DMIP – Exercise

Sinograms and Filtered Backprojection (FBP) for Parallel Beam

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Pattern Recognition Lab (CS 5)

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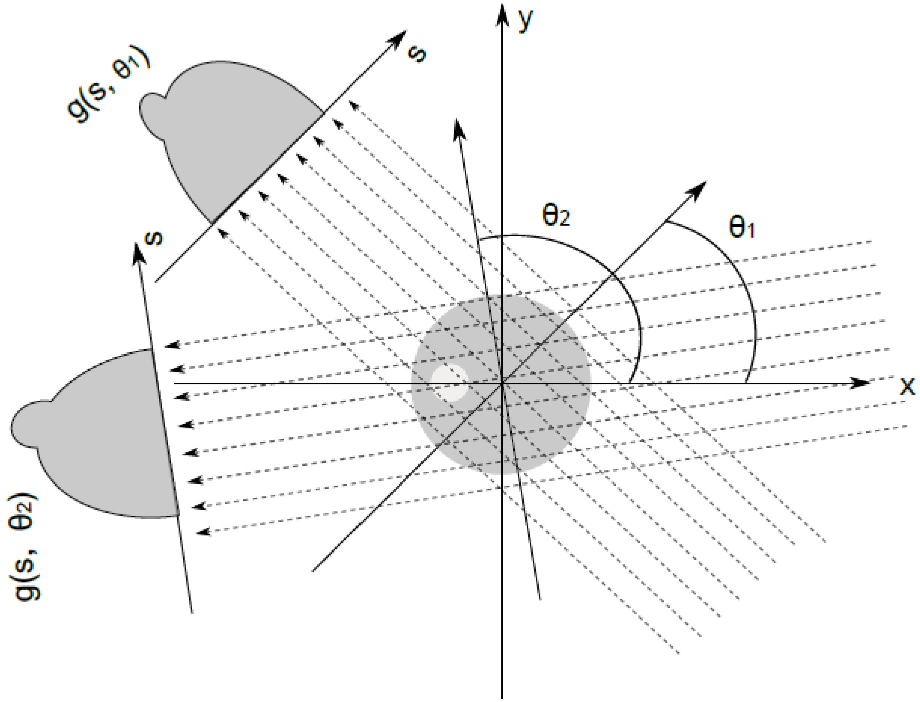
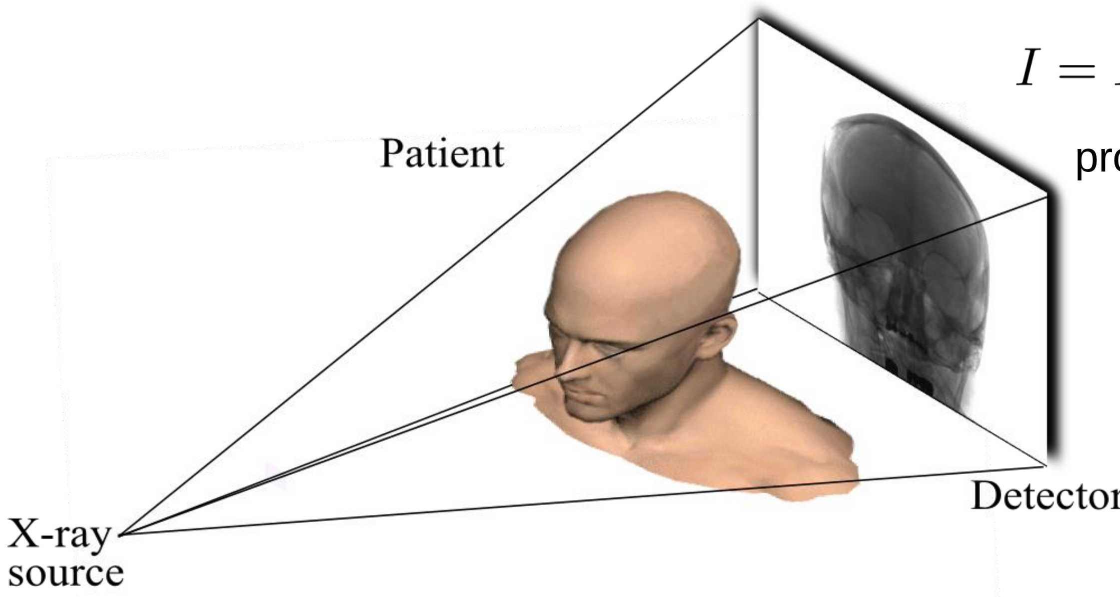
Sinograms

- What is a projection?
  - Mathematically, a projection is a line integral of a function



Sinograms

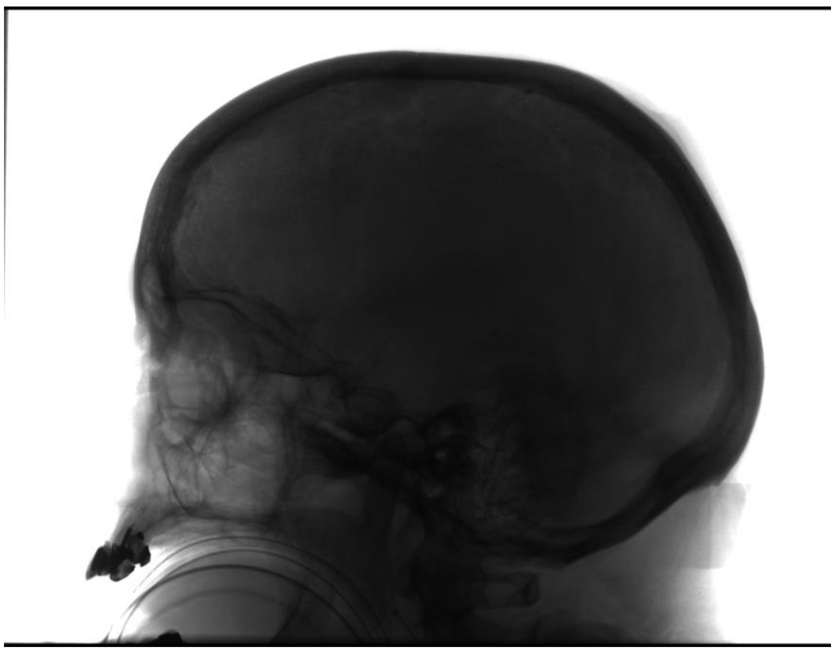
- What is a projection?
  - Mathematically, a projection is a line integral of a function
  - We use projection synonymous with X-ray projection



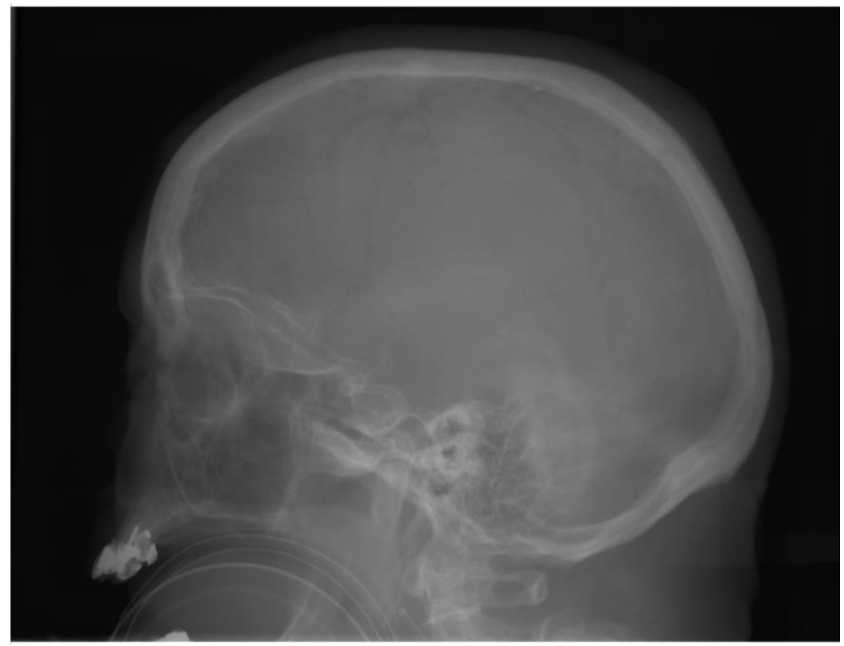
$$I = I_0 \mathrm{e}^{-\left(\int f(x,y)\,\mathrm{d}l\right)}$$

projection

$$I = I_0 \mathrm{e}^{-\left(\int f(x,y)\,\mathrm{d}l\right)} \Rightarrow \int f(x,y)\,\mathrm{d}l = -\ln\left(I/I_0\right)$$



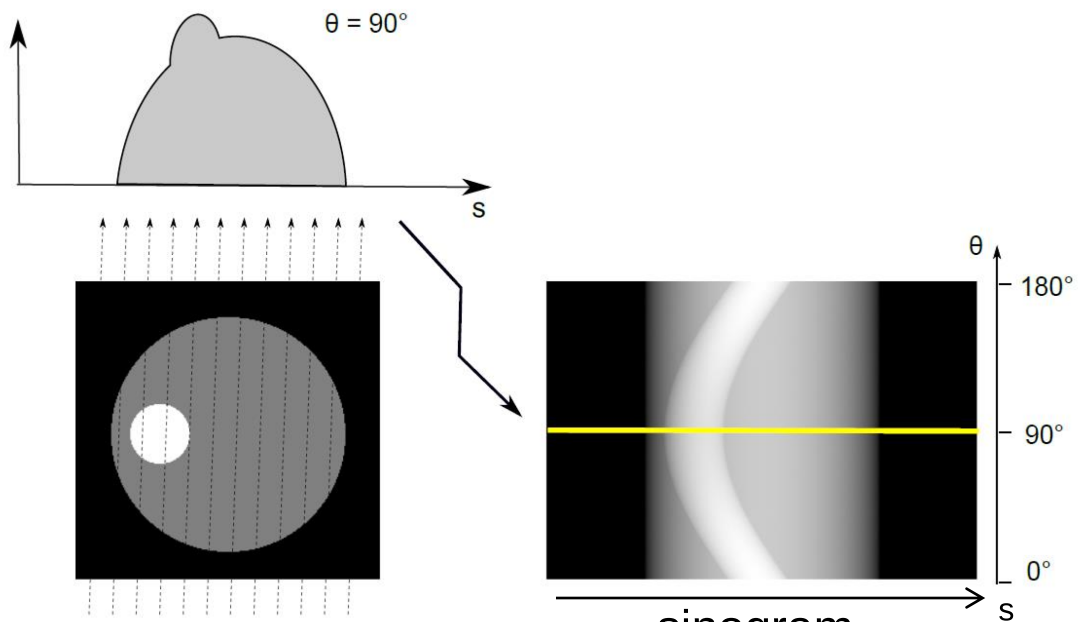
We get from detector



Line integral used for recon

Sinograms

- What is a sinogram and how does it relate to projections?
  - A stack of all acquired projections sorted by their angle
  - A 2-D sinogram contains information from 1-D projections, i.e. all necessary information to reconstruct one 2-D slice



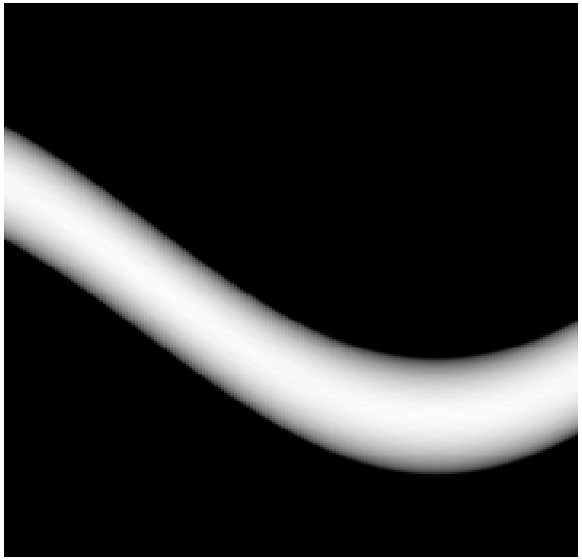
sinogram

Sinograms

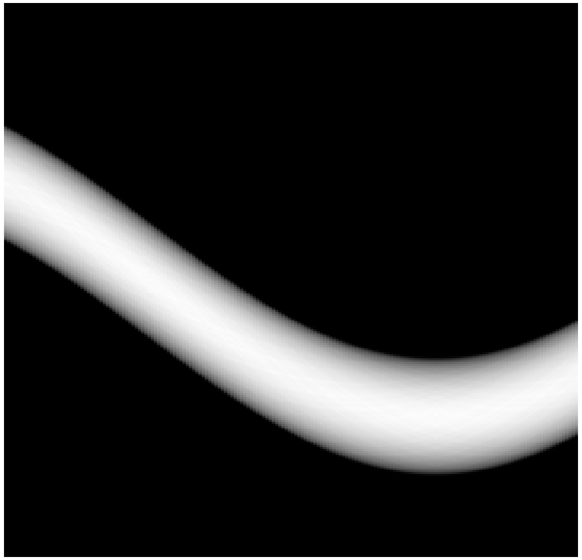
- What is a sinogram and how does it relate to projections?
  - A stack of all acquired projections sorted by their angle
  - A 2-D sinogram contains information from 1-D projections, i.e. all necessary information to reconstruct one 2-D slice

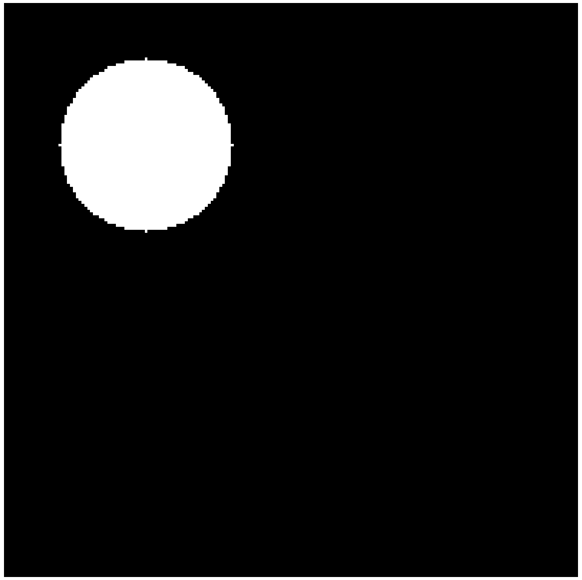
- Why is it called sinogram?
  - Because an off-centred object creates a trace that looks like a sine-wave

Sinograms

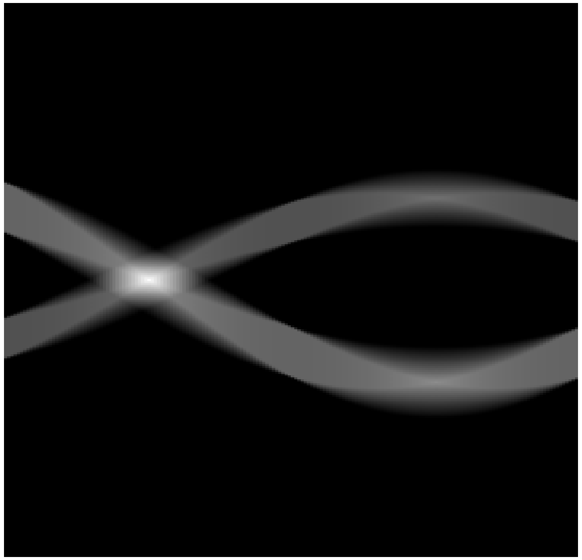


Sinograms

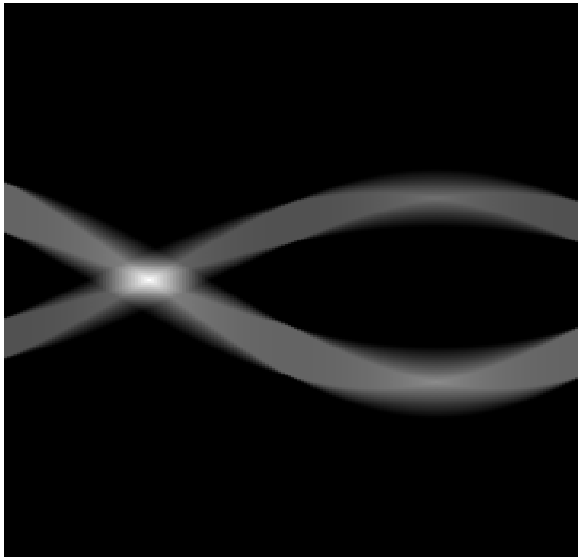


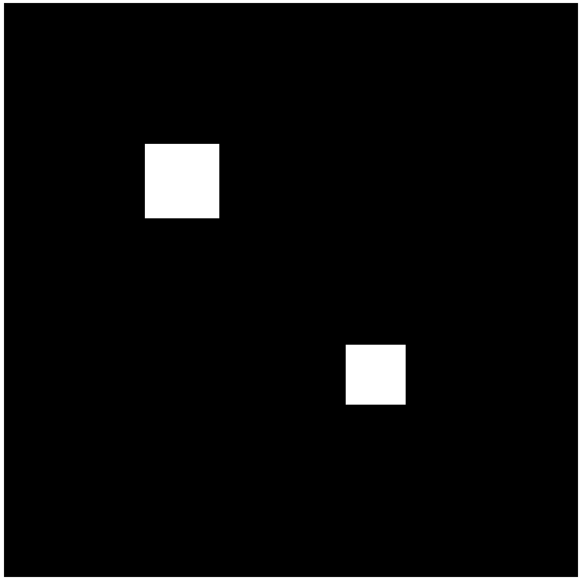


Sinograms

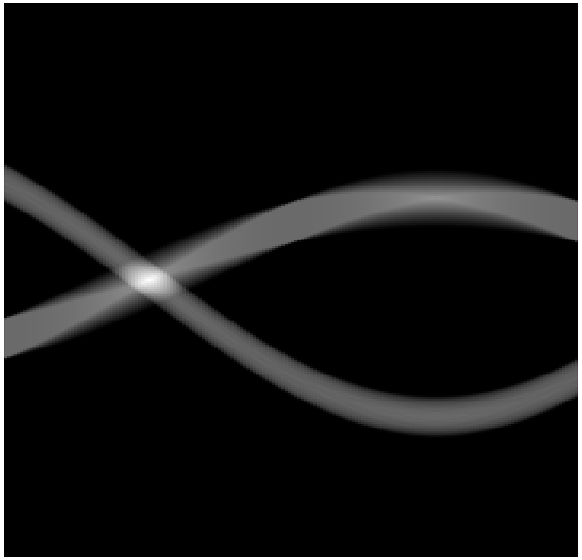


Sinograms

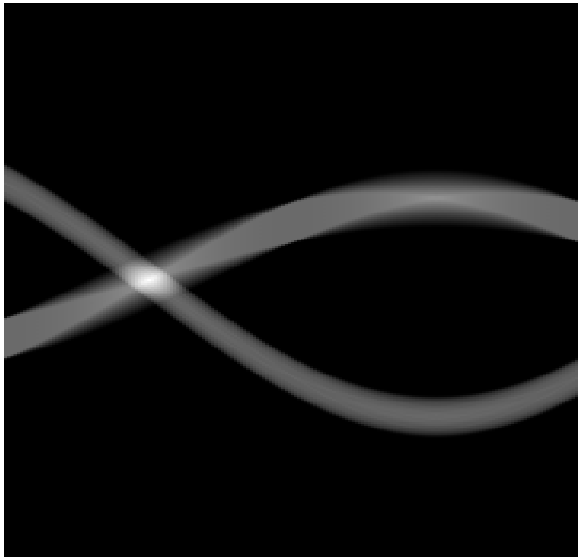


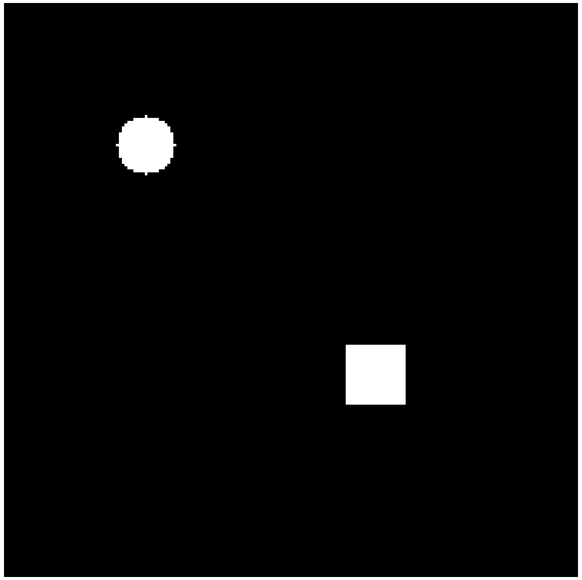


Sinograms

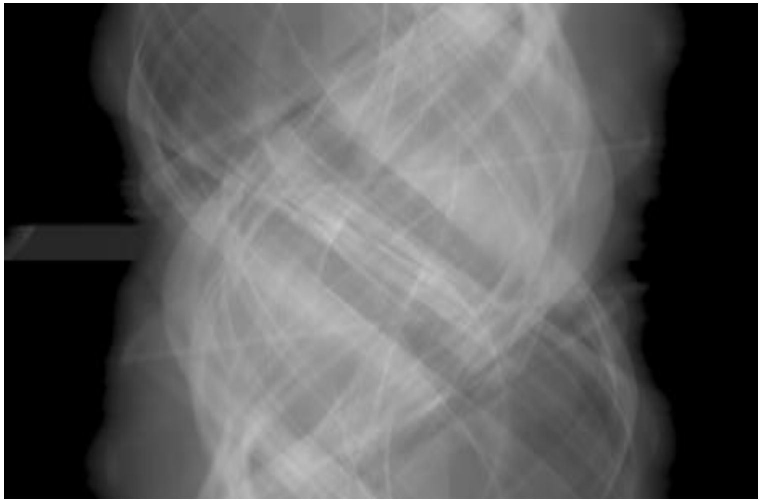


Sinograms

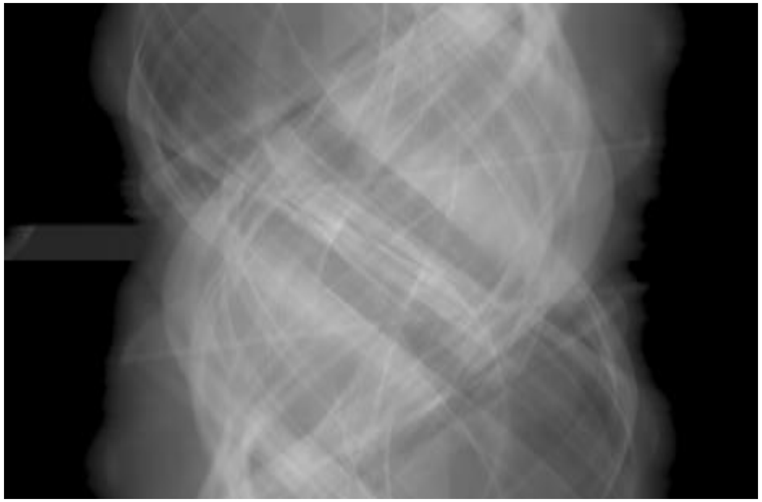


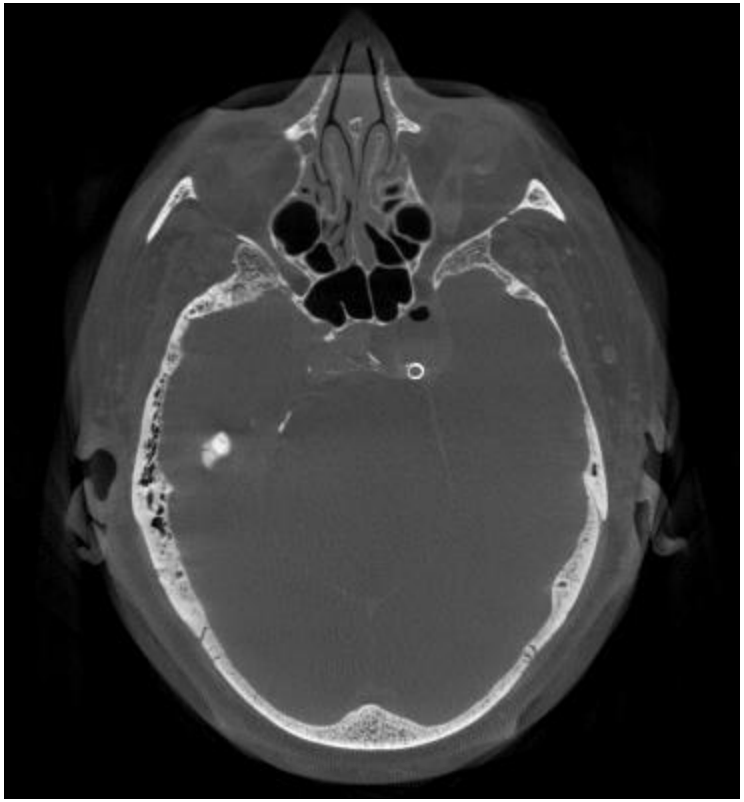


Sinograms

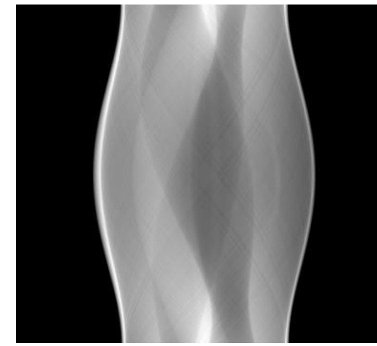


Sinograms

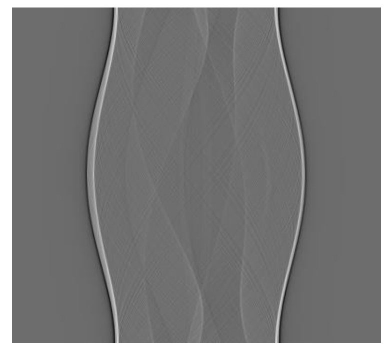




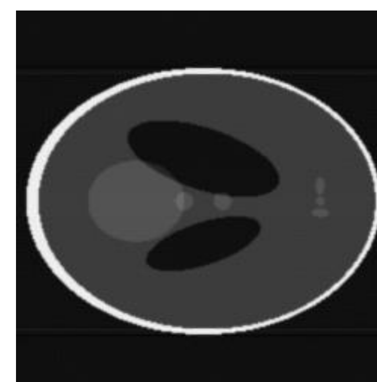
Exercise today:



1) We will scan a Shepp-Logan phantom and create a sinogam



2) Implementation of different kernels



3) Backprojection of the filtered sinogram to reconstruct the phantom

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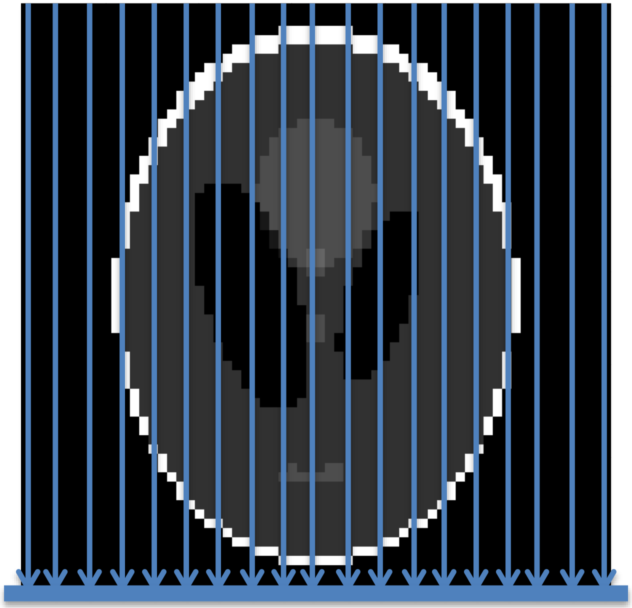
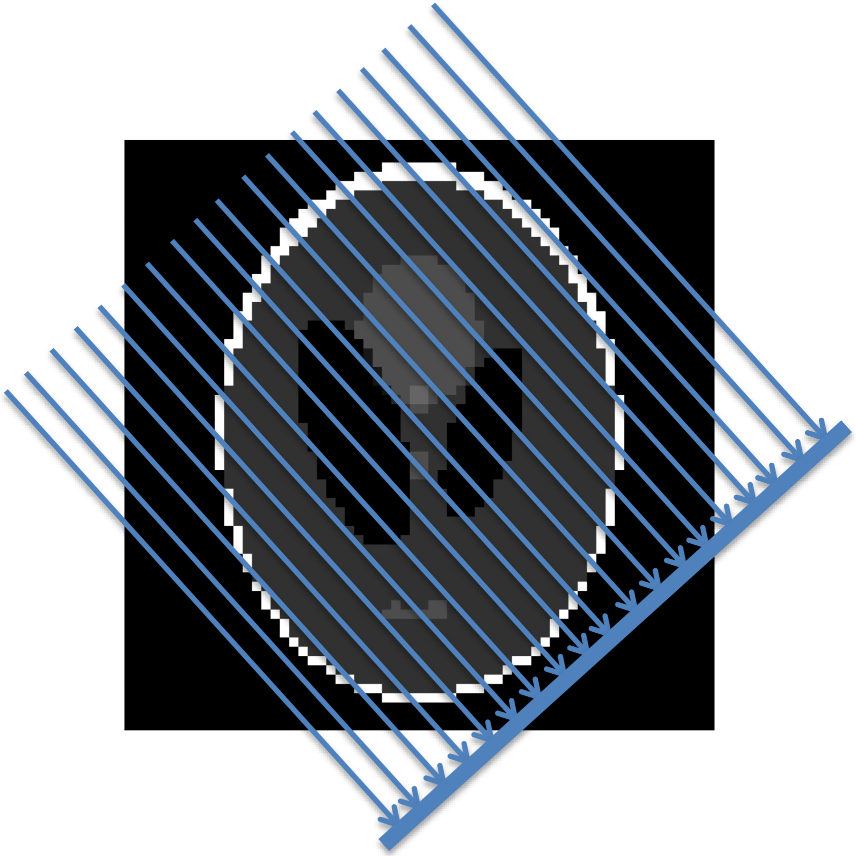
1) Scan Simulation

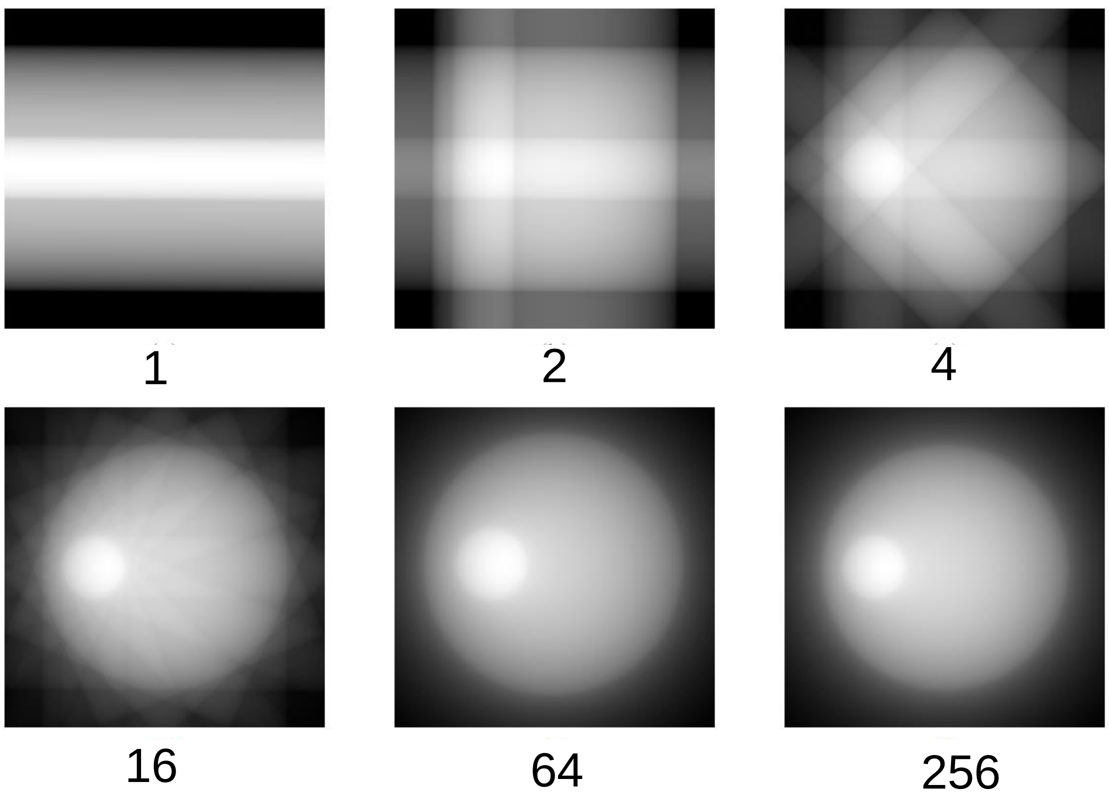
- Projections have to be acquired from different angles





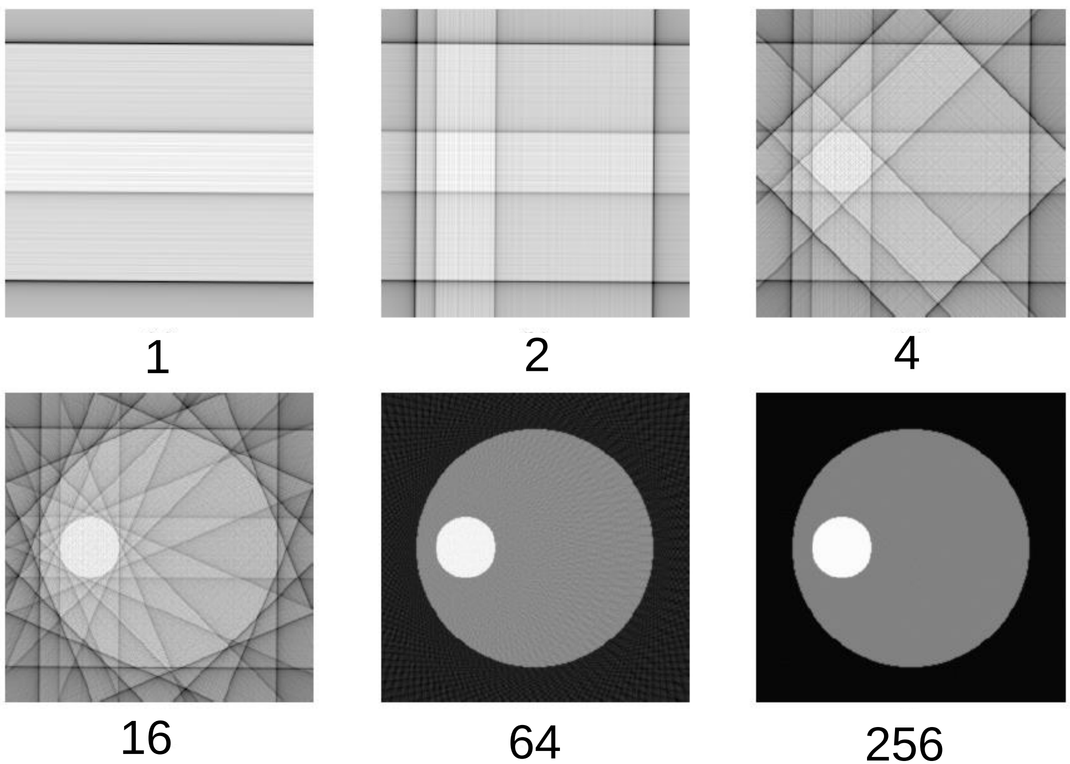
2) Filtering of the sinogram

- For the Filtered Backprojection, why do we need a high pass filter? What would the reconstruction look like without filter?



2) Filtering of the sinogram

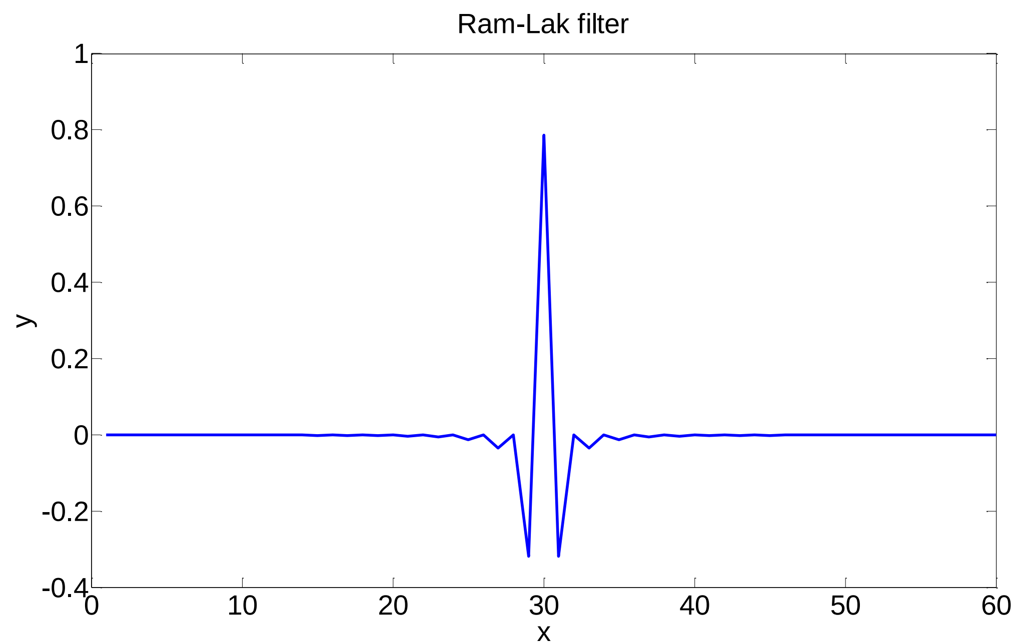
- For the Filtered Backprojection we can use different filter kernels. List them!
  - Most important are Ram-Lak and Shepp-Logan



Reconstruction

- **Task:** Implement the discrete spatial version of the RamLak filter.

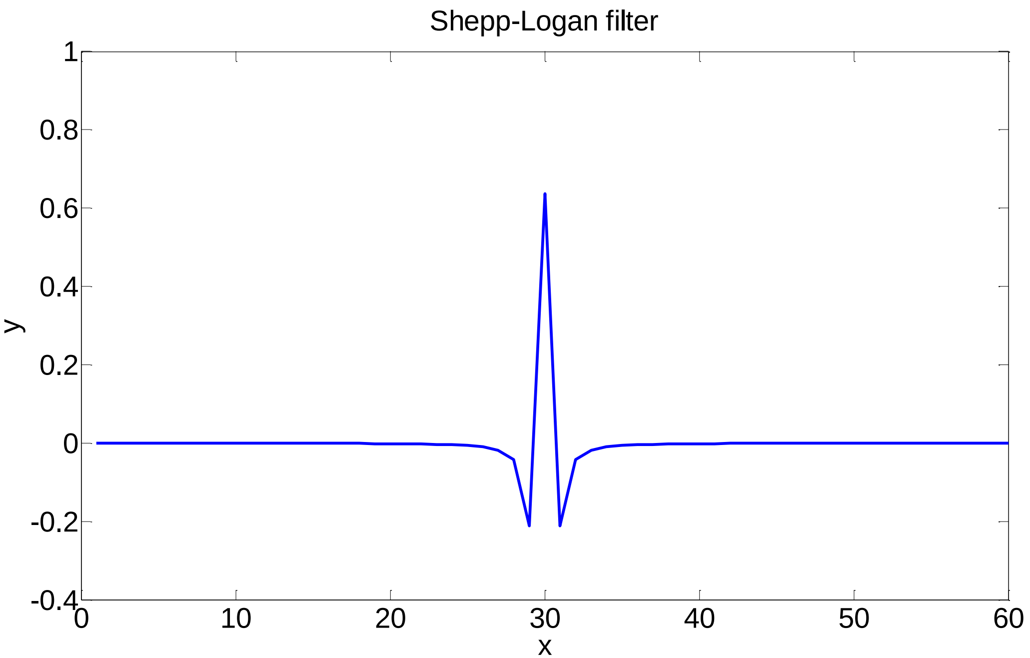
$$h_t = \begin{cases} \frac{1}{4} & t = 0 \\ 0 & t \text{ even}, \neq 0 \\ -\frac{1}{\pi^2 t^2} & t \text{ odd} \end{cases}$$



Reconstruction

- **Task:** Implement the discrete spatial version of the Shepp-Logan filter.

$$h_t = -\frac{2}{\pi^2} \cdot \frac{1}{(4t^2 - 1)}$$



3) Backprojection

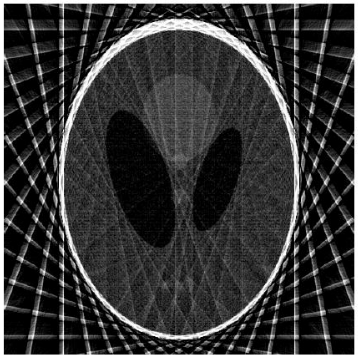
- Two different approaches are common

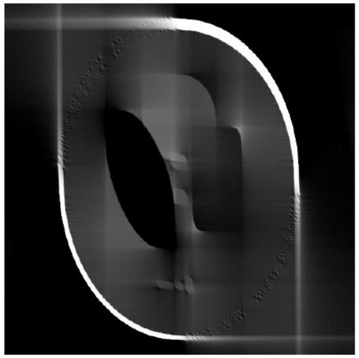
1. Detector driven: “Smear” detector values over the image.
   - Problem: Interpolation in 2D!

2. Pixel driven: Sample where you expect the outcome!
   - Go over all pixel centers
   - Project center points to the detector
   - Interpolate on the detector and assign to corresponding pixel

Filtered Backprojection for Parallel Beam

- What is the maximal angle that makes sense to acquire projections at?
  - 180° – after that, the same data is acquired twice
- Which artefacts appear if you use 110 projections at 1° increment?
  - View-undersampling artefacts
  - Manifestation in CT: Streaks, “rough” edges, wrong grey values and (most important) missing parts





Filtered Backprojection for Parallel Beam

- Which artefacts appear if data gets truncated?
  - Cupping artefacts, bright ring artefacts
  - Wrong grey values