Exercises for
Pattern Analysis
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Assignment 9, 07.07.2015

General Information:

Lecture (3 SWS):	Thu 14.15 – 15.45 (H16) and Tue 12.15 – 13.45 (H16)
Exercises (1 SWS):	Mo 12.15 – 13.45 (02.134-113) and Tue 12.15 – 13.45 (E1.12)
Certificate:	Oral exam at the end of the semester
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Hidden Markov Models

Exercise 1 Name the three central problems that can be solved with the help of Hidden Markov Models (HMMs)? Describe each issue and explain how it can be solved.

Exercise 2 Instruments are tracked during a minimally-invasive surgery. In total, four different objects can be tracked. Depending which objects are visible during the procedure, the surgery is in a different state. A Hidden Markov Model (HMM) can be used to model this.
Given an HMM with four hidden states and five visible symbols $v_0...v_4 \in V$, as well as the transition probabilities from state S_i to $S_j \in S$ are given by

$$\boldsymbol{A} = (a_{ij}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.8 & 0.1 & 0 & 0.1 \end{pmatrix}$$

and the output probabilities for symbol k at state S_j by

$$\boldsymbol{B} = (b_{jk}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0.1 & 0.7 & 0.1 \\ 0 & 0.5 & 0.2 & 0.1 & 0.2 \end{pmatrix}.$$

The priors for the initial state are given by

$$\boldsymbol{\pi} = (0.25, 0.25, 0.25, 0.25)^T .$$

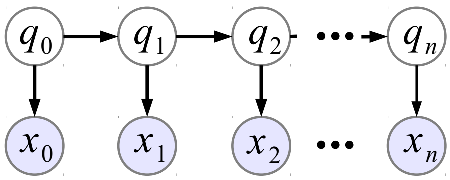
(a) Draw the HMM.

(b) What is the probability that this HMM generates a sequence S_1, S_3, S_2, S_0?

(c) What is the probability that this HMM generates a sequence v_1, v_3?

Exercise 3 The evaluation of an HMM can be done using the forward or the backward algorithm. Suppose an HMM transitioned through a sequence $q_0, ..., q_n \in \mathbf{S}$ of hidden states and produced the sequence $x_0, ..., x_n \in \mathbf{V}$ of observed variables from a set of observable events.

(a) Derive the forward algorithm to compute $p(q_k, x_0...x_k)$, which is the joint probability of observing the sequence $x_0, ..., x_k$ and reaching hidden state q_k.



Hint: Express $p(q_k, x_0...x_k)$ by the emission probability of x_k and the transition probability from $q_k - 1$ to q_k to find a recursive formulation.

(b) Write down the forward algorithms in pseudocode.