

## Exercise 1: Singular Value Decomposition (SVD) and Fourier Transform (FT)

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### 1 Singular Value Decomposition (SVD)

Have a look at the slides of the topic SVD

Create a matrix  $A = \begin{pmatrix} 11 & 10 & 14 \\ 12 & 11 & -13 \\ 14 & 13 & -66 \end{pmatrix}$ . Check the determinant of this matrix. Compute

the inverse matrix of  $A$  without using the command **A.inverse()**. Compare the result to **A.inverse()**. How do we get the condition number? What does the condition number express?

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If we set the threshold  $\epsilon = 10^{-3}$ , we get a rank deficiency. How can we get the nullspace and the range of the matrix  $B$ ?

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#### 1.1 Exercise problem

Show that a variation of the elements of  $b$  by 0.1% implies a change in  $x$  by 240%.

Consider the matrix  $A$ , which is non-singular. The equation  $Ax = b$ , where  $b = \begin{pmatrix} 1.001 \\ 0.999 \\ 1.001 \end{pmatrix}$  has the solution  $x = A^{-1}b$ .

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#### 1.2 Optimization Problems

- Implement the optimization problems 1 and 4 of the lecture slides.
- Optimization problem 2: Four 2-D vectors were given on the lecture slides. Implement the optimization problem for the general case, e.g. 5, 6, 20 or  $N$  vectors.
- Implement the third optimization problem using the image `yu_fill.jpg`. How many approximations do we have? Which rank- $l$ -approximations are sufficient?

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The rank approximation should look like this:

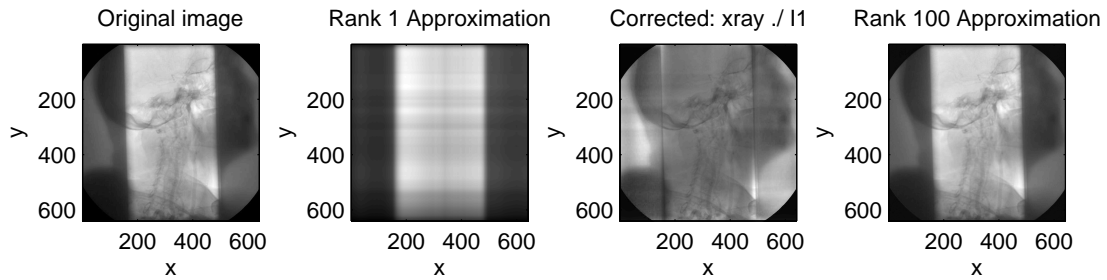


Figure 1: Rank approximation of image `yu_fill.jpg`.

## 2 Fourier Transform (FT)

Load a phantom image into your workspace. Compute the Fourier Transform. There are some possibilities of visualization. What's the difference?

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