# DMIP - Exercise:

*Defect Pixel Interpolation*

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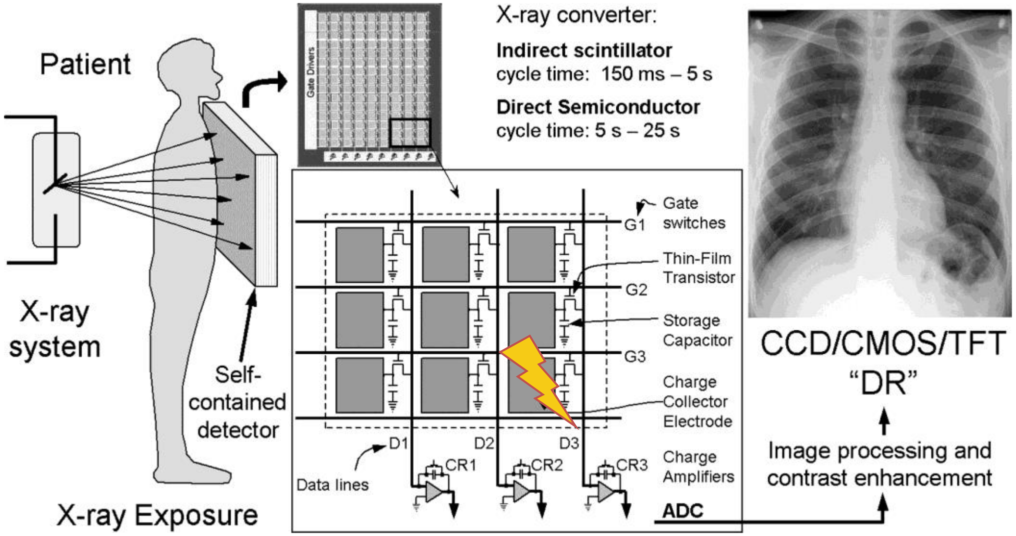
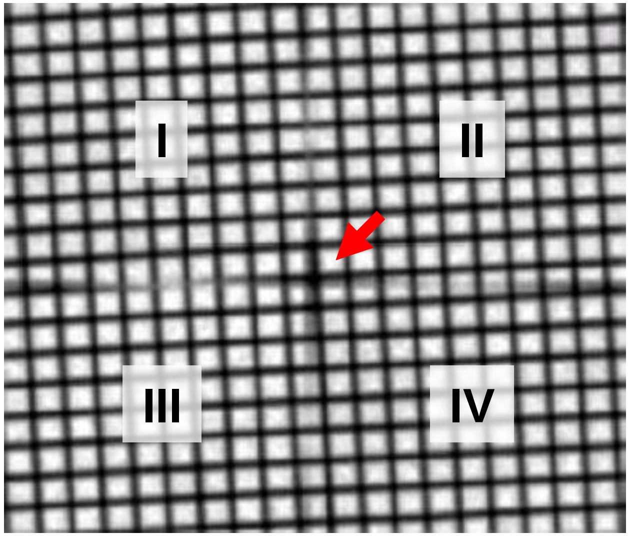
# Flat Panel Detectors

## Defect Pixels in Flat Panel Detectors

- Defect pixels are caused by defect detector cells.
- Small detectors are composed to generate a large one, which leads to butting cross effects.




Artifacts due to inactive pixels or rows




Butting Cross Artifact

Image taken from: Samei E et al. Radiographics 2004;24:313-334

Image taken from: Lecture DMIP (Maier, Hornegger)

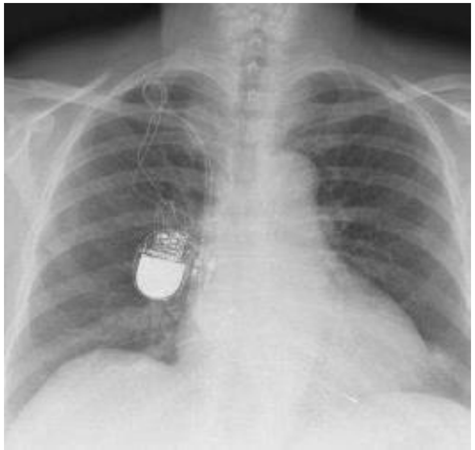
# Flat Panel Detectors

## Defect Pixels in Flat Panel Detectors

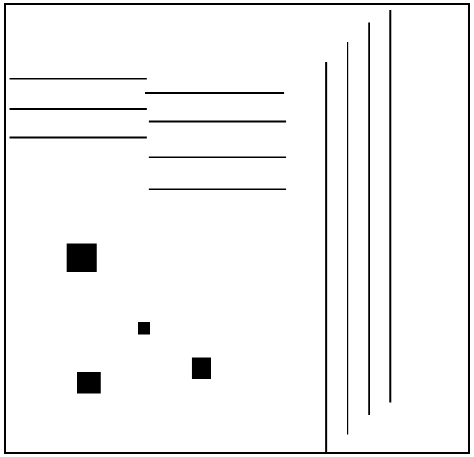
- Let $f_{i,j}$ denote the intensity value at grid point (i,j) of the **ideal image** f that has no defect pixels.
- Let $w_{i,j}$ denote the indicator value at (i,j) where w is **mask image** that indicates defect and uncorrupted pixels:

$$w_{i,j} = \begin{cases} 0 & \text{if pixel is defect} \\ 1 & \text{otherwise} \end{cases}$$

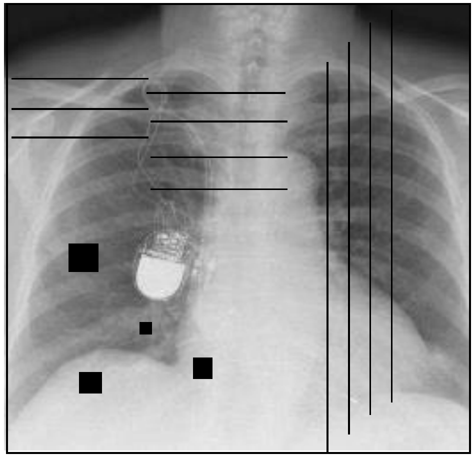
- Let $g_{i,j}$ denote the intensity value at grid point (i,j) of the **observed image** g that is acquired with the flat panel detector that has defect pixels.



Ideal image f



Mask image w



Observed image g

# Flat Panel Detectors

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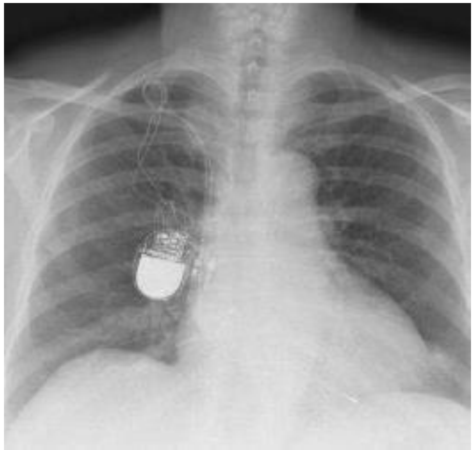
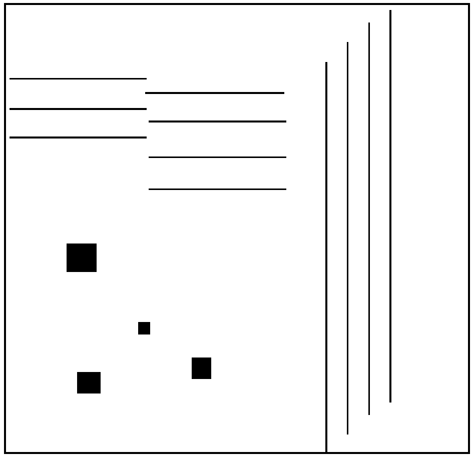


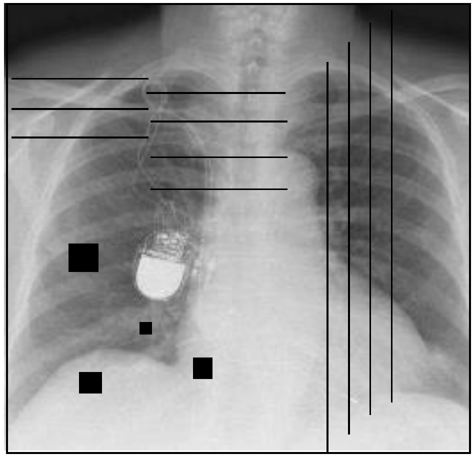
Ideal image f

$\bullet$



Mask image w

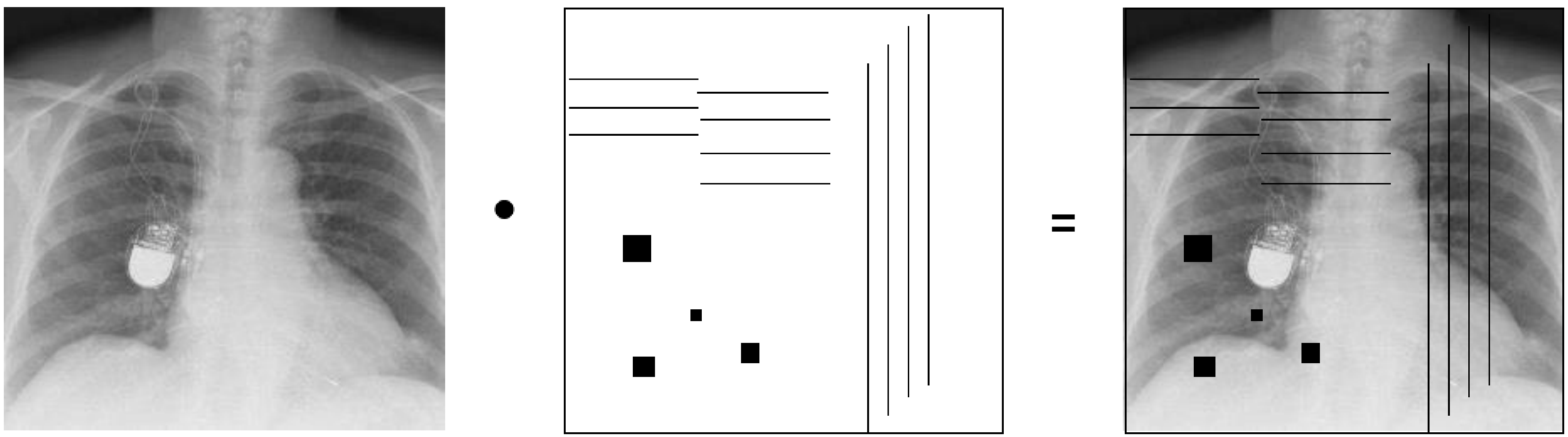
$=$



Observed image g

# Flat Panel Detectors

## Defect Pixels in Flat Panel Detectors



Ideal image f     Mask image w     Observed image g

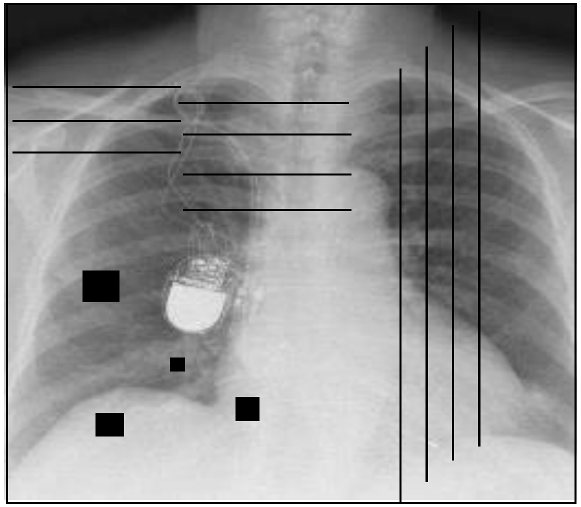
- Defect pixel problem: $f(n) \cdot w(n) = g(n)$
- Goal: Find $f(n)$, given the observed image $g(n)$ and the defect mask $w(n)$

# Defect Pixel Correction

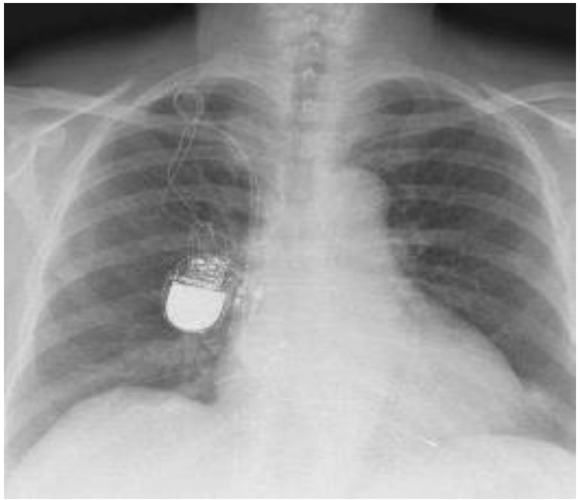
# Defect Pixel Correction

## Problem Statement

- Restore the **ideal image** based on the **observed image** and the known defect **pixel mask**.
- Defect pixel correction:
  - *Spatial Domain:* Interpolation
  - *Frequency Domain:* Band Limitation
  - *Frequency Domain:* Iterative Deconvolution



Observed image g



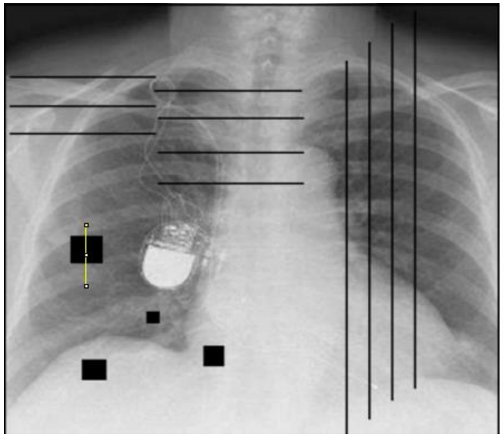
Ideal image f

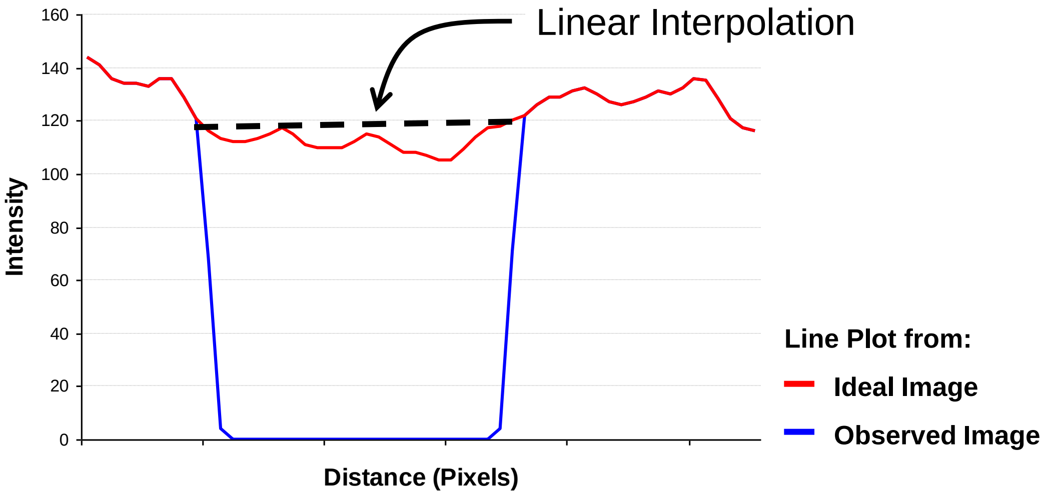
# Defect Pixel Correction

## Defect Pixel Correction by Spatial Interpolation

- Interpolate between active pixels to recover the inactive ones:
  - Bilinear interpolation
  - Median

- **However, this is only suitable for small defect areas!**





# Defect Pixel Correction

## Fourier Transform Revisited

- Convolution theorem
- Symmetry property of Fourier transform of real signals

# Defect Pixel Correction

## Fourier Transform Revisited

- **Convolution theorem**

$$
\begin{aligned}
FT(f \star h)(\xi) &= \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} f(k) h(n-k) e^{\frac{-2\pi i n \xi}{N}} \\
&= \sum_{k=0}^{N-1} f(k) \sum_{n=0}^{N-1} h(n-k) e^{\frac{-2\pi i n \xi}{N}} \\
&= \sum_{k=0}^{N-1} f(k) e^{\frac{-2\pi i k \xi}{N}} H(\xi) = F(\xi) H(\xi) = G(\xi)
\end{aligned}
$$

  The **convolution** of two signals in the **time domain**, corresponds to a **multiplication** in the **frequency domain**.

- **Symmetry property of Fourier transform of real signals**

# Defect Pixel Correction

## Fourier Transform Revisited

- **Convolution theorem**
- **Symmetry property of Fourier transform of real signals**
  If f(n) is a real valued discrete signal of length N, the Fourier transform F(ξ) fulfills the symmetry property:

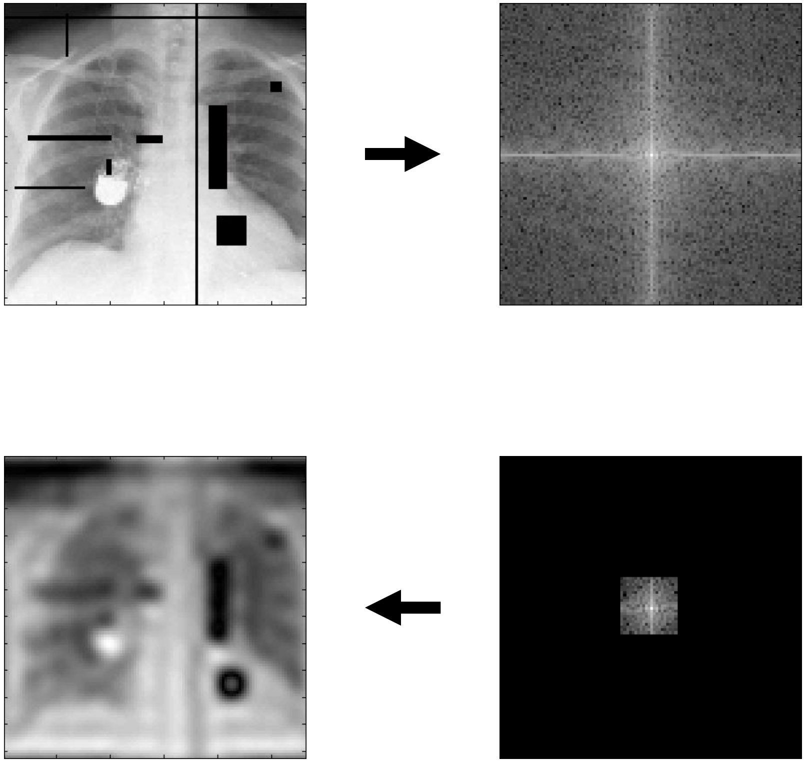
$$F(\xi) = F^*(N - \xi)$$

where ”* “ denotes the conjugate complex.

# Defect Pixel Correction

## Iterative Band Limitation

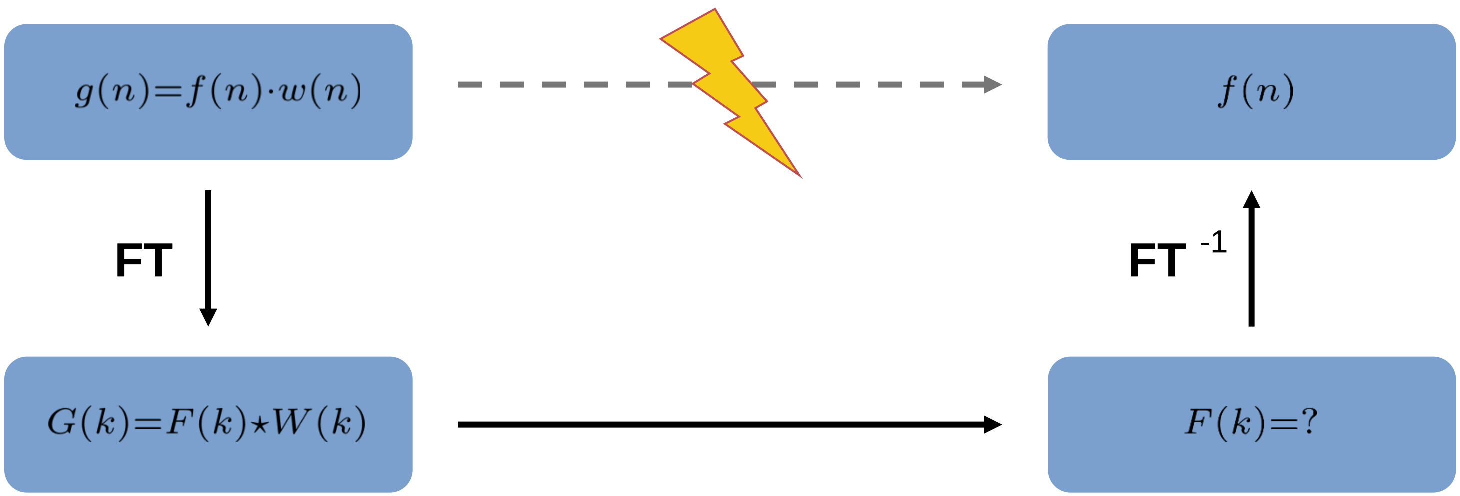
1. Fourier transform
2. Cut off high frequencies
3. Inverse Fourier transform
4. Replace only defect areas
5. Repeat from 1.



# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- Application of Fourier convolution theorem:




$$G(k) = \frac{1}{N} F(k) \star W(k) = \frac{1}{N} \sum_{l=0}^{N-1} F(l) \cdot W(k-l), \qquad 0 \leq n, k < N$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

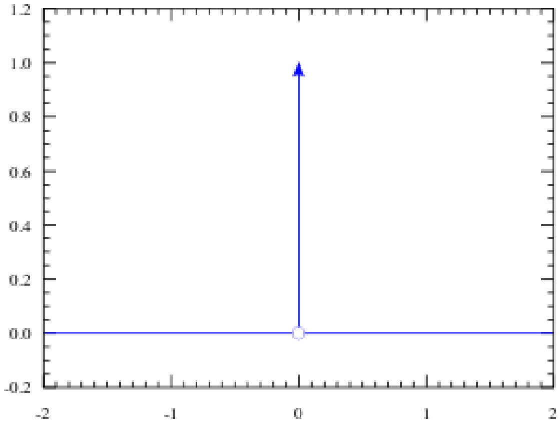
- **Application of Fourier symmetry properties**:

- Use Dirac's $\delta$- function to select a line pair F(s) and F(N-s):

$$\hat{F}(k) = \hat{F}(s)\delta(k-s) + \hat{F}(N-s)\delta(k-N+s)$$

where $\hat{F}$ denotes an estimate of F, and $\delta$- function is defined by:

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$




Dirac $\delta$-function

Image taken from: Wikipedia

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- After frequency selection, we convolve with the mask

$$\hat{G}(k) = \frac{1}{N}\hat{F}(k) \star W(k) = \frac{1}{N}\sum_{l=0}^{N-1} \hat{F}(l) \cdot W(k-l)$$

# Defect Pixel Correction

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$$\hat{G}(k) = \frac{1}{N}\left(\hat{F}(s)W(k-s) + \hat{F}(N-s)W\left(k-(N-s)\right)\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

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- We are only interested in $\hat{G}(k)$ at position $k = s$

# Defect Pixel Correction

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$$\hat{G}(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W\left(2s-N\right)\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

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# Defect Pixel Correction

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- We are only interested in $\hat{G}(k)$ at position $k = s$

$$\hat{G}(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W\left(2s\right)\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- **Application of Fourier symmetry properties**:
  Select a line pair G(s) and G(N-s) of the Fourier transform of the observed image. The observed image is described by a convolution of the ideal image and the known defect pixel mask:

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s)\right)$$

- And for the conjugate complex:

$$G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)$$

- Using these two equations, we can compute:

$$\hat{F}(s) = N\frac{G(s)W^*(0) - G^*(s)W(2s)}{|W(0)|^2 - |W(2s)|^2}$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- Special case without symmetry property (for s=0 and s = N/2)

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s)\right)$$

$$G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- Special case without symmetry property (for s=0 and s = N/2)

$$G(s) = \frac{1}{N}\left(\hat{F}(s)W(0) ~~+ \hat{F}^*(s)W(2s)~~\right)$$

$$~~G^*(s) = \frac{1}{N}\left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s)\right)~~$$

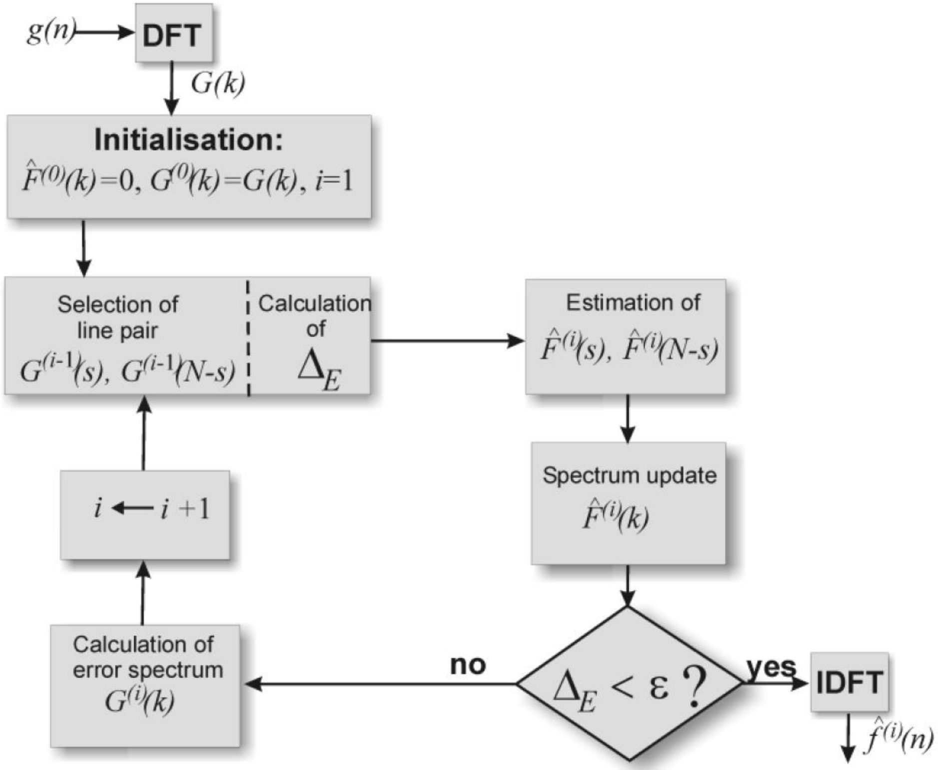
↓

$$\hat{F}(s) = N\left(\frac{G(s)}{W(0)}\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

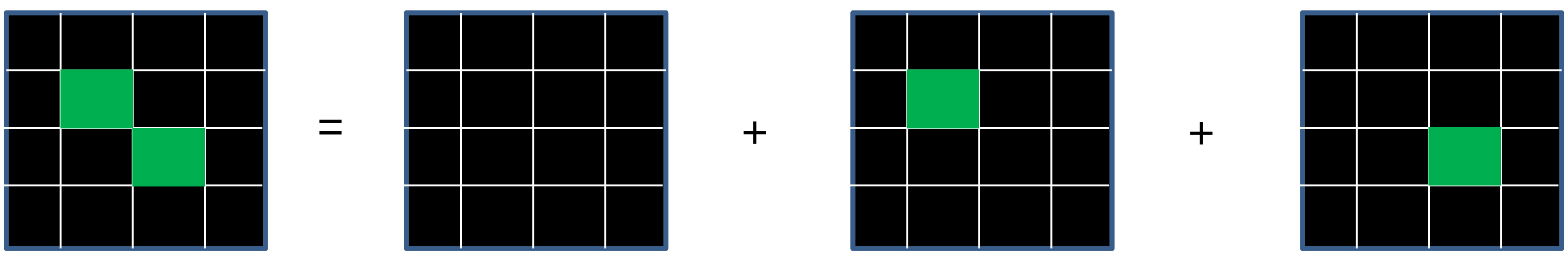
- So far only correction at line pair
  - ➔ Iterative correction of line pairs



# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- So far we only found estimates for a single or a pair of selected lines
- We also need to update the global estimate of the spectrum after each linepair computation




$$\hat{F}^{(1)}(k) \qquad \hat{F}^{(0)}(k) = 0 \qquad \hat{F}(s^{(1)}) \qquad \hat{F}(N - s^{(1)}) = \hat{F}^{*}(s^{(1)})$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- **How can we update the error spectrum G?**

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N}\left(\hat{F}(k) * W(k)\right)$$

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- **How can we update the error spectrum G?**

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N}\left(\hat{F}(k) * W(k)\right)$$

- Just subtract the new estimate from the previous estimate
- Requires convolution!!! → Same complexity as FFT?
- F(k) changed only at two positions → Convolution much easier to compute

# Defect Pixel Correction

## Defect Pixel Correction by Symmetry Properties

- **How to select the line pairs each iteration?**

- Just select the maximum of the error spectrum

$$s^{(i)} = \underset{\hat{s}^{(i)}}{\operatorname{argmax}} \; G^{(i)}(\hat{s}^{(i)})$$

- Detailed derivation can be found in the paper.
- It is based on Parseval's theorem.
- Approach minimizes the mean square error (MSE) in valid areas:

$$\text{MSE} = \sum_{n=0}^{N} (g(n) - \hat{f}(n)w(n))^2$$