Exercise Sheet 3 - Fan-Beam Reconstruction

Andreas Maier, Jennifer Maier, Bastian Bier,
Alexander Preuhs, Christopher Syben

November 28, 2017

In this exercise, we implement a fan-beam filtered back-projection algorithm and test it with the phantom that we created in Exercise 1.

Goals:

- Create a fan-beam sinogram (variable detector size, spacing, number of projections, angular range, $\mathrm{d_{SI}}$, $\mathrm{d_{SD}}$)
- Rebin a short-scan to 180° parallel-beam coverage and reconstruct using the parallel beam backprojector.

1. **Fan-Beam Projection:** Implement a ray-driven fan-beam projector for a linear-detector and use it to create a "fan-o-gram", $g(t, \beta)$, of your phantom. Be aware that in this case the angular spacing is not equidistant. The opening angle of the fan is determined by the maximum detector size $t_{\max}$ and the source-to-detector distance $\mathrm{d_{SD}}$. Remember that the projections have to be generated using a certain angular range that is provided to the projection function.
   The following parameters are needed:

   a) the detector spacing,
   b) the number of detector elements,
   c) the increment of the rotation angle β,
   d) the number of projections,
   e) the source-to-isocenter distance $\mathrm{d_{SI}}$, and
   f) the source-to-detector distance $\mathrm{d_{SD}}$.

   In contrast to parallel geometry, the last parameters have a methodical impact on the result. Ensure that during rotation the source and the detector will not hit the phantom.

   Further, note that the angular scan range is defined by your geometry! Determine the minimum fan-beam scan range and create a minimum "fan-o-gram" from your phantom. Recall that **each point** of your phantom needs to have at least 180° coverage. This is known as **Short Scan**.

2. **Rebinning:** Recall the rebinning formulation presented during the lecture. Implement the rebinning and use it to convert your "fan-o-gram", $g(t, \beta)$, into a parallel sinogram, $p(s, \theta)$. Remember that interpolation is required in order to read out the values from the "fan-o-gram". You can make use of the `InterpolationOperators` class to compute a bilinear interpolation.

   Hint: In a full 360° "fan-o-gram" each ray has been sampled twice, building a pair of two values that need to be the same. If your lookup in the short-scan "fan-o-gram" goes out of range, try to redirect the lookup to the corresponding sample in the range.

3. **Filtered Back-Projection (FBP):** Use the parallel sinogram that you generated in the previous exercise to reconstruct your phantom. Use the filtered back-projection algorithm that you implemented in Exercise 2.

◇ **Fan-Beam Reconstruction (optional):** Implement a fan-beam back-projector and cosine weights for a fan-beam filtered back-projection. Which differences do you observe compared to the rebinning approach?

Think you are done? Checklist:

☐ Fan-o-gram created (have you varied the parameters?)

☐ Rebinned short scan to 180 degrees

☐ Backprojected the rebinned fan-o-gram

☐ Validated by a supervisor