

# **Generative Adversarial Networks (GANs)**

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**FERIENAKADEMIE 2018**

1. Motivation
2. Principles of Information Theory & Machine Learning
3. Generative Adversarial Networks
4. Photographic Image Synthesis
5. MR to CT Synthesis

# 1. Motivation

**What can generative models do?**

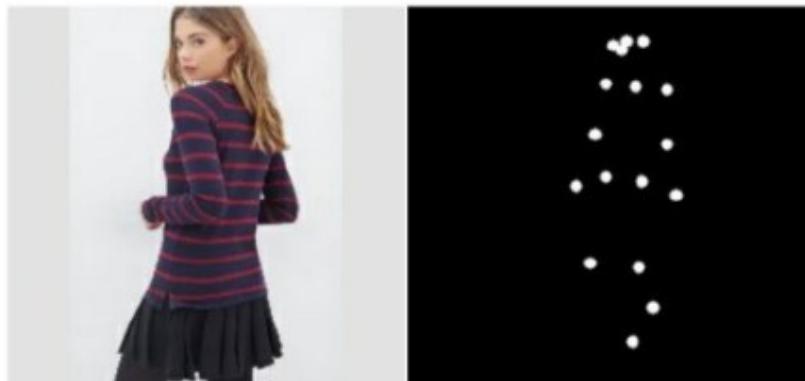
## Quiz: Which ones are real ?



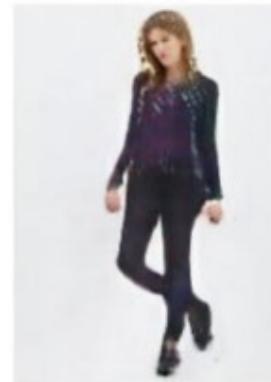
Progressive GAN  
10/2017  
1024x1024

# 1. Motivation

## Pose Guided Person Image Generation



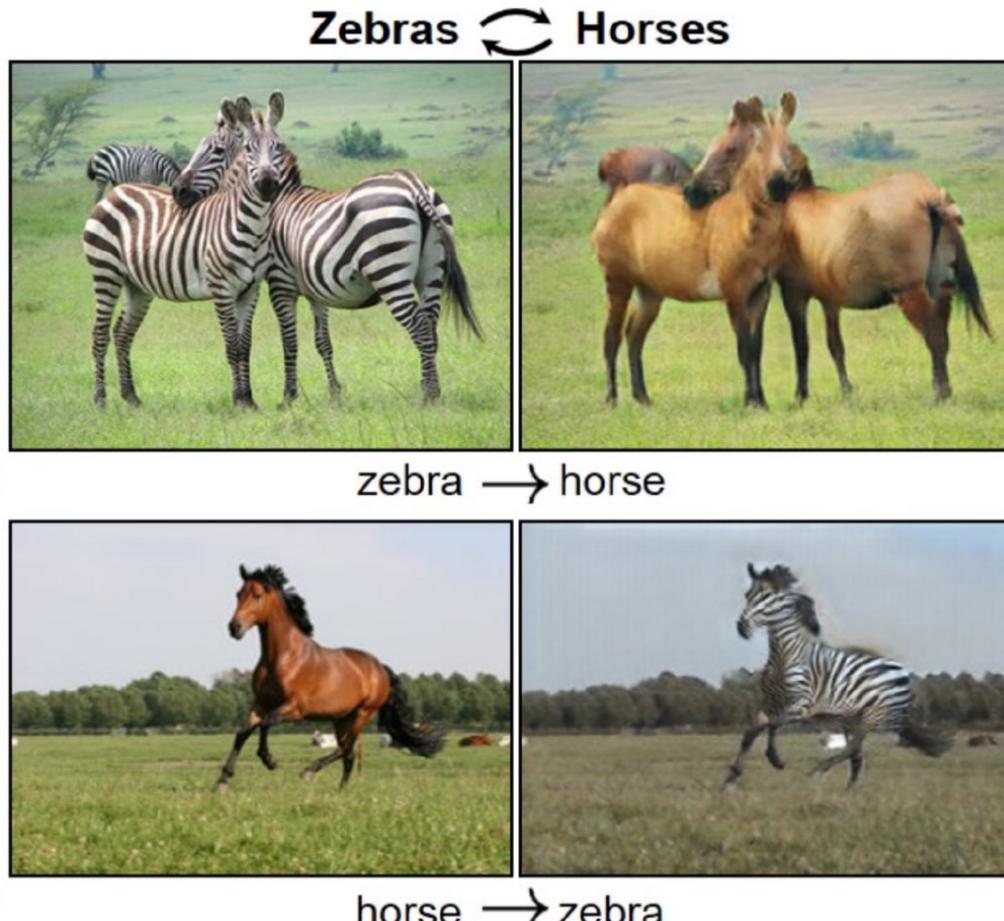
Ground truth



Generated

# 1. Motivation

Cross-domain transfer: e.g. style transfer



CycleGAN

# 1. Motivation

Low resolution to high resolution

bicubic  
(21.59dB/0.6423)



SRResNet  
(23.53dB/0.7832)



SRGAN  
(21.15dB/0.6868)



original



Super resolution  
GAN (SRGAN)

# 1. Motivation

Text to image

This flower has long thin yellow petals and a lot of yellow anthers in the center

Stage-I



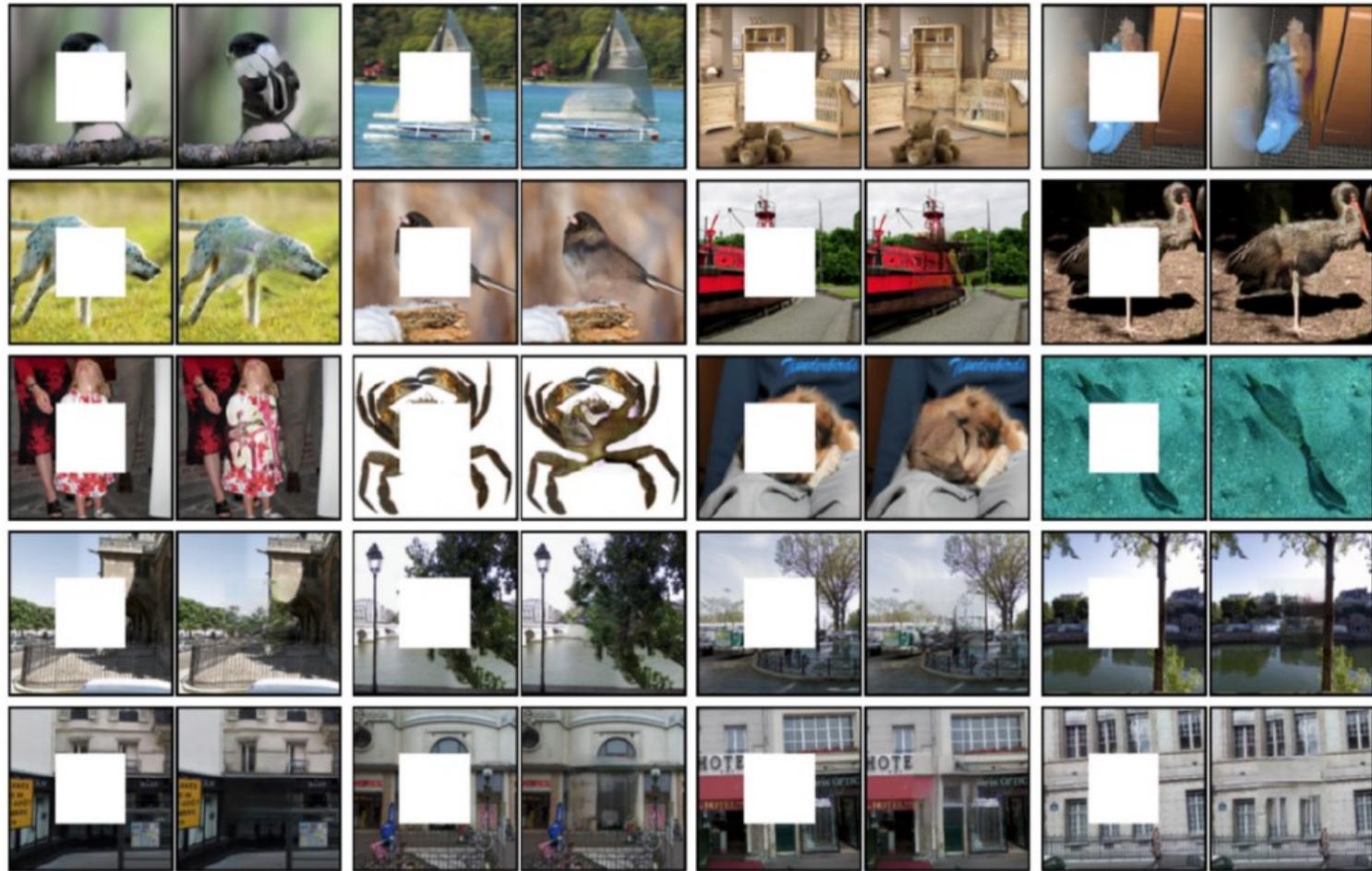
Stage-II



StackGAN

# 1. Motivation

## Image inpainting



# 1. Motivation

Maching style

INPUT



OUTPUT

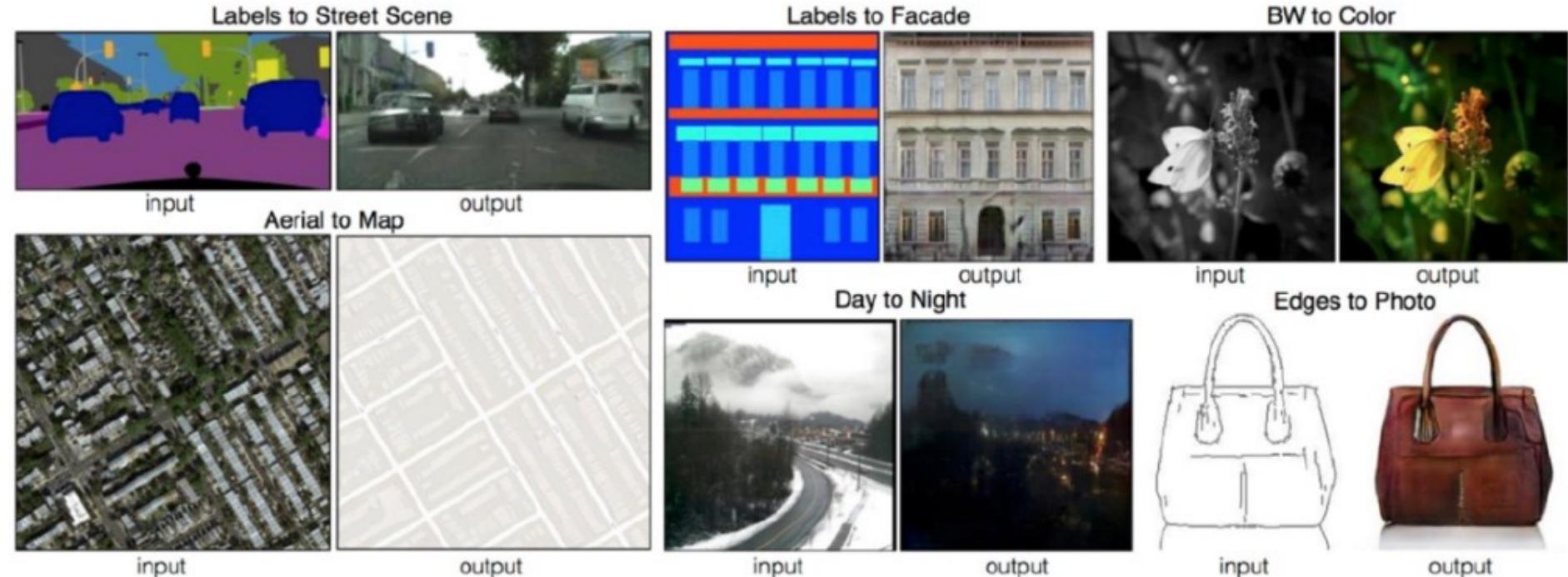


(b) Handbag images (input) & **Generated** shoe images (output)

DiscoGAN

# 1. Motivation

## image-to-image translation



pix2pix

# 1. Motivation

## Face aging

0-18

19-29

30-39

40-49

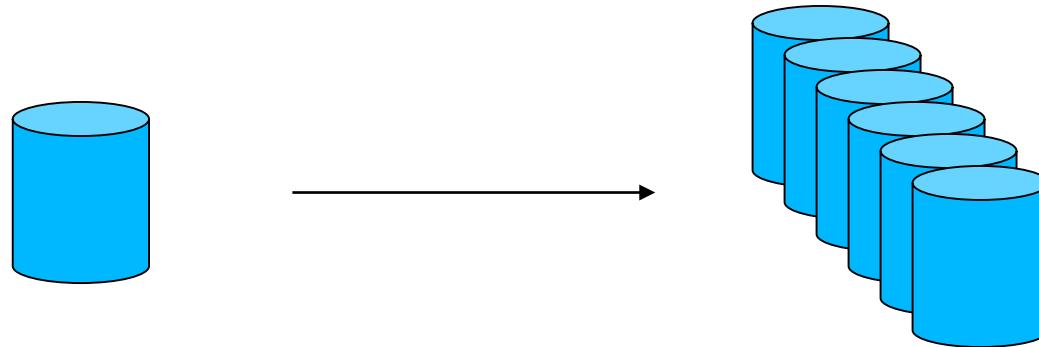
50-59

60+



Age-cGAN

## Data augmentation



## 2. Principles of Information Theory & Machine Learning

## 2.1. Information Theory

Shannon information or the self-information content:

$$\mathbb{I}(X) := -\log p(X) \in [0, \infty)$$

measures the likeliness of an event

Entropy:

$$\mathbb{H}(X) := \mathbb{E}_{x \sim p_X} [\mathbb{I}(X)] = - \sum_{i=1}^n p_X(x_i) \log p_X(x_i).$$

measure of the “uncertainty”

Binary cross entropy:

$$\mathbb{H}(X, Y) := \mathbb{E}_{x \sim p_X} [\mathbb{I}(Y)] = - \sum_{i=1}^n p_X(x_i) \log p_Y(y_i).$$

measures the amount of certainty how similar two random variables are

f-divergence:

$$\mathbb{D}_f(p_X \| p_Y) := \sum_{i=1}^n f\left(\frac{p_X}{p_Y}\right) dY$$

where X and Y are random variables and f is a convex function such that  $f(1) = 0$ .

Kullback–Leibler divergence (also called relative entropy):

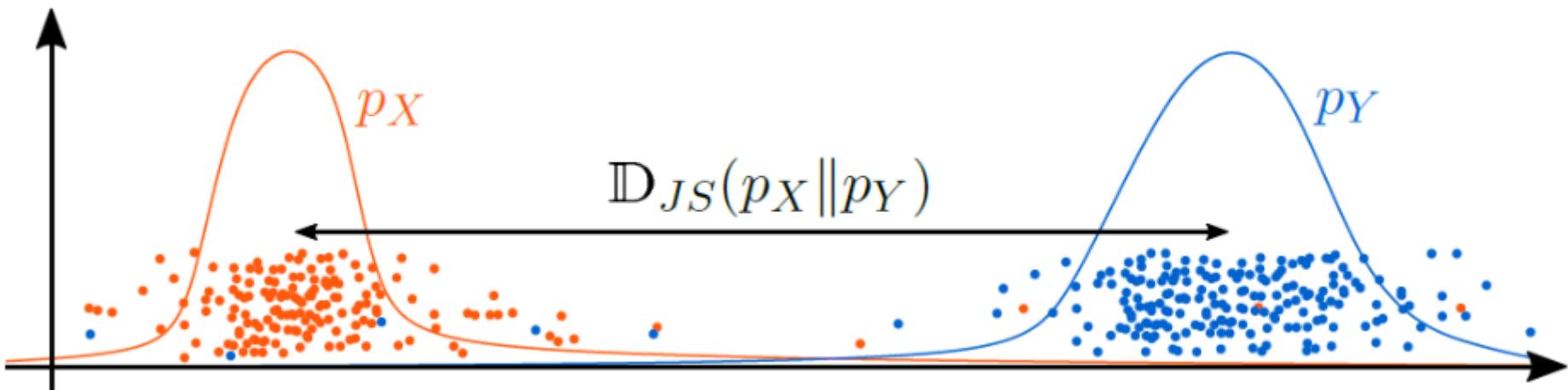
$$\mathbb{D}_{KL}(p_X \| p_Y) := \sum_i p_X(x_i) \cdot \log \frac{p_X(x_i)}{p_Y(y_i)}$$

Not symmetric

Jensen–Shannon divergence:

$$\mathbb{D}_{JS}(p_X \| p_Y) = \mathbb{D}_{JS}(p_Y \| p_X) = \frac{1}{2} \mathbb{D}_{KL}(p_X \| \frac{p_X + p_Y}{2}) + \frac{1}{2} \mathbb{D}_{KL}(p_Y \| \frac{p_X + p_Y}{2})$$

Smoothed version of KL-divergence



## 2.2. Generative vs. Discriminative Models

- A discriminative model  $D$  describes the discrete mapping function

$$x \mapsto \hat{y} := D(x; \boldsymbol{\theta}_D) \sim p_{\boldsymbol{\theta}_D}$$

$x$  : features  $\hat{y}$ : predictions  $y$  : labels  
 $\boldsymbol{\theta}_D$ : parameters (biases & weights)

- Goal: To find a good representation for  $p(y|x)$  without explicitly modeling the generative process, such that

$$p_{\boldsymbol{\theta}_D} \approx p(y|x)$$

- Example techniques: K nearest neighbors, logistic regression, linear regression, etc.

- A generative model  $G$  describes the mapping function

$$y \mapsto \hat{x} := G(y; \boldsymbol{\theta}_G) \sim p_{\boldsymbol{\theta}_G}$$

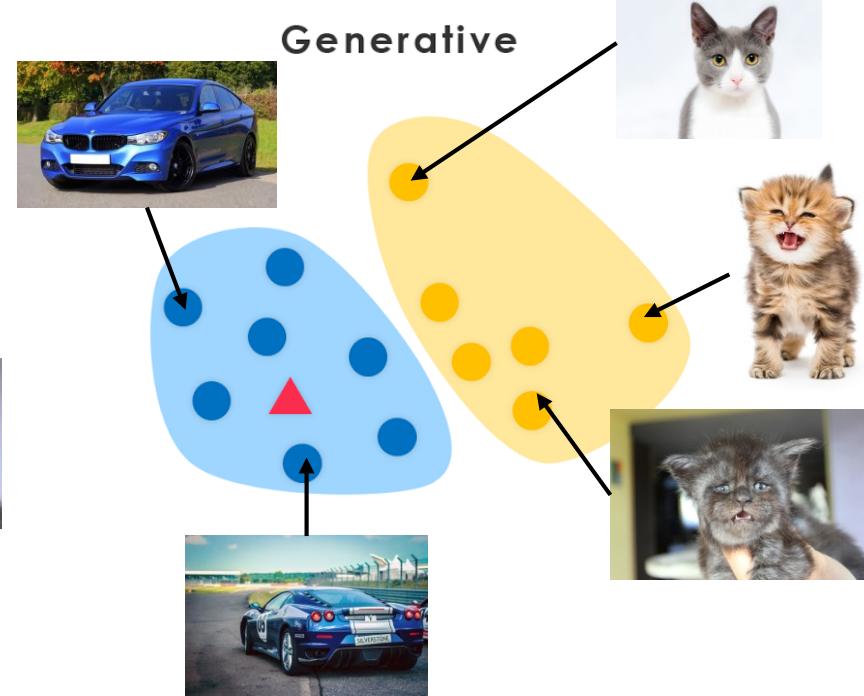
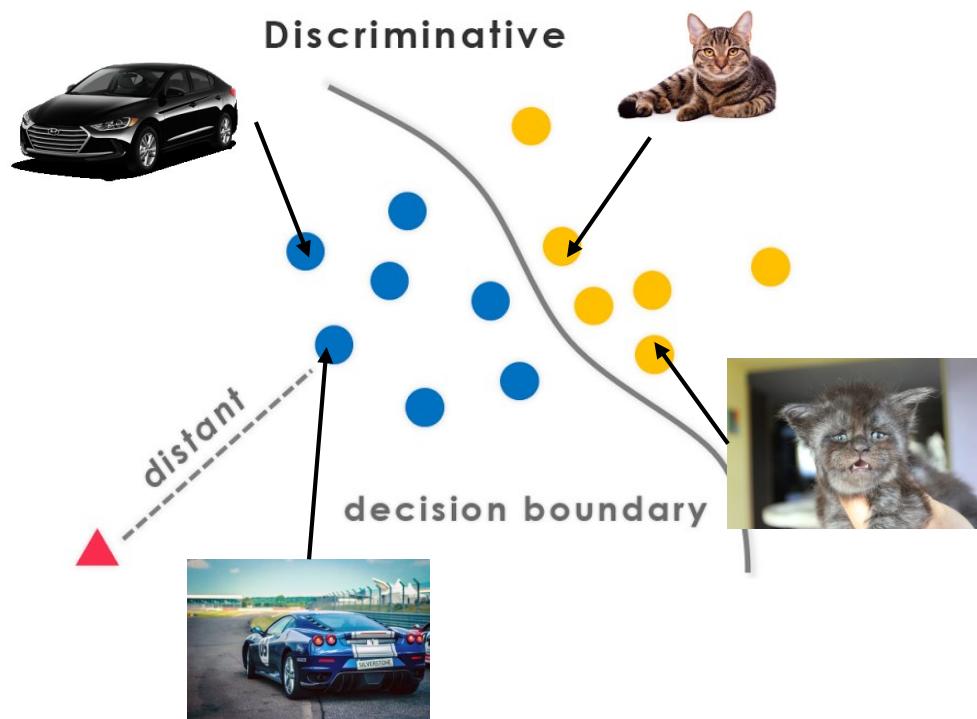
$x$  : features  $\hat{x}$ : outputs  
 $y$  : latent variable  $\boldsymbol{\theta}_G$ : parameters

- Goal: To find a probabilistic model that explicitly models the distribution of the features, such that

$$p_{\boldsymbol{\theta}_G} \approx p(x) = \int p(x|y) p(y) dy$$

- Example techniques: Hidden Markov models, Mixture models, etc.

## 2.2. Generative vs. Discriminative Models



The objective of a generative model is to find the optimal  $\theta_G$  such that:

$$p_{\theta_G} \approx p(x) = \int p(x|y) p(y) dy$$

How to estimate  $p(x)$  ?

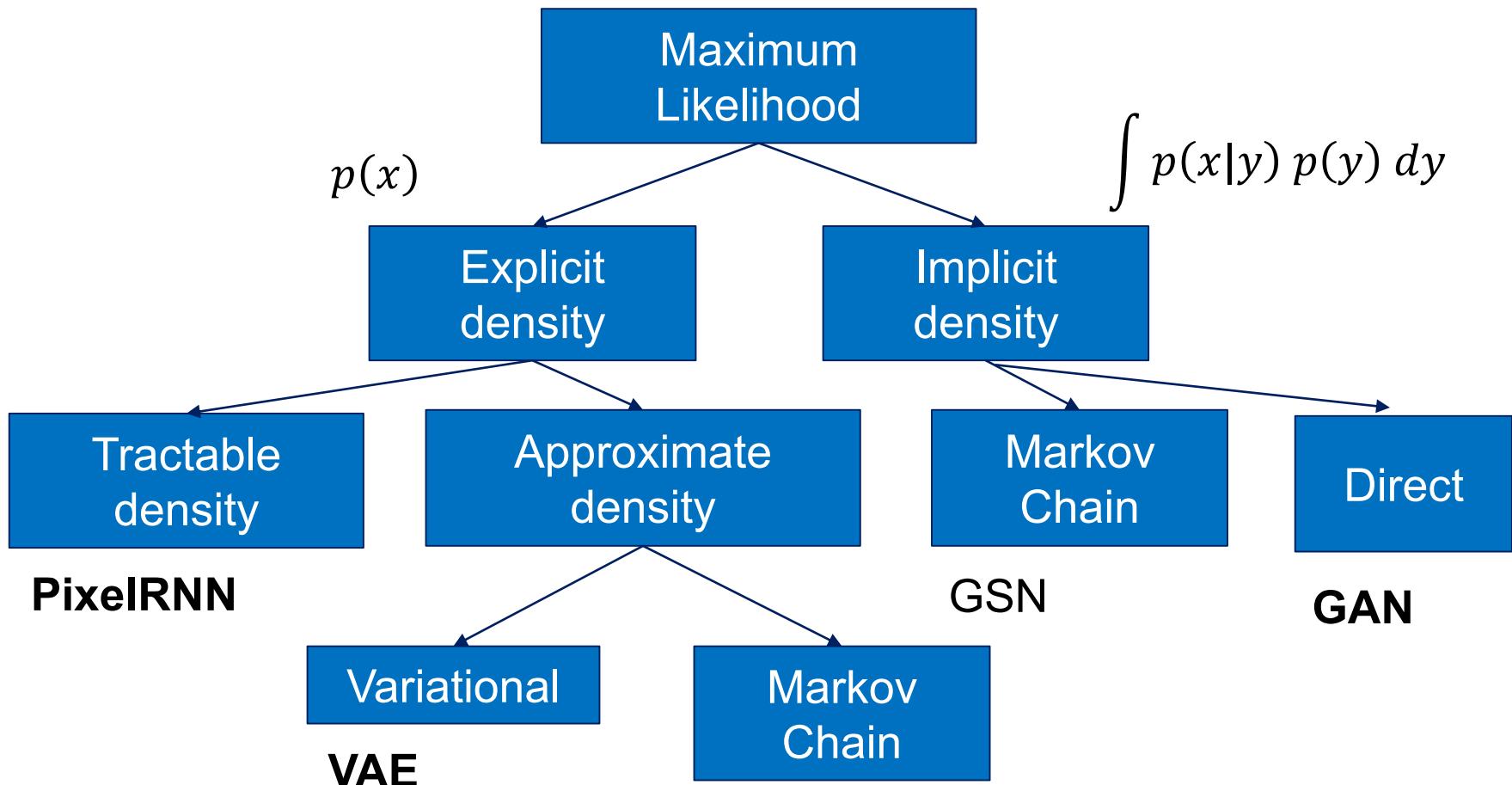
- In a high dimensional space, estimating  $p(x)$  is not easy!
- Neural networks are the best models that can estimate high dimensional distributions by providing a high number of parameters and thus represent complex transformations.

**But how to update  $\theta_G$  in order to represent  $p(x)$  ?**

$$\begin{aligned}\theta_G^* &= \operatorname{argmax}_{\theta_G} p(x) \\ &= \operatorname{argmax}_{\theta_G} \log p(x)\end{aligned}$$

**log Maximum Likelihood**

## 2.3. Generative Models

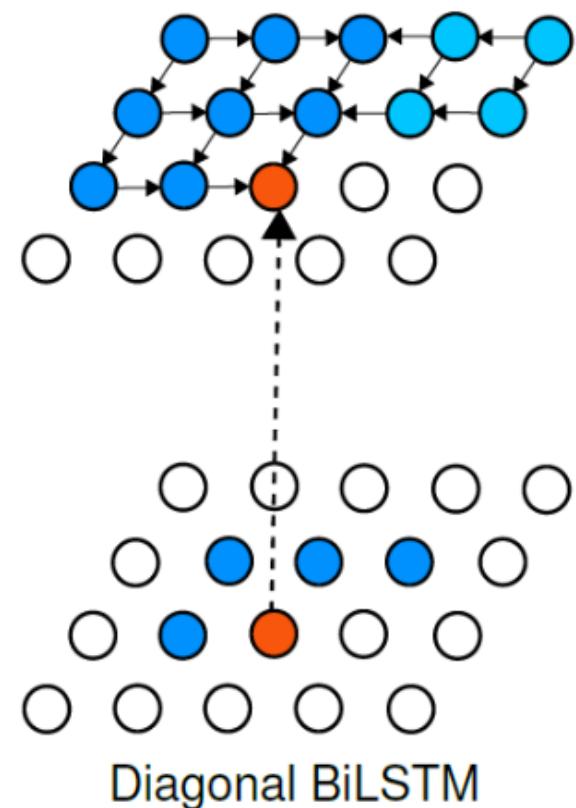


## 2.4. Pixel RNN

- Describe the Likelihood of an image as the joint distribution of all pixels:

$$p(x) = \prod_{i=1}^n p(x_i | x_1, \dots, x_{i-1}) = \prod_{i=1}^n p(x_i | x_{<i})$$

- A **sequence problem** wherein the next pixel value is determined by all the previously generated pixel values.
- Use LSTM to describe the recurrence → BiLSTM
  - **Drawbacks:** sequential generation slow to train
  - **Alternative:** Use CNN to reduce the computational cost => PixelCNN and PixelCNN++



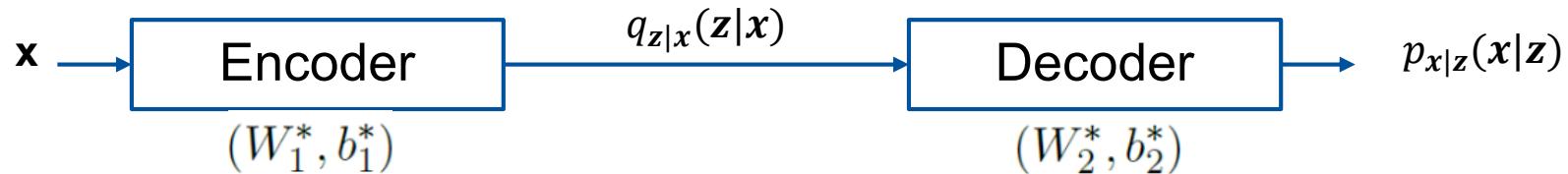
## 2.5. Variational Autoencoders VAEs

- Describe the Likelihood of an image using a latent vector  $\mathbf{z}$ :

$$p_{\mathbf{x}}(\mathbf{x}) = \int p_{\mathbf{z}}(\mathbf{z}) \cdot p_{\mathbf{x}|\mathbf{z}}(\mathbf{x}|\mathbf{z}) dz = \int p_{\mathbf{x}}(\mathbf{x}) \cdot p_{\mathbf{z}|\mathbf{x}}(\mathbf{z}|\mathbf{x}) dz$$

$$\begin{aligned} \log p_{\mathbf{x}}(\mathbf{x}) &= \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}|\mathbf{x}}(\mathbf{z}|\mathbf{x})} [\log p_{\mathbf{x}}(\mathbf{x})] \\ &\stackrel{A.1}{=} \underbrace{\mathbb{E}_{\mathbf{z}} [\log p(\mathbf{x}|\mathbf{z})]}_{\mathcal{L}_{VAE}(\mathbf{x}, \theta, \phi)} - \underbrace{\mathbb{D}_{KL}(q(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\geq 0} + \underbrace{\mathbb{D}_{KL}(q(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}|\mathbf{x}))}_{\text{Intractable!}} \end{aligned}$$

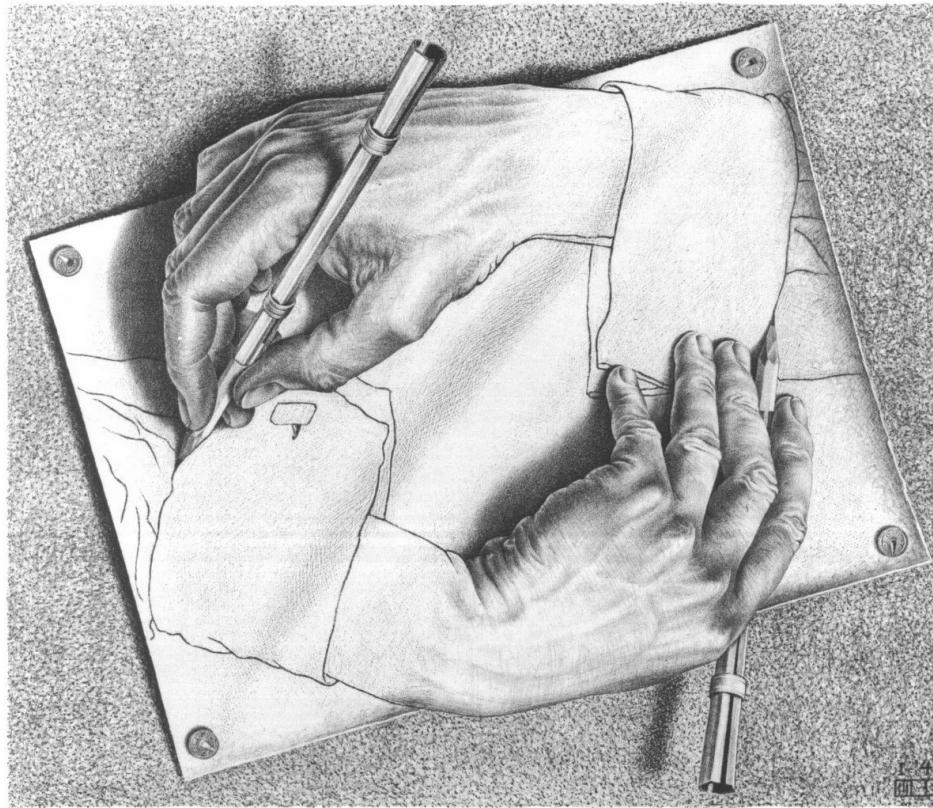
$p_{\mathbf{z}}(\mathbf{z})$  is known, e.g. Gaussian



$$(W_1^*, b_1^*), (W_2^*, b_2^*) = \arg \max_{(W_1, b_1), (W_2, b_2)} \sum_{i=1}^N \mathcal{L}_{VAE}(x_i, \theta, \phi) \text{ with } \mathcal{L}_{VAE}(x_i, \theta, \phi) \leq \log p_{\mathbf{x}}(\mathbf{x})$$

Low quality, since VAE maximizes the so-called Evidence Lower Bound (ELBO)  $\mathcal{L}_{VAE}(x_i, \theta, \phi)$

# 3. Generative Adversarial Networks



# 3.0. Idea

Optimization problem : zerosum/minimax game

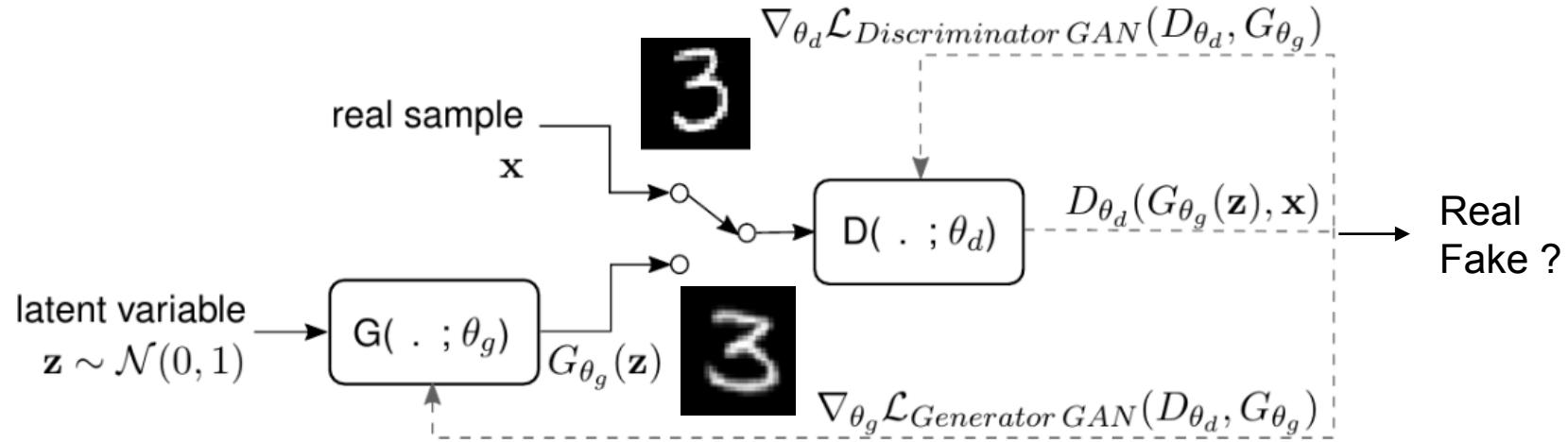


- |                                    |                                       |
|------------------------------------|---------------------------------------|
| 1 Type of Note                     | 8 Inscribed Security Thread           |
| 2 Portrait                         | 9 Federal Reserve Letter/Number       |
| 3 Microprinting                    | 10 Series                             |
| 4 Fine-Line Printing Pattern       | 11 Treasury Seal                      |
| 5 Serial Number                    | 12 Check Letter and Face Plate Number |
| 6 Check Letter and Quadrant Number | 13 Back Plate Number                  |
| 7 Federal Reserve Seal             |                                       |



[Wikiwand]

### 3.1. Network Structure



$z$  is a latent variable (e.g. random variable),  $x$  is a sample from the dataset to learn

General objective:  $G_\theta(z) \sim p_g \approx p_x$

$$z \sim p_z(z) \mapsto G(z; \theta_g) \sim p_g$$

$$(x \sim p_{data}(x), G(z; \theta_g) \sim p_g) \mapsto D(x, G(z; \theta_g); \theta_d) \sim p_d \in \{0, 1\}$$

**How to learn sampling from complex and high-dimentional distribution ?**

Game-theory approach: learn to generate from training distribution through 2-player game

### 3.1. Network Structure

- Training CelebA & interpolating over  $z$

CelebA-HQ  
 $1024 \times 1024$

Progressive growing

<https://www.youtube.com/watch?v=XOxxPcy5Gr4>

## 3.2. Optimization Problem

Optimization problem : zerosum/minimax game

$$\theta_g^*, \theta_d^* = \arg \min_{\theta_g} \arg \max_{\theta_d} \mathcal{L}_{GAN}(D_{\theta_d}, G_{\theta_g}) \quad (2.29)$$

$$= \arg \min_{\theta_g} \arg \max_{\theta_d} \mathbb{E}_{\mathbf{x} \sim p_{data}} [\log(D(\mathbf{x}; \theta_d))] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}} [\log(1 - D(G(\mathbf{z}; \theta_g); \theta_d))] \quad (2.30)$$

$$= \arg \min_{\theta_g} \arg \max_{\theta_d} \mathbb{E}_{\mathbf{x} \sim p_{data}} [\log(D(\mathbf{x}; \theta_d))] + \mathbb{E}_{\mathbf{x} \sim p_g} [\log(1 - D(\mathbf{x}; \theta_d))] \quad (2.31)$$

$$\theta_g^* = \arg \min_{\theta_g} \mathcal{L}_G(D_{\theta_d}, G_{\theta_g}) \quad \text{gradient descent on generator} \quad (2.32)$$

$$= \arg \min_{\theta_g} \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}} [\log(1 - D_{\theta_d}(G(\mathbf{z}; \theta_g)))] \quad \begin{matrix} \text{Minimize likelihood of discriminator} \\ \text{being correct} \end{matrix} \quad (2.33)$$

$$\theta_d^* = \arg \min_{\theta_d} \mathcal{L}_D(D_{\theta_d}, G_{\theta_g}) \quad \text{gradient ascent on discriminator} \quad (2.34)$$

$$= \arg \max_{\theta_d} [\mathbb{E}_{\mathbf{x} \sim p_{data}(\mathbf{x})} [\log D(\mathbf{x}; \theta_d)] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{x})} [\log(1 - D(G_{\theta_g}(\mathbf{z}); \theta_d))]] \quad (2.35)$$

**Maximize likelihood of discriminator  
being correct**

## 3.2. Optimization Problem

**Problem:** In practice, optimizing the generator objective does not work well!

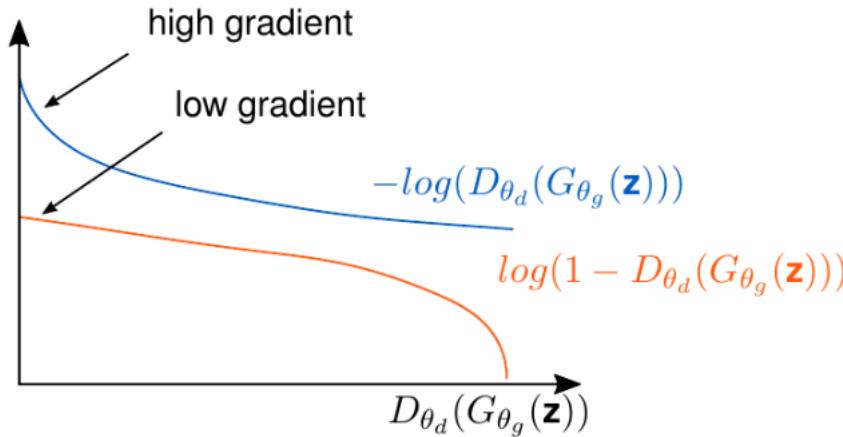


Figure 2.13.: Generator loss gradient

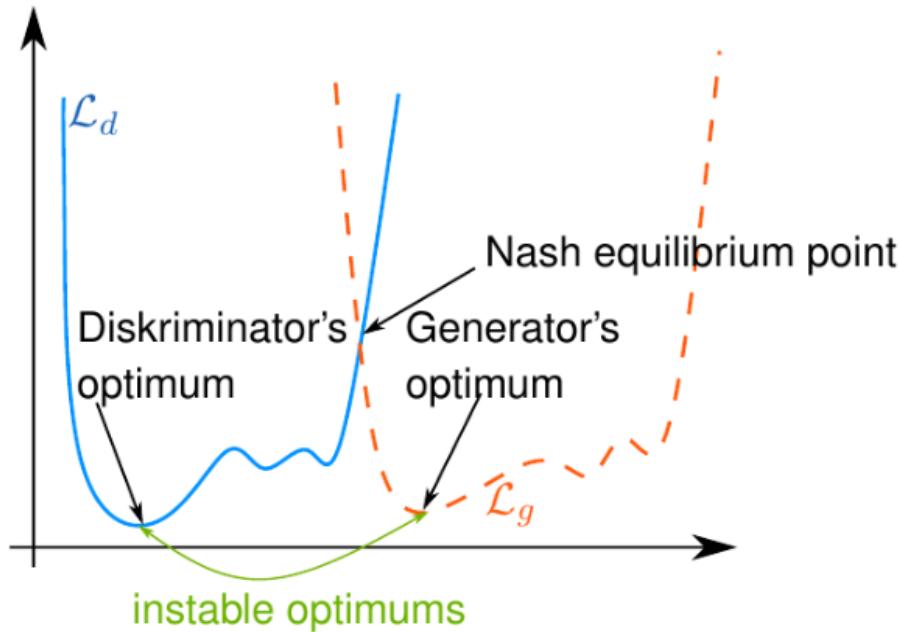
**Non-saturating heuristic game:**

$$\begin{aligned}\theta_g^* &= \arg \min_{\theta_g} \mathcal{L}_G(D_{\theta_d}, G_{\theta_g}) \\ &= \arg \max_{\theta_g} \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}} [\log (D_{\theta_d}(G(\mathbf{z}; \theta_g)))]\end{aligned}$$

**gradient ascent on generator**  
**Maximize likelihood of discriminator  
being wrong**

## 3.2. Optimization Problem

**Problem:** GANs may be very unstable since they are sensible to hyperparameters such as the learning rate of the optimizer



**Figure 2.16.:** Generator's and Discriminator's loss functions and general instability

## 3.2. Optimization Problem

Minimizing the overall loss function  $\Leftrightarrow$  Minimizing the JS(Jenson-Shannon)-Divergence:

$$\begin{aligned}
 \mathcal{L}_{GAN} &= \mathbb{E}_{\mathbf{x} \sim p_{data}} [\log D_{\theta_d}(\mathbf{x})] + \mathbb{E}_{\mathbf{x} \sim p_g} [\log(1 - D_{\theta_d}(\mathbf{x}))] \\
 &\stackrel{2.38}{=} \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[ \log \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} \right] + \mathbb{E}_{\mathbf{x} \sim p_g} \left[ \log \left(1 - \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})}\right) \right] \\
 &= \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[ \log \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} \right] + \mathbb{E}_{\mathbf{x} \sim p_g} \left[ \log \frac{p_g(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} \right] \\
 &= \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[ \log \frac{p_{data}(\mathbf{x})}{2 * \frac{1}{2}(p_{data}(\mathbf{x}) + p_g(\mathbf{x}))} \right] + \mathbb{E}_{\mathbf{x} \sim p_g} \left[ \log \frac{p_g(\mathbf{x})}{2 * \frac{1}{2}(p_{data}(\mathbf{x}) + p_g(\mathbf{x}))} \right] \\
 &= \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[ \log \frac{p_{data}(\mathbf{x})}{\frac{1}{2}(p_{data}(\mathbf{x}) + p_g(\mathbf{x}))} \right] - \log 2 + \mathbb{E}_{\mathbf{x} \sim p_g} \left[ \log \frac{p_g(\mathbf{x})}{\frac{1}{2}(p_{data}(\mathbf{x}) + p_g(\mathbf{x}))} \right] - \log 2 \\
 &= \mathbb{E}_{\mathbf{x} \sim p_{data}} \left[ \log \frac{p_{data}(\mathbf{x})}{p_{average}(\mathbf{x})} \right] + \mathbb{E}_{\mathbf{x} \sim p_g} \left[ \log \frac{p_g(\mathbf{x})}{p_{average}(\mathbf{x})} \right] - 2\log 2 \\
 &\stackrel{2.14}{=} 2 \mathbb{D}_{JS}(p_{data} \| p_g) - 2\log 2
 \end{aligned}$$

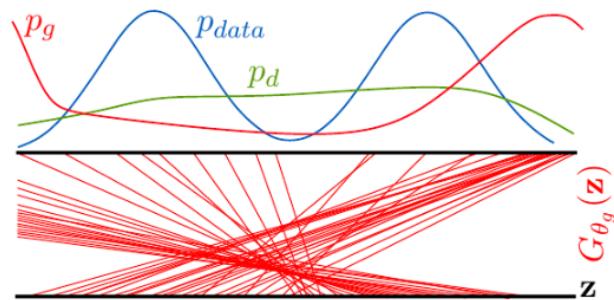
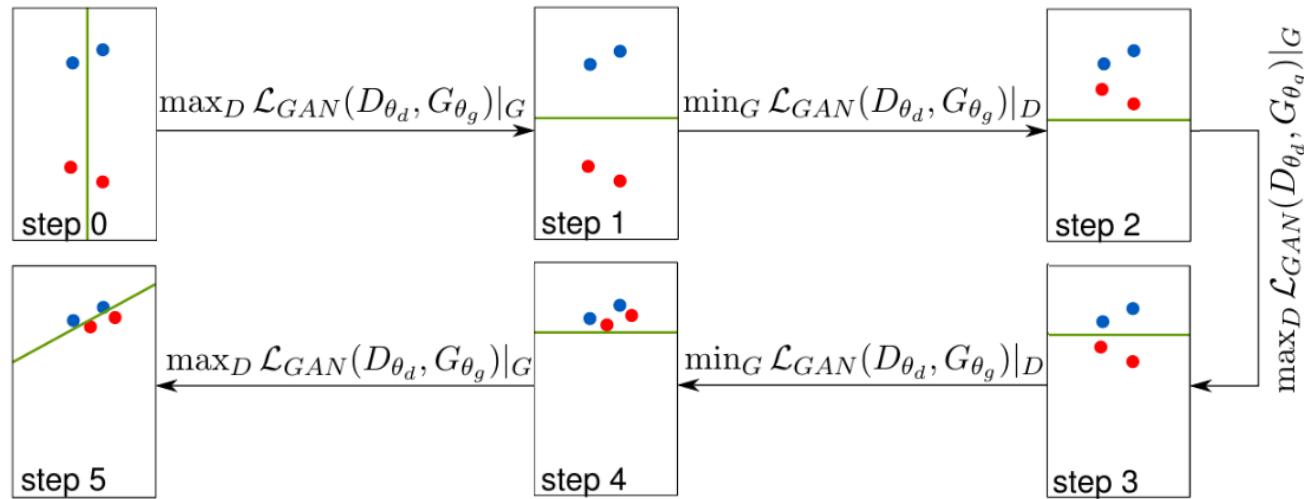
Global optimum (Nash equilibrium) is reached for:

$$D_G^*(\mathbf{x}) \stackrel{A.2}{=} \frac{p_{data}(\mathbf{x})}{p_{data}(\mathbf{x}) + p_g(\mathbf{x})} \quad \text{for G fixed}$$

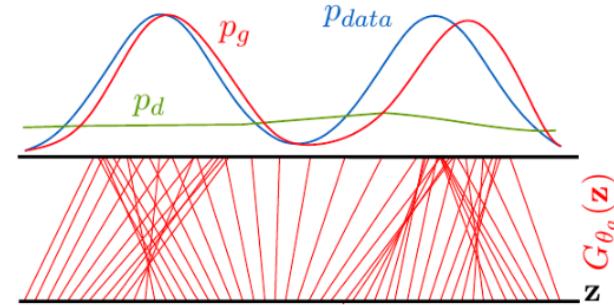
$$p_g \rightarrow p_{data}$$

if G and D have enough capacity

### 3.3. Training



(b) Mapping function of Generator in step 1: The objective of the generator is to find the mapping function  $\mathbf{z} \mapsto G_{\theta_g}(\mathbf{z})$  such that  $p_g \approx p_d$ . The objective of the discriminator is to distinguish between the ground truth data and the generated samples, i.e. to find a distribution  $p_d$  that is non-uniform.



(c) Mapping function of Generator in step 5: The generator outputs almost similar samples to the ground truth data. The discriminator cannot distinguish between the two distributions anymore and its PDF becomes nearly uniform.



### 3.3. Training

GAN learning a 2D distribution:



## GAN learning a 2-dimensional distribution

- Source data
- Generator's output
- Contours around Discriminator's view



<https://www.youtube.com/watch?v=a1fjBkwRDY8>

### 3.3. Deep Convolutional GAN (DCGAN)

The original paper (GAN) uses Fully connected layer to describe the generator and the discriminator

#### Drawbacks:

- very slow
- instable to train

#### Alternative:

Use convolutional layer to learn and evaluate only relative features (e.g. Deep Convolutional GAN (DCGAN) and all recent GANs) instead of using fully connected hidden layers

- + Replace any pooling layers with strided convolutions (discriminator) and fractional-strided convolutions (generator)
- + Use batchnorm in both the generator and the discriminator
- + Use ReLU activation in generator for all layers except for the output, which uses Tanh
- + Use LeakyReLU activation in the discriminator for all layers
  - faster
  - much more stable to train

### 3.3. Deep Convolutional GAN (DCGAN)

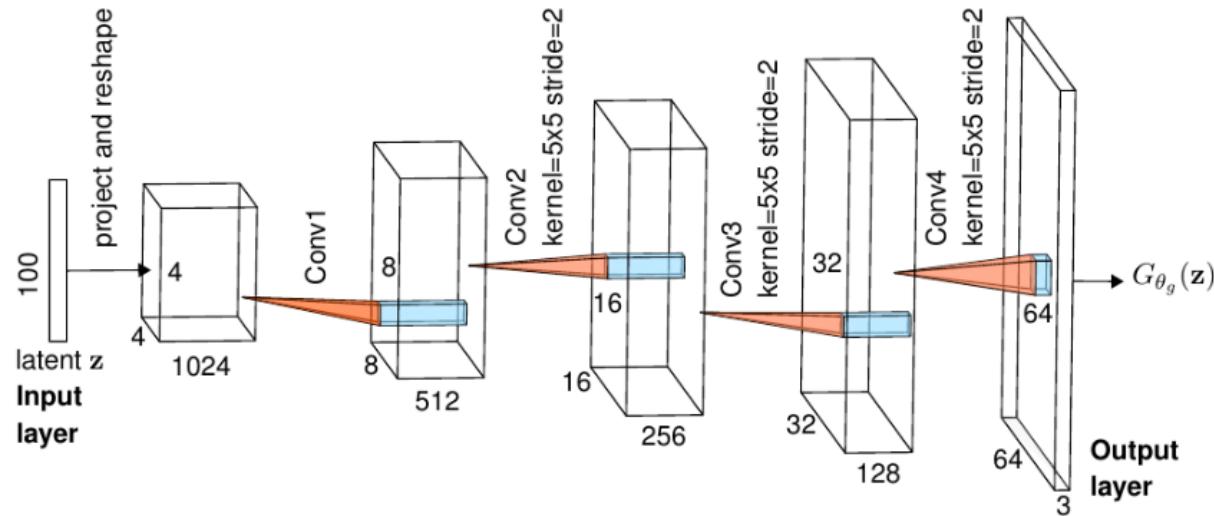


Figure 2.14.: Architecture of the generator in DCGAN

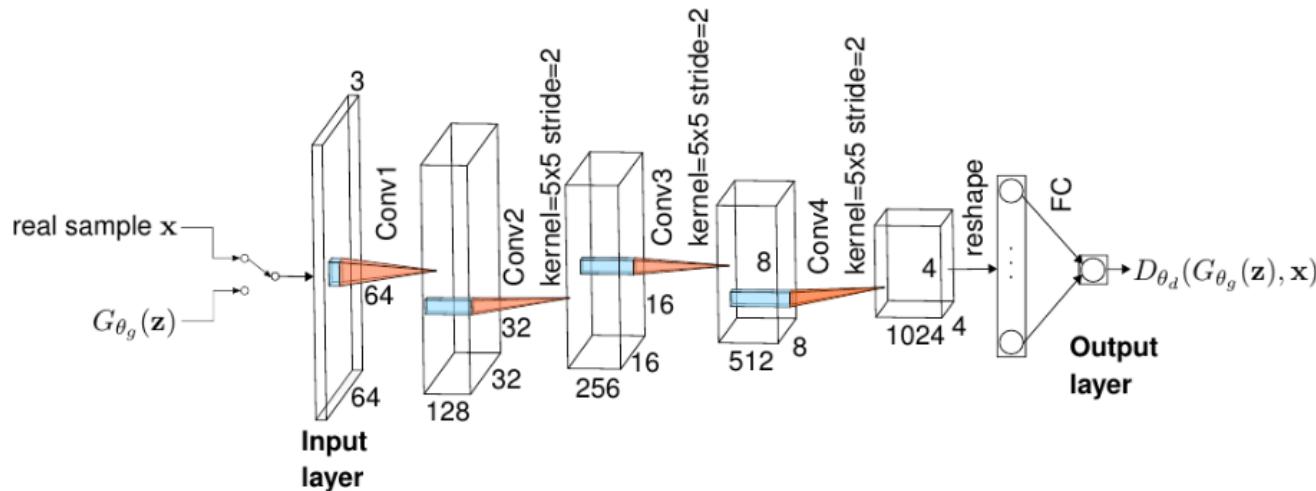


Figure 2.15.: Architecture of the discriminator in DCGAN

## 3.4. Pros / Cons

### Pros:

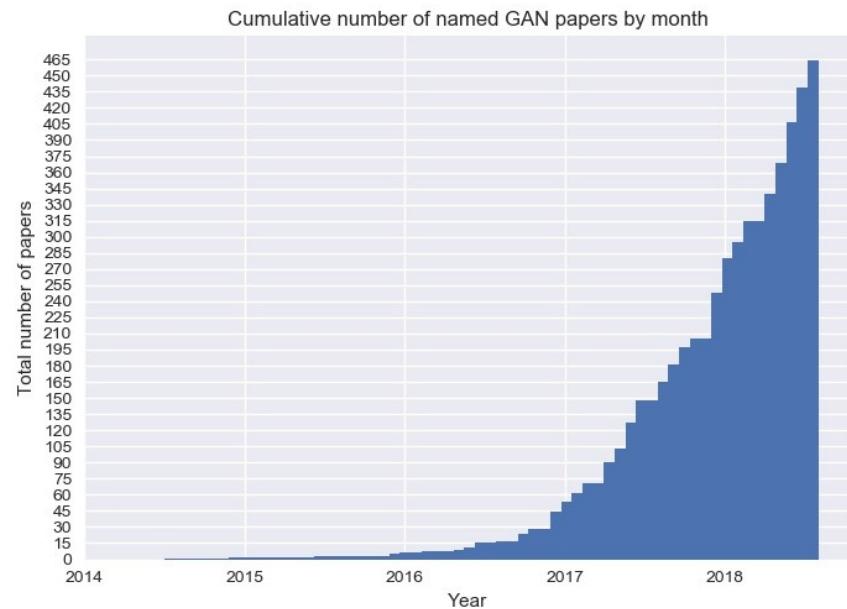
- Beautiful, state-of-the-art samples!

### Cons:

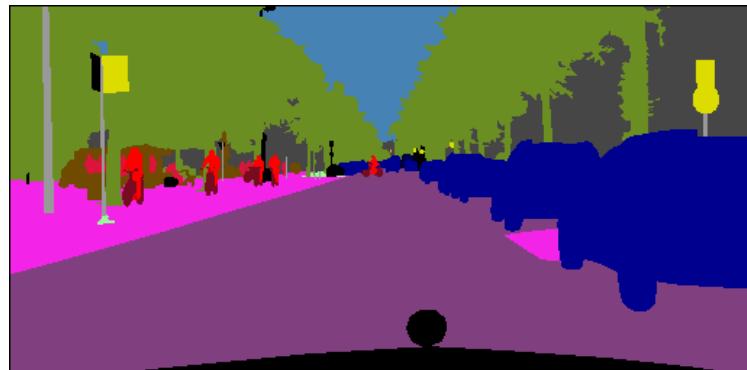
- Trickier / more unstable to train
- Hard to generate discrete data, like text

## Improvement methods and active areas of research:

- Better loss functions to improve stability  
(Wasserstein GAN)
- Novel architecture of the discriminator and/or generator  
(e.g. Capsule GAN)
- Changing in the global structure of the GAN  
(e.g. Multi-Generator GAN)



# 4. Photographic Image Synthesis



Input semantic layouts

Image synthesis

Semantic  
segmentation



Synthesized images

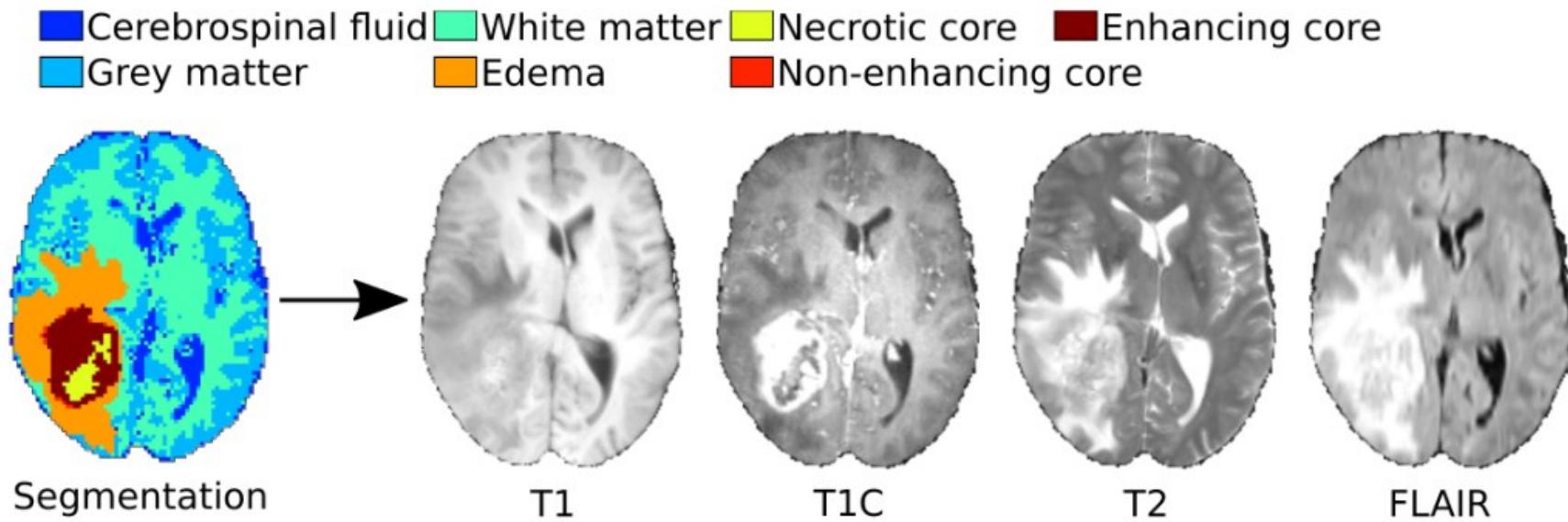
## 4.1. Motivation



Computer graphics:

- Alternative route to photorealism
- Capture photographic appearance
- Fast image synthesis

## 4.1. Motivation



Medicine:

- Medical imaging: semantic labels → MRI / CT / MRI / CT → photographic image
- Data augmentation ??

## 4.2. Photographic Image Synthesis with Cascaded Refinement Networks (CRN)

Technische Universität München



- Cascaded Refinement Network (CRN)
- Perceptual loss
- Diversity (synthesis of a set of images)

**Important characteristic for synthesizing photorealistic images:**

- Global coordination (e.g. symmetry)
- High resolution (depending on the application)
- Memory/ high model capacity (generalization)

## 4.2. Photographic Image Synthesis with Cascaded Refinement Networks (CRN)

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### CRN

A single refinement module in a CRN

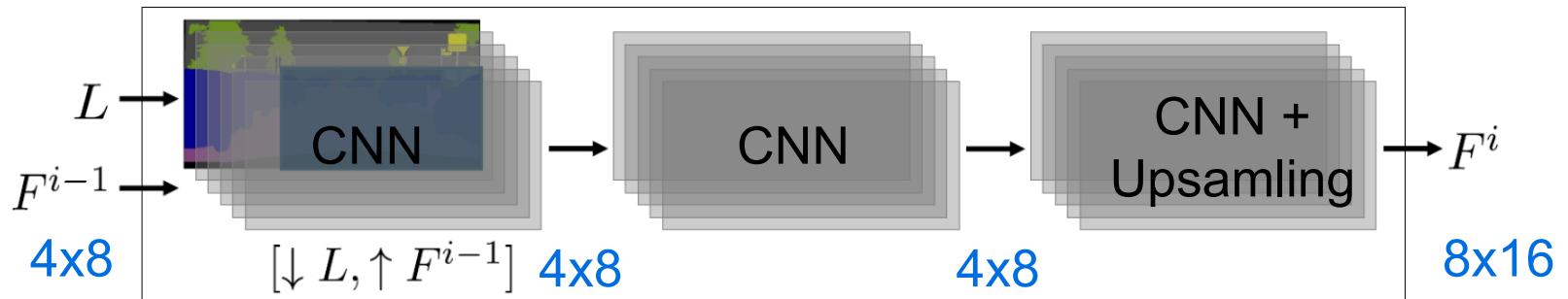


Figure 3. A single refinement module.

## 4.2. Photographic Image Synthesis with Cascaded Refinement Networks (CRN)

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### Perceptual Loss

$$\mathcal{L}_{I,L}(\theta) = \sum_l \lambda_l \|\Phi_l(I) - \Phi_l(g(L; \theta))\|_1.$$

Match activation in a pretrained visual perception network VGG

$\Phi_l$  Activations of the layer l in the VGG netw $\Phi_k$

$I$  Ground truth image

$g$  The mapping function performed by the CRN

$\lambda_l$  hyperparameters in order to balance the contribution of each layer l to the loss

## 4.2. Photographic Image Synthesis with Cascaded Refinement Networks (CRN)

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### Results

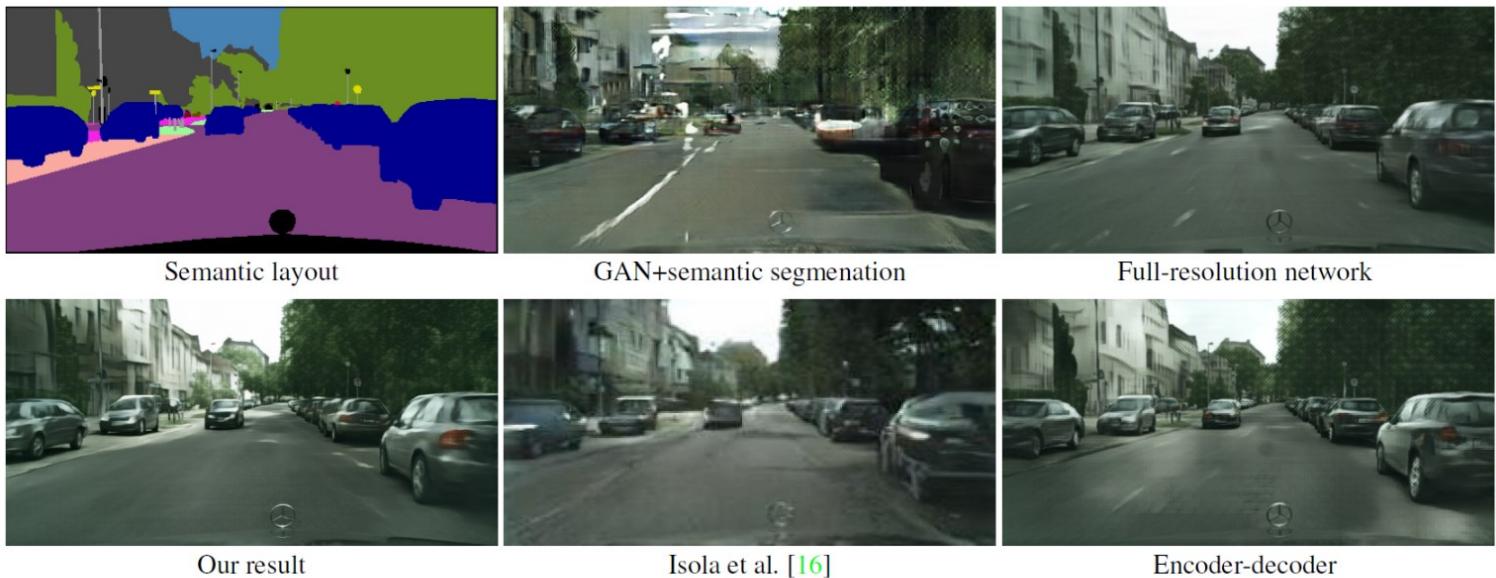


Figure 5. Qualitative comparison on the Cityscapes dataset.



Figure 6. Qualitative comparison on the NYU dataset.

An attempt to train a image synthesis system based on **GANs** was not successful

FIRNAKADEMIE

# 4. MR to CT Synthesis

### Similarities to computer vision:

- Object detection → organ detection
- Object segmentation → organ segmentation
- Object tracking → organ tracking

### Challenges:

1. Images are often 3D or 4D → dimensionality reduction
2. Number of images for training is often limited
3. Training data is expensive (annotation of data by hand:  
manpower, cost, time)
4. Training data is sometimes imperfect (e.g. diseases such  
as Alzheimer's require confirmation through pathology:  
difficult and costly to obtain)

1. Learning the right features
2. Detecting when it goes wrong
3. Going beyond human-level performance

## 4.2. CT vs MR

	CT	MR
Basic principles of scanning	X-rays - slices	Magnetic field + radio waves Identify hydrogen atoms
Harmful radiation	Yes for long exposure	No
Type of tissues scanned	Tumors Lungs Brain	Ligaments Heart Liver Blood vessels

## 4.2. CT vs MR

	CT	MR
Noise	No noise	noisy
Time	Seconds → Minutes	Minutes → >Hours
Metallic implants	No impact	High impact
Cost	cheap	expensive

### Challenges:

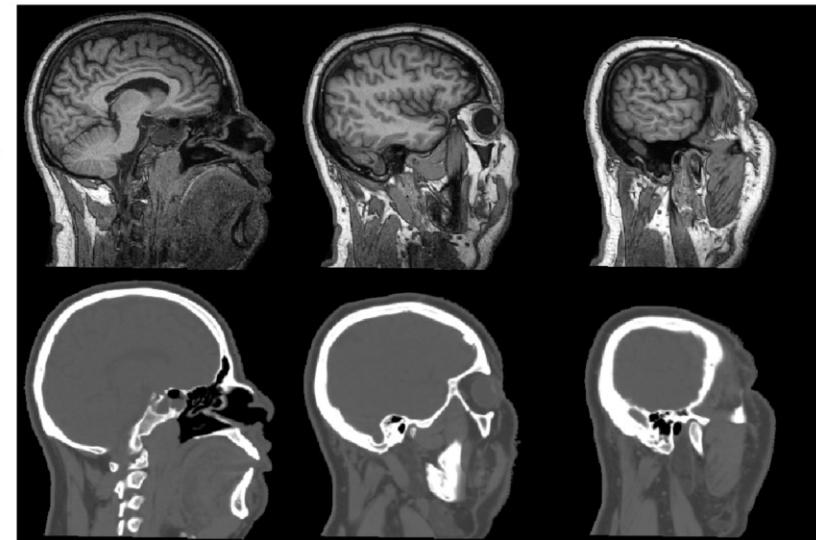
- 2D slices to 3D transformation:  
MR is problematic for moving objects
- CT can capture structures that MR is not able to.  
CT uses x-rays, which may harm the fetal  
**→ CT to MR**



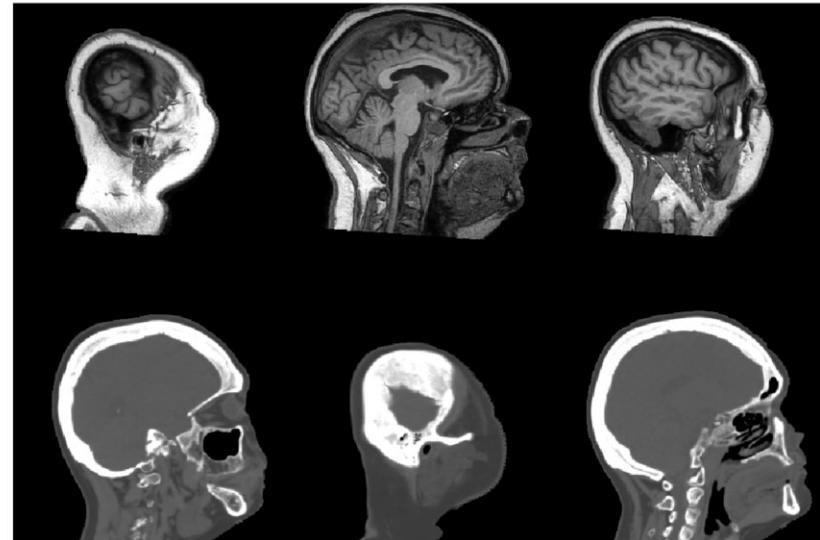
## 4.3. MR to CT Synthesis so far

### Data

Paired data



Unpaired data



- same patient
- same anatomical location

- different patient
- different anatomical location in the brain

## 4.3. MR to CT Synthesis so far

### Local misalignment

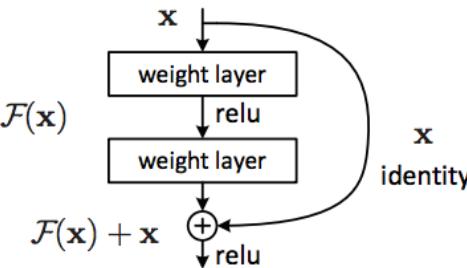


- The skull is generally wellaligned
- Misalignments in the thorat, mouth, vertbrae and nasal cavities

## 4.4. Deep MR to CT Synthesis using Unpaired Data

### Architecture

- CycleGAN (Zhu et al.)
- Consists of:
  - forward cycle (3 separate CNNs):
    - $\text{Syn}_{\text{CT}}: \mathbf{I}_{\text{MR}} \rightarrow \text{Syn}_{\text{CT}}(\mathbf{I}_{\text{MR}})$
    - $\text{Syn}_{\text{MR}}: \mathbf{I}_{\text{CT}} \rightarrow \text{Syn}_{\text{MR}}(\mathbf{I}_{\text{CT}})$
    - $\text{Dis}_{\text{CT}}: [\text{Syn}_{\text{CT}}(\mathbf{I}_{\text{MR}}), \mathbf{I}_{\text{CT}}] \rightarrow [\text{synthesized}, \text{real}]$
  - backward cycle (to improve training stability):
    - $\text{Syn}_{\text{MR}}: \mathbf{I}_{\text{CT}} \rightarrow \text{Syn}_{\text{MR}}(\mathbf{I}_{\text{CT}})$
    - $\text{Syn}_{\text{CT}}: \mathbf{I}_{\text{MR}} \rightarrow \text{Syn}_{\text{CT}}(\mathbf{I}_{\text{MR}})$
    - $\text{Dis}_{\text{MR}}: [\text{Syn}_{\text{MR}}(\mathbf{I}_{\text{CT}}), \mathbf{I}_{\text{MR}}] \rightarrow [\text{synthesized}, \text{real}]$
- $\text{Syn}_{\text{CT}}$  and  $\text{Syn}_{\text{MR}}$  are identical: DeConvolutional Network
  - 2D ConvLayers, strides=2x2, 9 ResBlocks, Upsampling
  - Input: 256x256 image, output: 256x256 image
- $\text{Dis}_{\text{CT}}$  and  $\text{Dis}_{\text{MR}}$  are identical: Convolutional Network
  - 2D ConvLayers
  - input: overlapping 70x70 image patches, output: scalar (0 or 1)



## 4.4. Deep MR to CT Synthesis using Unpaired Data

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### Losses

On  $\text{Dis}_{\text{CT}}$

$$\mathcal{L}_{\text{CT}} = (1 - \text{Dis}_{\text{CT}}(I_{\text{CT}}))^2 + \text{Dis}_{\text{CT}}(\text{Syn}_{\text{CT}}(I_{\text{MR}}))^2$$

On  $\text{Dis}_{\text{MR}}$

$$\mathcal{L}_{\text{MR}} = (1 - \text{Dis}_{\text{MR}}(I_{\text{MR}}))^2 + \text{Dis}_{\text{MR}}(\text{Syn}_{\text{MR}}(I_{\text{CT}}))^2$$

On  $\text{Syn}_{\text{CT}}$  and on  $\text{Syn}_{\text{MR}}$

Forward cycle:

$$L_{\text{SynCT}} = -\text{Dis}_{\text{CT}}(\text{Syn}_{\text{CT}}(I_{\text{MR}}))^2$$

$$L_{\text{SynMR}} = -\text{Dis}_{\text{MR}}(\text{Syn}_{\text{MR}}(I_{\text{CT}}))^2$$

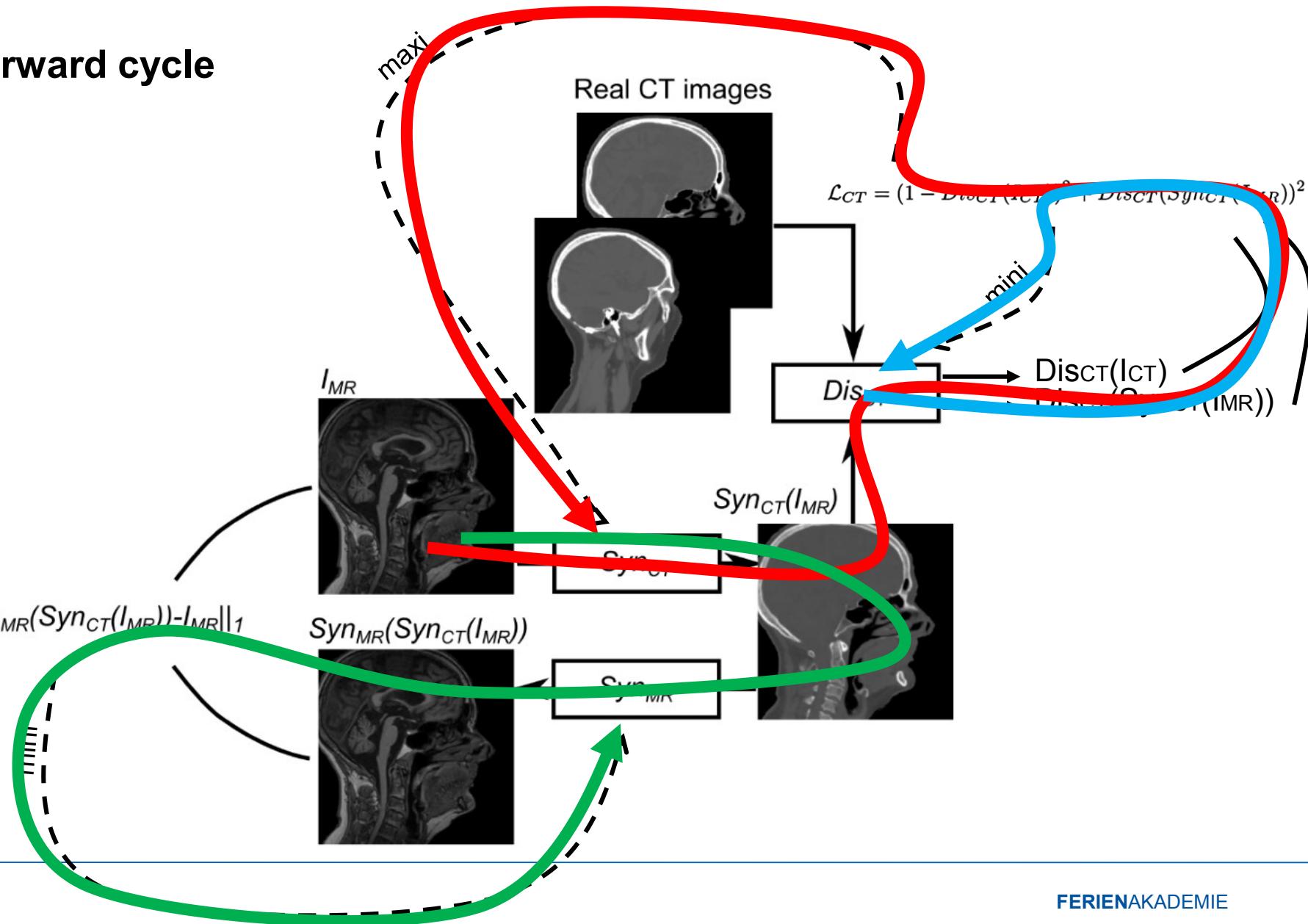
Backward cycle:

$$L_{\text{SynCT}} = -\lambda \|\text{Syn}_{\text{CT}}(\text{Syn}_{\text{MR}}(I_{\text{CT}})) - I_{\text{CT}}\|_1$$

$$L_{\text{SynMR}} = -\lambda \|\text{Syn}_{\text{MR}}(\text{Syn}_{\text{CT}}(I_{\text{MR}})) - I_{\text{MR}}\|_1$$

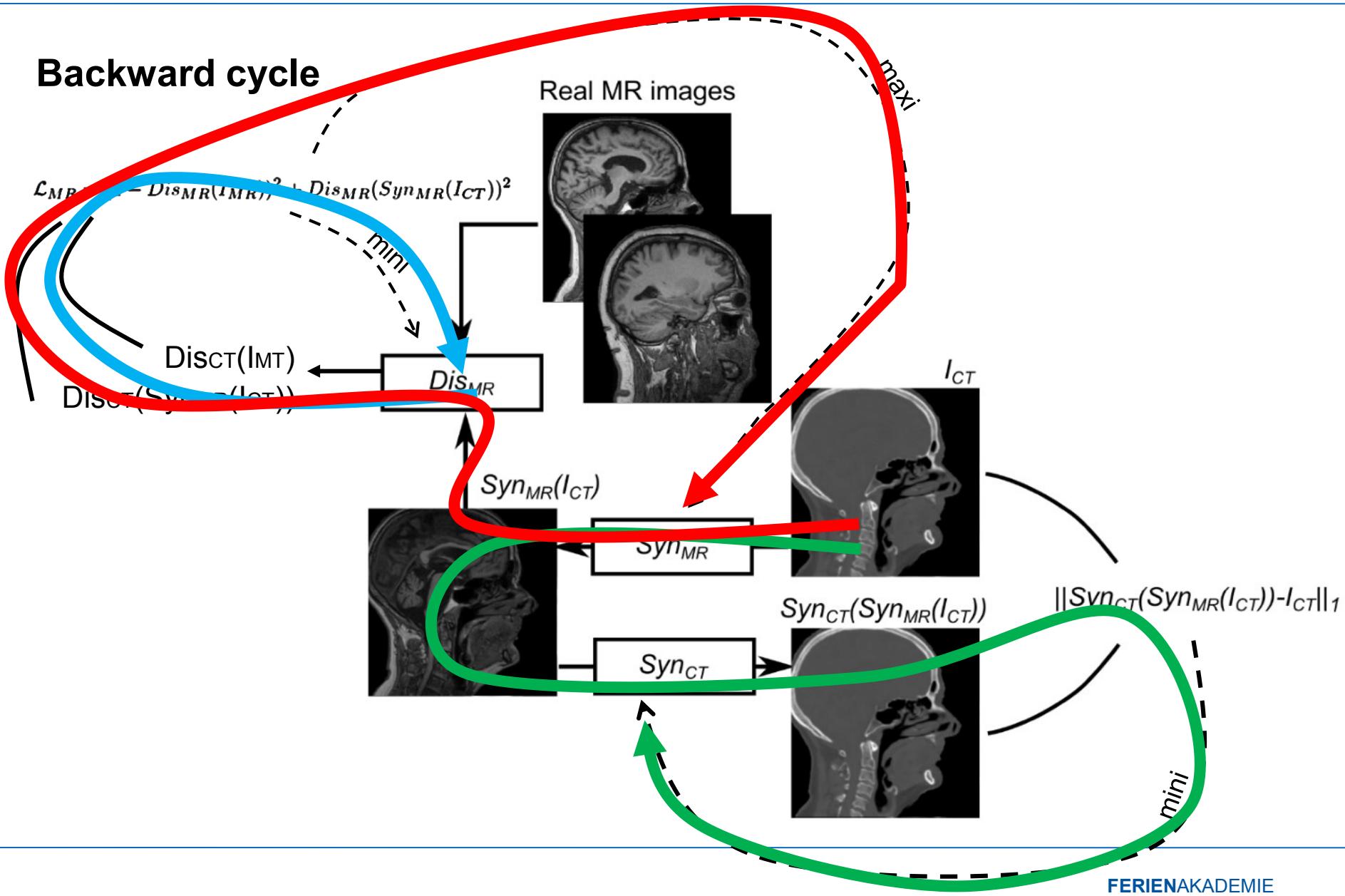
## 4.4. Deep MR to CT Synthesis using Unpaired Data

### Forward cycle



## 4.4. Deep MR to CT Synthesis using Unpaired Data

Backward cycle



### Evaluation

The mean absolute error

$$MAE = \frac{1}{N} \sum_{i=1}^N |I_{CT}(i) - Syn_{CT}(I_{MR}(i))|,$$

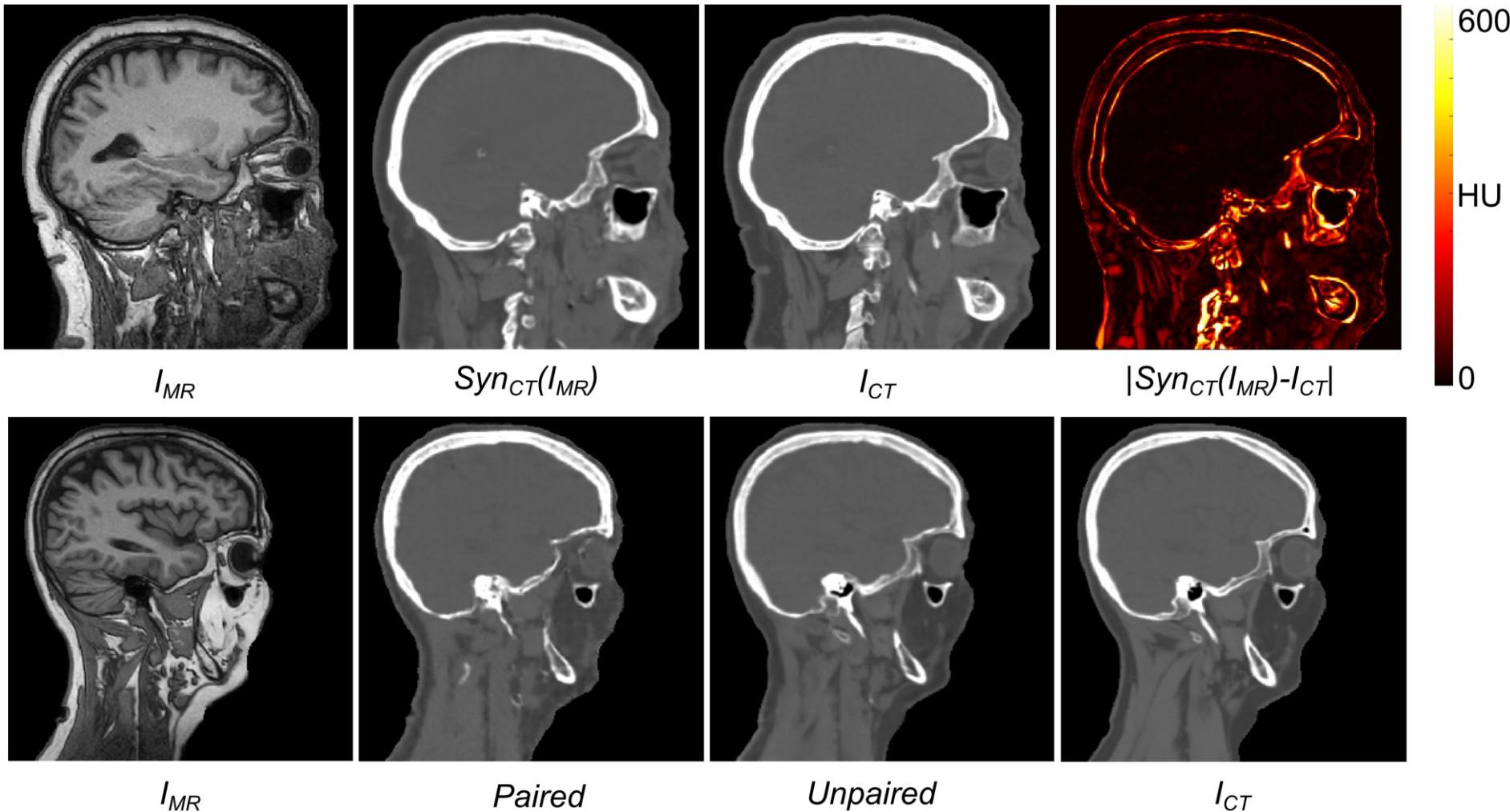
The peak-signal-to-noise-ratio

$$PSNR = 20 \log_{10} \frac{4095}{MSE},$$

$$MSE = \frac{1}{N} \sum_{i=1}^N (I_{CT}(i) - Syn_{CT}(I_{MR}(i))^2$$

## 4.4. Deep MR to CT Synthesis using Unpaired Data

### Results



## 4.4. Deep MR to CT Synthesis using Unpaired Data

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### Results

	MAE		PSNR	
	Unpaired	Paired	Unpaired	Paired
Patient 1	70.3	86.2	31.1	29.3
Patient 2	76.2	98.8	32.1	30.1
Patient 3	75.5	96.9	32.9	30.1
Patient 4	75.2	86.0	32.9	31.7
Patient 5	72.0	81.7	32.3	31.2
Patient 6	73.0	87.0	32.5	30.9
Average $\pm$ SD	$73.7 \pm 2.3$	$89.4 \pm 6.8$	$32.3 \pm 0.7$	$30.6 \pm 0.9$

## 4.4. Deep MR to CT Synthesis using Unpaired Data

### Notes

Tricks that help the network learning a generalization :

- Unpaired data (because the network was trained with random unpaired data).
- Images fed into the discriminator are randomly cropped : cancels the effects of rigid registration.

Limitation:

- using images of the same patients in the MR set and the CT set may affect training.

[1] Generative Adversarial Networks - Ian Goodfellow – Jun 2014 - arXiv:1406.2661

[2] Photographic Image Synthesis with Cascaded Refinement Networks - Qifeng Chen et al. - Jul 2017 - arXiv:1707.09405

[3] Deep MR to CT Synthesis using Unpaired Data - Jelmer M. Wolterink and Anna M. Dinkla and Mark H. F. Savenije and Peter R. Seevinck and Cornelis A. T. van den Berg and Ivana Isgum - Aug 2017 – arXiv:1708.01155

[4] Extended Modality Propagation: Image Synthesis of Pathological Cases. Cordier N, Delingette H, Le M, Ayache N. – Jul 2016 - IEEE Trans Med Imaging

[5] Lecture 13 | Generative Models - Stanford University School of Engineering

[6] Novel approach for generative modelling using capsule generative adversarial networks – Oussema Dhaouadi – BSc. Thesis – LDV & IN6 TUM