Exercise 5

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Perspective Factorization & Homography

1 Introduction

Consider a situation in which a set of 3D points $p^w_{i,j}$ is viewed by a moving camera. We wish to solve the following reconstruction problem: given the set of image coordinates $q^i_{i,j}$ find the set of camera matrices (motion), P_i, and the 3D points $p^w_{i,j}$ (structure) such that

$$\lambda_{i,j}\tilde{q}^i_{i,j} = P_i\tilde{p}^w_{i,j}. \quad P_i \in \mathbb{R}^{3\times 4}, \tag{1}$$

where i determines the frame number and j the point. We will work in homogeneous coordinates $\tilde{()}$ with respect to arbitrary projective coordinate frames. If one knows the **projective depth** $\lambda_{i,j}$ of each of the points then the structure and camera parameters may be estimated by a simple factorization algorithm (orthographic factorization by C. Tomasi 1991) $M = PX$. M is the measurement matrix, containing all known 2D image points, P is the motion and X the shape/structure.

2 Isotropic scaling

To improve accuracy of results we perform a normalization of the given coordinates in each image.

1 The points are translated that their centroid is at the origin.

2 The points are then scaled that the average distance from the origin is equal to $\sqrt{2}$.

3 This transformation is applied to each of the images independently.

To compute the factorization we have to compute the measurement matrix:

3 Measurement matrix

We need the point correspondences of N_P image points $\tilde{q}^i_{i,j} = (u^i_{i,j}, v^i_{i,j}, 1)^T$, which are all tracked over the same frame sequence ($i = 1, ..., N_F$, $j = 1, ..., N_P$). N_P and number of frames N_F are given by the Matlab program.

Form the $3N_F \times N_P$ measurement matrix as follows:

4 Compute for each frame $\bar{u}_i = \frac{1}{N_P} \sum_{j=1}^{N_P} u^i_j$ and $\bar{v}_i = \frac{1}{N_P} \sum_{j=1}^{N_P} v^i_j$. u, v are the coordinates of the image points and can be picked up through the structure "points".

5 Compute for each point $\hat{u}^i_j = u^i_j - \bar{u}_i$, $\hat{v}^i_j = v^i_j - \bar{v}_i$ and $\hat{q}^i_j = (\hat{u}^i_j, \hat{v}^i_j, 1)^T$ to get centered data.

6 Build the measurement matrix $\hat{M}$ (Initialize $\lambda_{i,j} = 1$):

$$\hat{M}_n = \begin{pmatrix} \lambda_{1,1}\hat{q}_{1,1} & \lambda_{1,2}\hat{q}_{1,2} & \cdots & \lambda_{1,N_P}\hat{q}_{1,N_P} \\ \lambda_{2,1}\hat{q}_{2,1} & \lambda_{2,2}\hat{q}_{2,2} & \cdots & \lambda_{2,N_P}\hat{q}_{2,N_P} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N_F,1}\hat{q}_{N_F,1} & \lambda_{N_F,2}\hat{q}_{N_F,2} & \cdots & \lambda_{N_F,N_P}\hat{q}_{N_F,N_P}. \end{pmatrix} \quad (2)$$

Notice that with the correct projective depths $\lambda_{i,j}$, the 3 $N_F \times N_P$ **rescaled measurement matrix** $\hat{M}$ has rank at most 4. Question: Why is the rank at most four? Hint: What is the rank of P and X?

4 Normalizing the depths

Projective depths as defined here are not unique. In other words, the projective depths $\lambda_{i,j}$ may be replaced by multiplying the i-th row by a factor α_i and the j-th column by a factor β_j: $(\alpha_i\beta_j\lambda_{i,j})q_{i,j} = (\alpha_i P^i)(\beta_j q_{i,j})$. A simple heuristic manner of doing this is to scale the measurement matrix such that column and row vectors have norm 1.

7 Multiply each row by a factor α_i so that it has unit norm,

8 followed by a similar pass normalizing the columns. Start with initial depths of 1.

5 Estimating projective depths $\lambda_{i,j}$

9 Compute the SVD of $\hat{M}_n$: $\hat{M}_n = U\ diag(\sigma_1, \sigma_2, ..., \sigma_s)\ V^T$, with $s = \min(3N_F, N_P)$ and singular values $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_s \geq 0$.

10 Since $\hat{M}$ is of rank 4, the σ_i for $i > 4$ vanish. Thus enforce the rank of 4 and set all $\sigma_i = 0$ for $i > 4$. Thus, only the first 4 columns (rows) of U (V) contribute to this matrix product.

11 $P_i = U\ diag(\sqrt{\sigma_1}, \sqrt{\sigma_2}, \sqrt{\sigma_3}, \sqrt{\sigma_4}, 0, 0, ...)$ $\quad X = diag(\sqrt{\sigma_1}, \sqrt{\sigma_2}, \sqrt{\sigma_3}, \sqrt{\sigma_4}, 0, 0, ...)\ V^T$ or you can use $P_i = U\Sigma \quad X = V^T$

12 Reproject the estimated points into each frame and update the $\lambda_{i,j}$.
$\hat{M}_{n+1} = U\ diag(\sigma_1, \sigma_2, \sigma_3, \sigma_4, 0, 0, ...)\ V^T$.

13 Get new $\lambda_{i,j}$ by dividing the measurement matrix $\hat{M}_{n+1}$ through the measurement matrix $\hat{M}_n$ element wise and

14 there will be in total three new $\lambda_{i,j}$ (one for each component of $q_{i,j}$). Compute the mean of these three to estimate the new $\lambda_{i,j}$ in each frame for each of the N_P points.

15 If the projective depths changes significantly go back to step 7.

6 Homography

The projective reconstruction does not give us a good idea how the reconstructed object does look like. Therefore we will jump from the projective reconstruction to a metric reconstruction using ground truth. In part II of this exercise we will compute the **homography** between the projective and the metric reconstruction. In our case we know the ideal structure and can therefore easily use ground truth. To jump from the projective reconstruction to the metric reconstruction we compute the homography $X_{metric} = HX_{proj}, \quad H \in \mathbb{R}^{4\times4}$.

16 Compute the Homography $X_{metric} = HX_{proj}$. Solve the nonlinear curve-fitting problem in the least-squares sense. Use e.g. the Matlab function **lsqcurvefit** to minimize the function: $f(H) = \sum_i d(X^i_{metric}, HX^i_{proj})^2$

17 Jump to the metric space and plot X_{metric}.

The parts should be completed in **imip3_ex.m**.