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625	0.587335
-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255	0.614315
0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076
-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126	0.000367659
0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533	-0.0000441657
-2.52436 × 10 ⁻⁶	-4.60984 × 10 ⁻⁶	0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119
0	0	-2.52436 × 10 ⁻⁶	-4.60984 × 10 ⁻⁶	0.0000441657	0.0000881605	-0.000367659
0	0	0	0	-2.52436 × 10 ⁻⁶	-4.60984 × 10 ⁻⁶	0.0000441657
0	0	0	0	0	0	-2.52436 × 10 ⁻⁶
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

The eigenvector for the eigenvalue 1

- 1.33974 × 10⁻⁹ + 0. i
- 1.06411 × 10⁻⁶ + 0. i
- 0.000067639 + 0. i
- 0.000975503 + 0. i
- 0.0105753 + 0. i
- 0.0410692 + 0. i
- 0.0880722 + 0. i
- 1.11781 + 0. i
- 0.102635 + 0. i
- 0.0585417 + 0. i
- 0.0214361 + 0. i
- 0.00490709 + 0. i
- 0.000724169 + 0. i
- 0.0000752093 + 0. i
- 1.38855 × 10⁻⁶ + 0. i
- 3.25612 × 10⁻⁷ + 0. i
- 6.94007 × 10⁻⁹ + 0. i
- 1.38763 × 10⁻⁹ + 0. i
- 5.43309 × 10⁻¹² + 0. i
- 2.36781 × 10⁻¹⁴ + 0. i
- 1.36065 × 10⁻¹⁷ + 0. i
- 3.43476 × 10⁻²³ + 0. i



Out[815]=

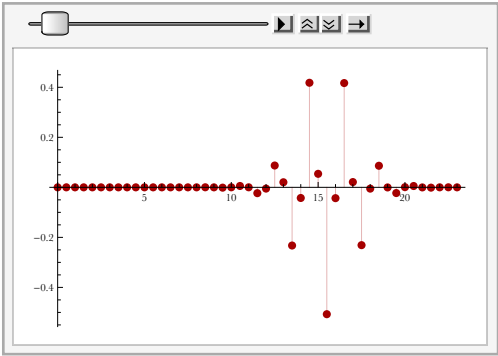
In[816]= psianimate[coif24, 5]

The matrix for the eigenvalue problem

-0.00230445	0.00126192	0	0	0	0	0	0
0.0227249	-0.0103891	-0.00230445	0.00126192	0	0	0	0
-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192	0	0
0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891	-0.00230445	0.00126192
0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345	0.0227249	-0.0103891
-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053	-0.114928	0.0377345
0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625	0.587335	-0.0793053
-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255	0.614315	1.10625
0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273	-0.136076	-0.0942255
-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126	0.0354717	0.0556273
0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533	-0.00800202	-0.0215126
-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659	-0.000833	0.00179119	0.00530533
0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605	-0.000367659	-0.000833
0	0	0	0	-2.52436×10^{-6}	-4.60984×10^{-6}	0.0000441657	0.0000881605
0	0	0	0	0	0	-2.52436×10^{-6}	-4.60984×10^{-6}
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

The eigenvector for the eigenvalue 1

- $1.33974 \times 10^{-9} + 0. i$
- $1.06411 \times 10^{-6} + 0. i$
- $0.000067639 + 0. i$
- $0.000975503 + 0. i$
- $-0.0105753 + 0. i$
- $0.0410692 + 0. i$
- $-0.0880722 + 0. i$
- $1.11781 + 0. i$
- $-0.102635 + 0. i$
- $0.0585417 + 0. i$
- $-0.0214361 + 0. i$
- $0.00490709 + 0. i$
- $-0.000724169 + 0. i$
- $0.0000752093 + 0. i$
- $-1.38855 \times 10^{-6} + 0. i$
- $-3.25612 \times 10^{-7} + 0. i$
- $6.94007 \times 10^{-9} + 0. i$
- $1.38763 \times 10^{-9} + 0. i$
- $5.43309 \times 10^{-12} + 0. i$
- $-2.36781 \times 10^{-14} + 0. i$
- $-1.36065 \times 10^{-17} + 0. i$
- $3.43476 \times 10^{-23} + 0. i$



Out[816]=