

Smoothing and Edge Detection

1. Describe how separability and cascading can help to speed up Gaussian smoothing and design a fast algorithm for computing a 3-step gaussian pyramid (filtered with σ , $\sqrt{2}\sigma$, 2σ) of a 2D image using pseudo-code.
2. Compare the time complexity of convolution with a $n \times n$ kernel when using
 - (a) direct convolution with the 2-D mask,
 - (b) a separable kernel, and
 - (c) cascading with a separable kernel.
3. Prove that convolving a 1-D signal twice with a Gaussian kernel of standard deviation σ is equivalent to convolving the signal with a Gaussian kernel of $\sigma_c = \sqrt{2} \cdot \sigma$, scaled by the area of the Gaussian filter.

Hint: Make use of the identity

$$\int_{-\infty}^{+\infty} e^{-\frac{1}{2}Ax^2+Zx} dx = \sqrt{\frac{2\pi}{A}} e^{\frac{Z^2}{2A}}$$

with $A > 0$).