Image Formation

Lens Optics, Photometry, Geometric Optics

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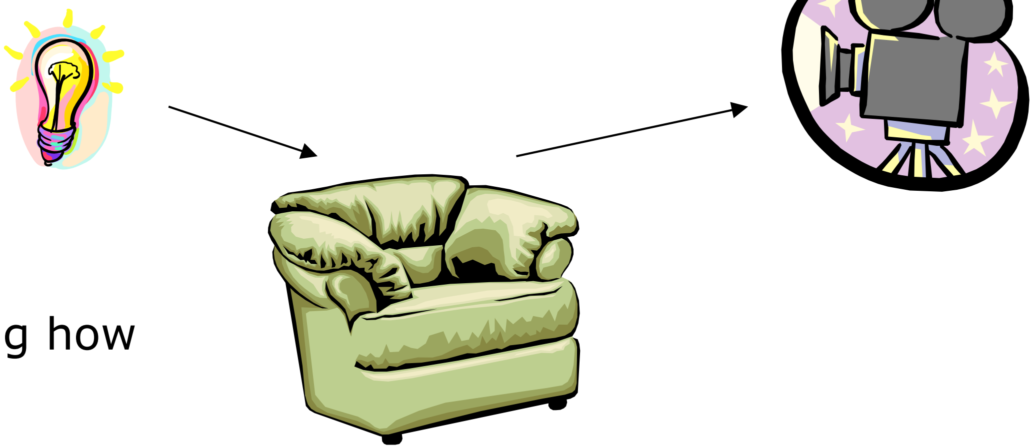
Image Formation

- There are three major components that determine the appearance of an image
  - Geometry
    - Geometry of the scene
    - Geometry of the projection to the camera
  - Optical properties
    - Optical properties of the materials in the scene
    - Optical properties of the sensor
  - Illumination conditions



Note: At this point we are ingnoring how the sensor itself records the data.

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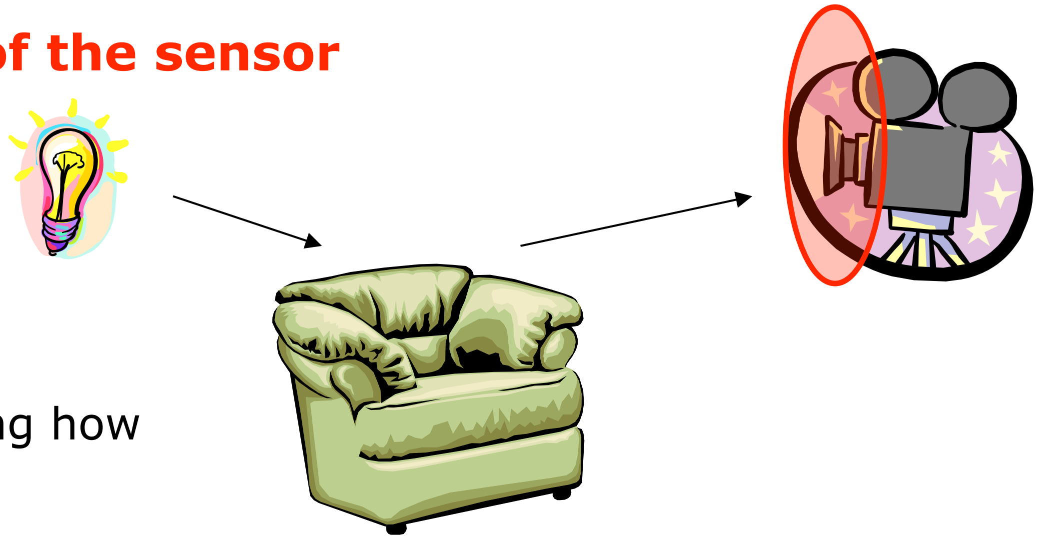

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Pinhole Camera

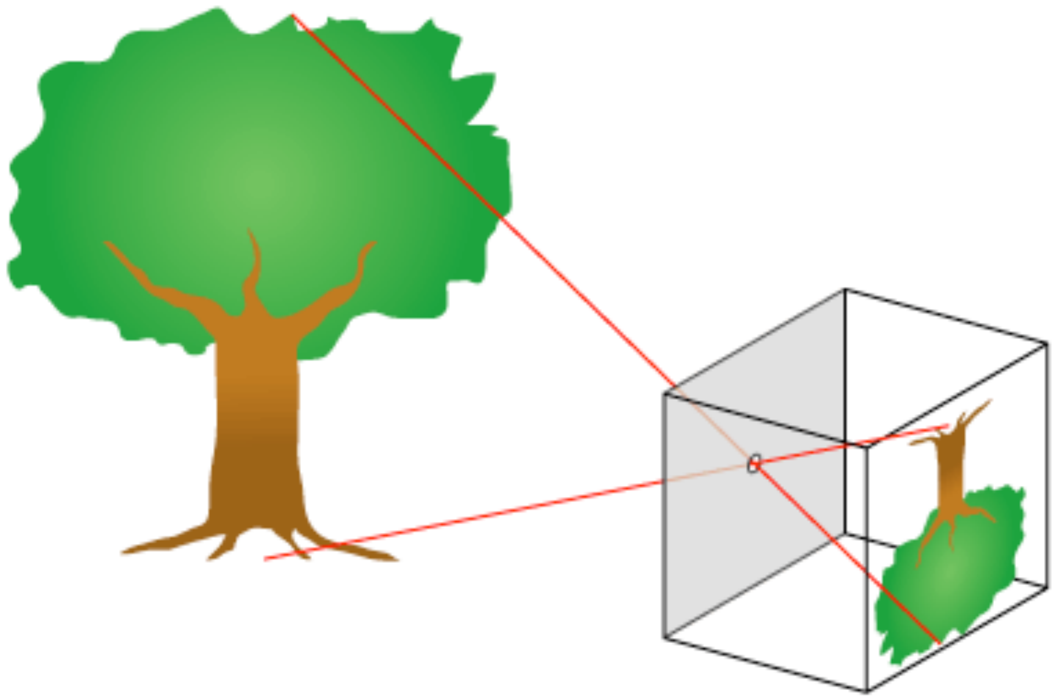


Image courtesy of wikipedia, http://upload.wikimedia.org/wikipedia/commons/8/81/Pinhole-camera.png

Pinhole + Lens

- Pinhole cameras can produce very crisp images of stationary scenes.
- They require long exposure time, since all the incoming light has to go through a single hole.

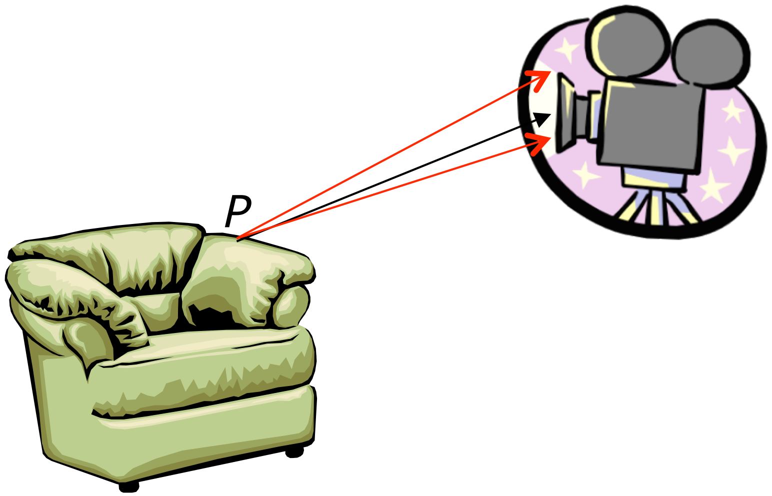
- Solution: Open up the hole ⇒ blurred images

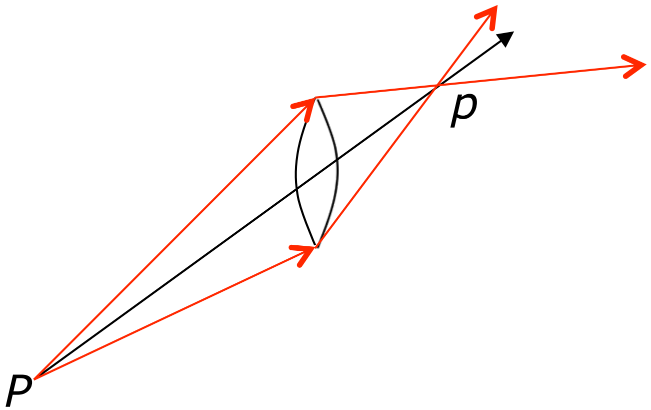
- Solution: Use a lens to focus the light rays
  ⇒ crisp, bright image obtained at shorter exposure times

Lens

From a point *P* on the scene, a cone of rays reaches the lens and then converges to a point *p* on the other side of the lens.

Where does the point *p* lie?





Thin (Convex-Convex) Lens

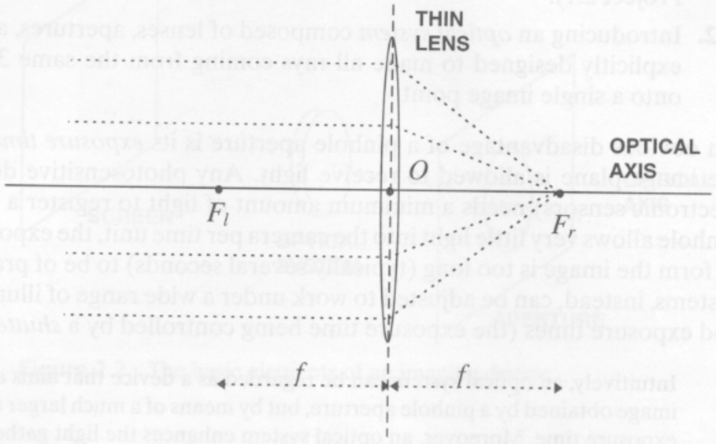


Figure 2.3 Geometric optics of a thin lens (a perpendicular view to the plane approximating the lens).

Center of projection (COP), O: center of lens

Optic axis: axis perpendicular to the lens that passes through the COP.

Focus of the lens, F_r: point on the optic axis where all the parallel rays incident on the lens converge.

Focal length, f: the perpendicular distance between the lens and the focus point of the lens F_r .

Imaging with Thin Lens

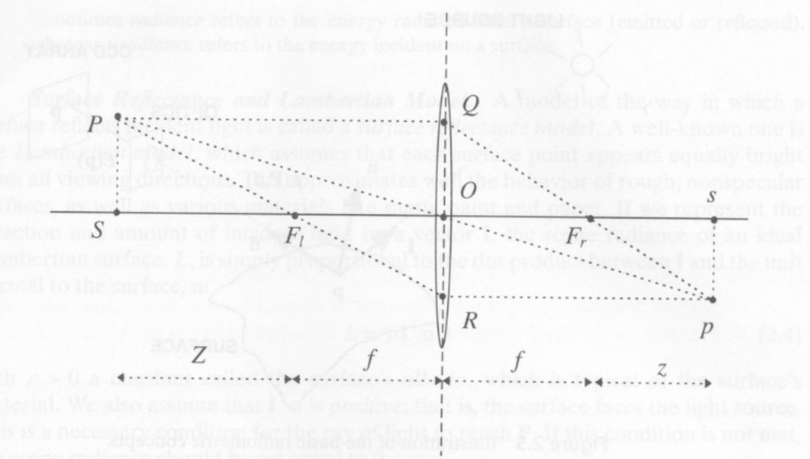


Figure 2.4 Imaging by a thin lens. Notice that, in general, a real lens has two different focal lengths, because the curvatures of its two surfaces may be different. The situation depicted here is a special case, but it is sufficient for our purposes. See the Further Readings at the end of this chapter for more on optics.

From the similar triangles $P\ F_l\ S$ and $R\ F_l\ O$ => $\frac{Z}{f} = \frac{\overline{PS}}{\overline{OR}}$

and the similar triangles $p\ F_r\ s$ and $Q\ F_r\ O$ => $\frac{z}{f} = \frac{\overline{sp}}{\overline{QO}}$

But $\overline{PS} = \overline{QO}$ and $\overline{OR} = \overline{sp}$ => $Zz = f^2$

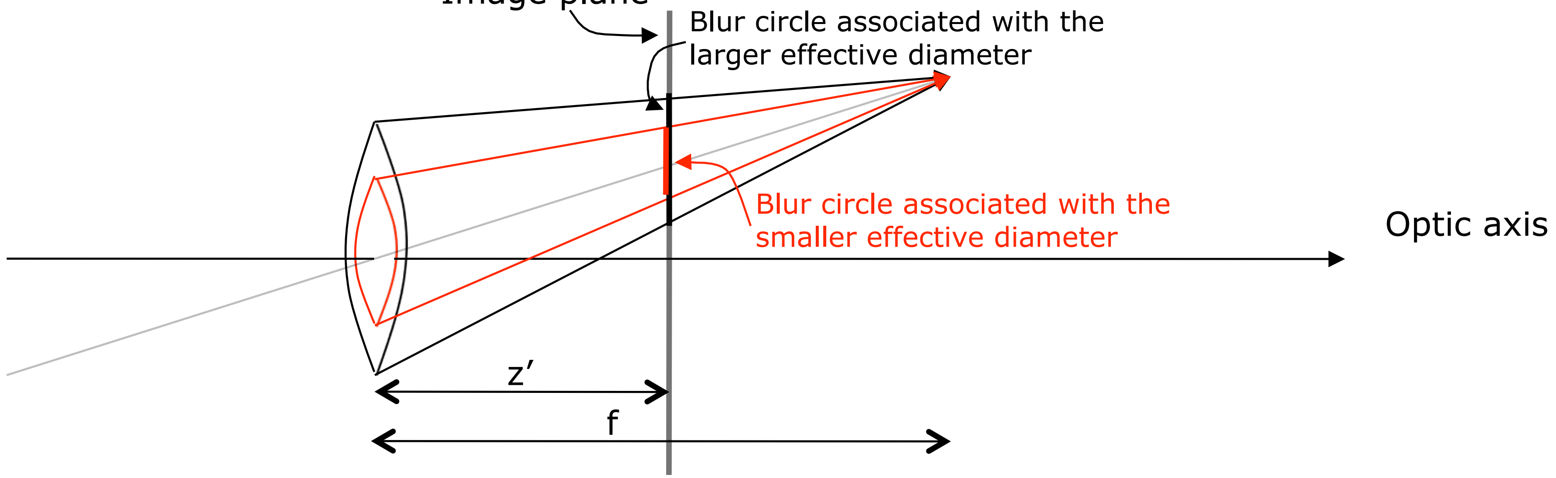
Let $\hat{Z} = Z + f$ and $\hat{z} = z + f$. Then $\boxed{\frac{1}{f} = \frac{1}{\hat{z}} + \frac{1}{\hat{Z}}}$ ⇐ Thin Lens Law

More on Thin Lenses

- As $\hat{Z} \rightarrow \infty \Rightarrow \hat{z} \rightarrow f$
- We assume that $\hat{z} = f$
- Effective diameter of a lens, *d*: diameter of the lens reachable by light rays (an adjustable iris changes the effective diameter of a lens).
- Field of view, *w*: the half-angle subtended by the effective lens diameter, *d*, on the focus, *F*:

$$w = \tan^{-1} \frac{d}{2f}$$

Image Blur



- Depending on the distance *Z* of a point *P* in the scene, the focus point *p* may not lie on the image plane.
- Thus, a <u>blur circle</u> instead of a point is formed on the image plane.
- The diameter, *b*, of the blur circle depends on the effective lens diameter, *d*.

$$b = \frac{d}{f}(f - z')$$

Depth of Field

- The depth of field is the distance between the nearest and farthest points in the scene that appear in focus. It is defined as the difference between the far and near planes: $\hat{Z}_{DOF} = \hat{Z}_f - \hat{Z}_n$
- The far plane is the plane parallel to the lens plane that is farthest away from the COP and still has an imperceptible blur circle. Assume it is located at distance $\hat{Z}_f$.

$$\hat{Z}_f = \frac{\tilde{Z}f(d-\tilde{b})}{df - \tilde{b}\tilde{Z}}$$

where $\tilde{Z}$ is the scene distance at which we want to focus and $\tilde{b}$ is the diameter of the tolerable, or imperceptible blur circle.

- The near plane is the plane parallel to the lens plane that is the closest possible to the COP and still has an imperceptible blur circle. Assume it is located at distance $\hat{Z}_n$.

$$\hat{Z}_n = \frac{\tilde{Z}f(d+\tilde{b})}{df + \tilde{b}\tilde{Z}}$$

Depth of Field – continued

- Depth of field: range of depth values that appear in-focus in an image.
- F-stop number: the ratio of the focal length over the effective diameter f/d
- Recall that the radius of the blur disk, *b* is given by

$$b = \frac{d}{f}(f - z')$$

- Dilemma:
  - Large F-stop (i.e. small effective diameter)
    => crisp but dim image (weak signal)
  - Small F-stop (i.e. large effective diameter)
    => more blurred regions but bright image (strong signal)

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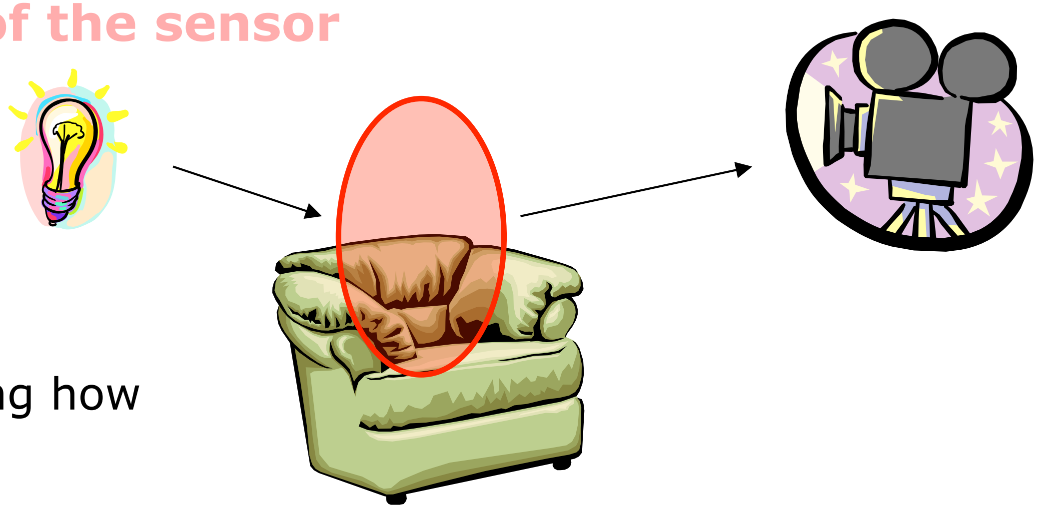

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Light-surface-camera

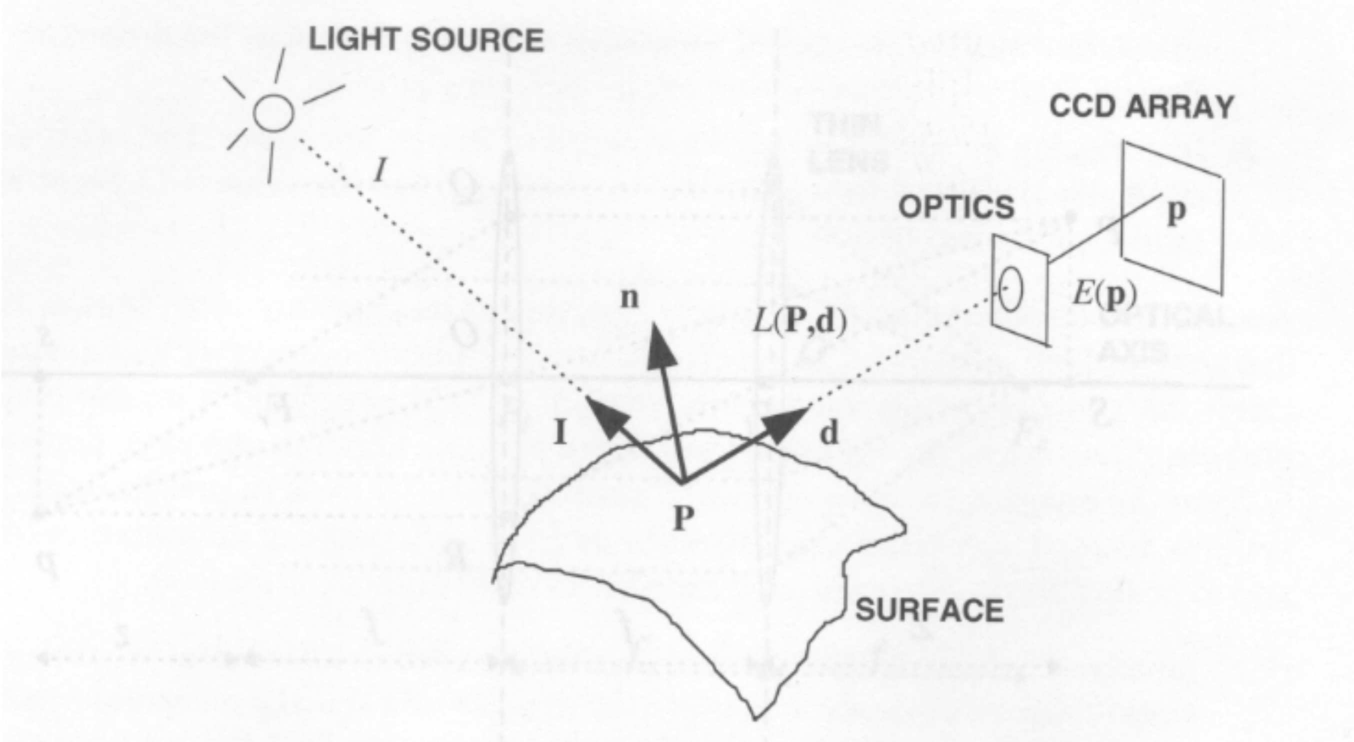


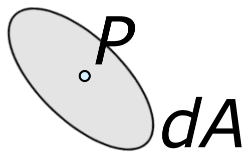
Figure 2.5 Illustration of the basic radiometric concepts.

$$I(p) \propto E(p) \propto L(P)$$

where $I(p)$ is the camera response at pixel p, $E(p)$ is the amount of light that falls on pixel ***p***, and $L(P)$ is related to how bright is the scene point ***P*** that corresponds to pixel ***p***.

Radiometry

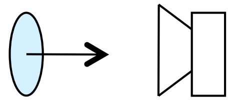
- Informal terms like "brightness" and "amount of light" need to be accurately defined.
- Light is a form of Electromagnetic Energy.
- Radiometry is a field of Physics that measures light. We will use metrics and measurement units established in radiometry.
- In all the radiometric discussions we will always consider an infinitesimal area patch *dA* centered around a point ***P***.



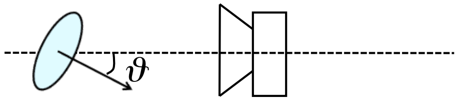
$$dA \rightarrow 0$$

Foreshortening

- A patch *dA* can have different orientations with respect to the image plane.
  - Parallel



  - Tilted

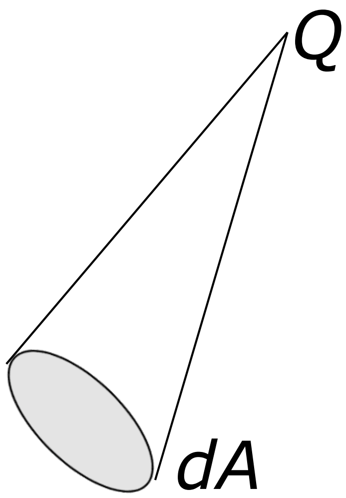


Patch appears smaller to the viewer, i.e. **foreshortened**

- Foreshortening affects
  - How bright a patch appears to the viewer, $E(p)$
  - How much light falls on a patch and hence how bright it is, $L(P)$
- A patch with area *dA* whose surface normal forms an angle ϑ with the viewing direction (optic axis) has a **foreshortened area** of $dA\cos\vartheta$

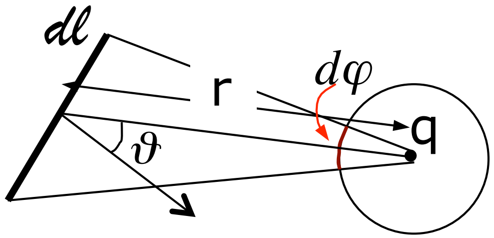
3D Angle (Solid Angle)

- So far we have talked about the patch *dA*.
- The patch *dA* is illuminated by a point light source, positioned at *Q*.
- We need to consider the light that travels through the cone with its tip at *Q* and its base at *dA*.
- We will refer to the shape of this cone as the solid angle.



2D Angle

- The angle **subtended** by the line dl at a point q.



- Derivation:
  - Draw a unit circle centered at q
  - Project dl on the perimeter of that circle.
  - The length of that projection is the angle in radians. It depends on:
    - The angle ϑ between the normal to line dl and the radius that reaches the center of dl.
    - The distance r between the center of the line dl and the center of the circle q.

$$d\varphi = \frac{dl \cos\vartheta}{r}$$

Solid Angle

- The solid angle **subtended** by the patch dA at a point q.

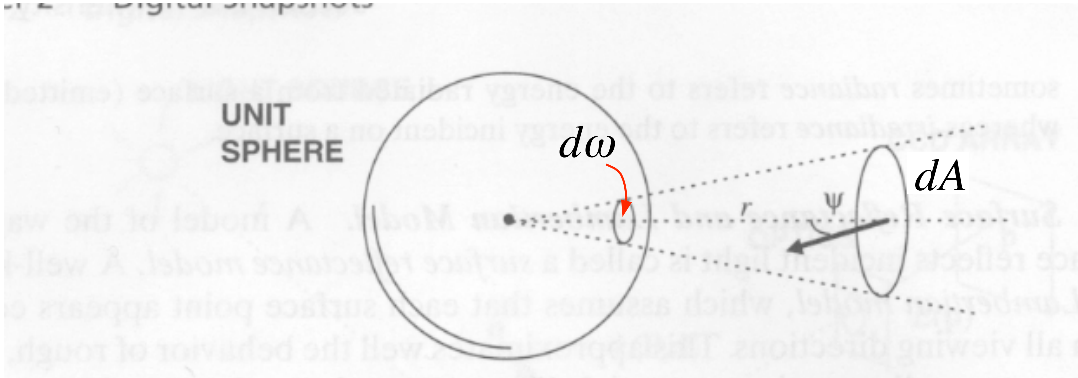


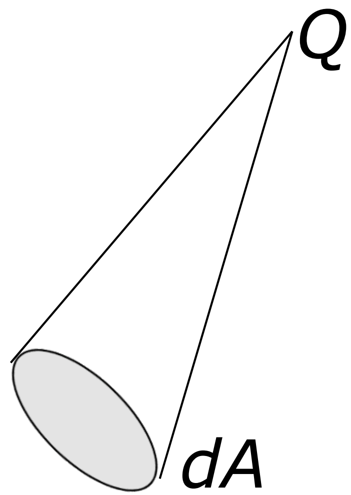
Figure 2.6 The definition of solid angle.

- Derivation:
  - Draw a unit sphere centered at q
  - Project dA on the perimeter of that sphere.
  - The area of that projection is the angle in steradians. It depends on:
    - The angle ψ between the normal to the patch dA and the radius that reaches the center of dA.
    - The distance r between the center of the patch dA and the center of the circle q.

$$d\omega = \frac{dA \cos\psi}{r^2}$$

Light Measurement

- Light: Electromagnetic energy, Q
- Flux: Power carried by the EM radiation, $P = \frac{dQ}{dt}$
- Intensity: Power of light traveling in a specific direction $I = \frac{dP}{d\omega}$

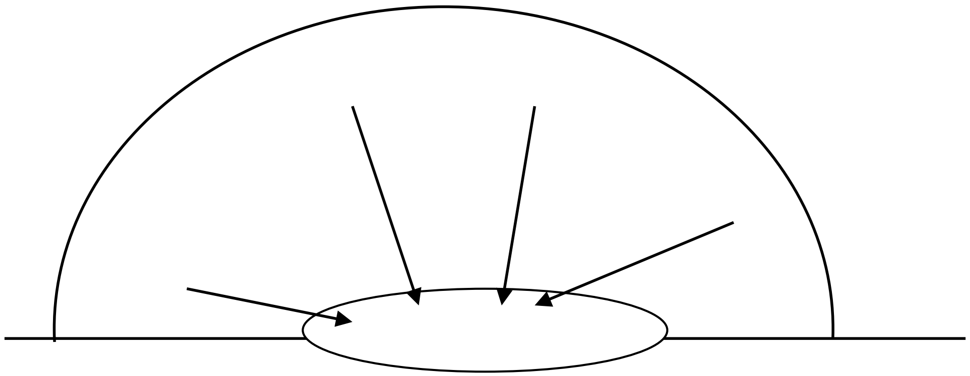


Irradiance

- Irradiance: power of light falling on a surface patch

$$E = \frac{dP}{dA}$$

- Measured in W/m^2
- It's a measure of concentration of power.
- It is independent of direction (direction is **irr**elevant)

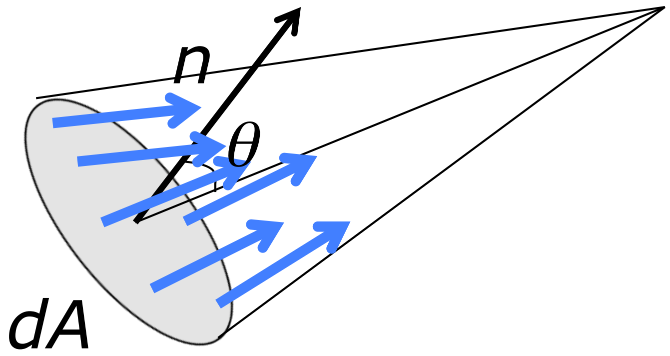


Radiance

- Radiance: power of light falling on a surface patch from a specific direction

$$L = \frac{d^2 P}{d\omega dA \cos\vartheta}$$

- Measured in W/sr*m^2
- It is a measure of the distribution of light in space.
- It is directional light



Light-surface-camera

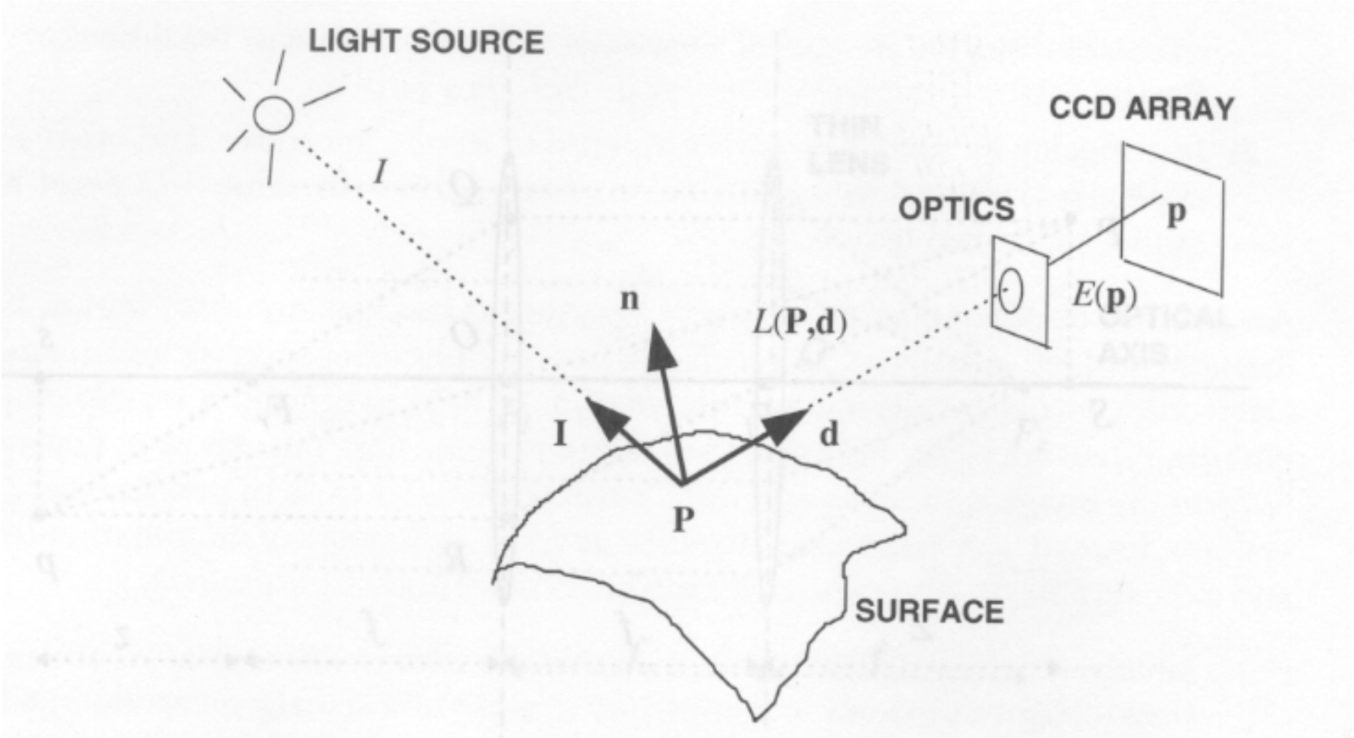


Figure 2.5 Illustration of the basic radiometric concepts.

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Solid Angle Subtended by Scene Patch

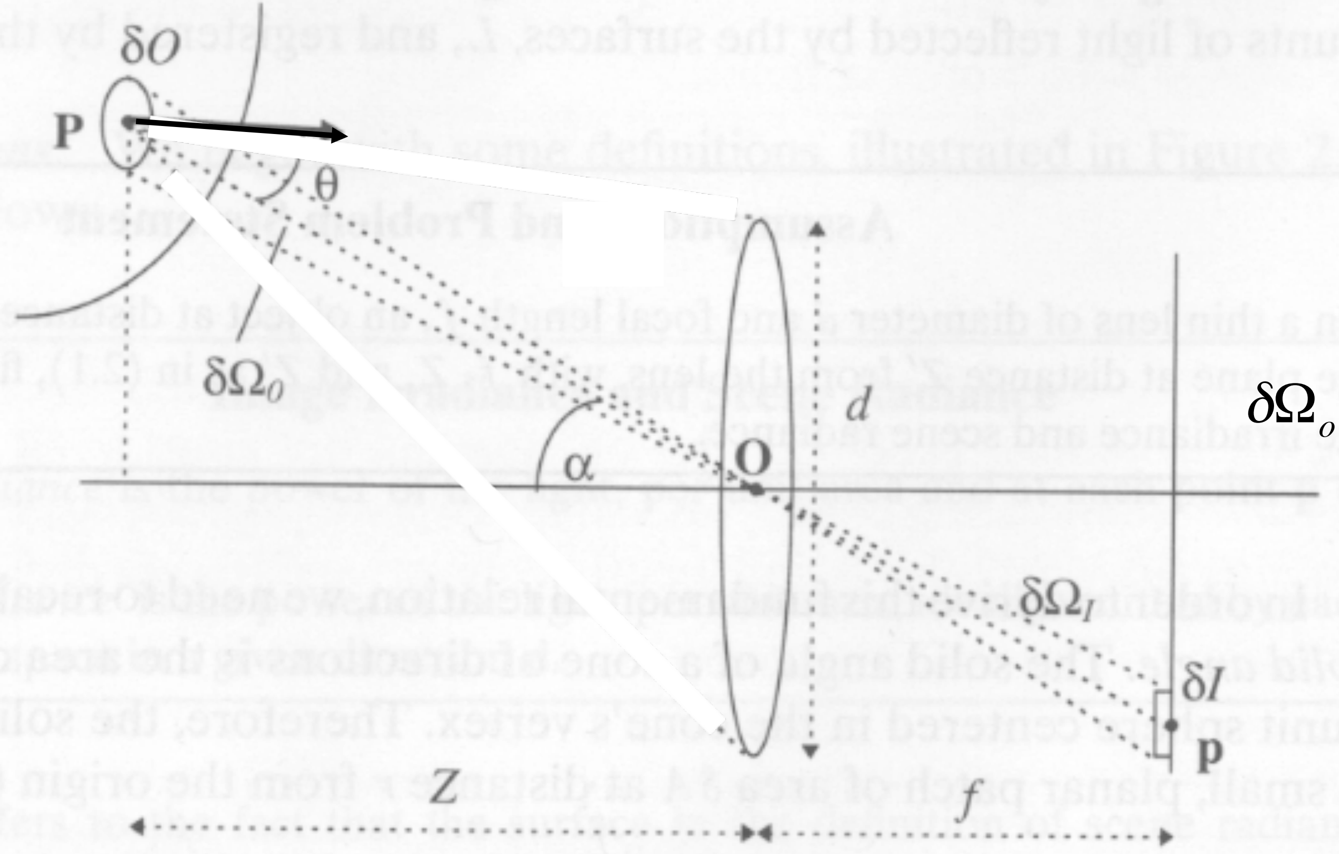


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_o = \frac{\delta O \cos\vartheta}{R^2} = \frac{\delta O \cos\vartheta}{\left(Z/\cos\alpha\right)^2}$$

Solid Angle Subtended by Image Patch

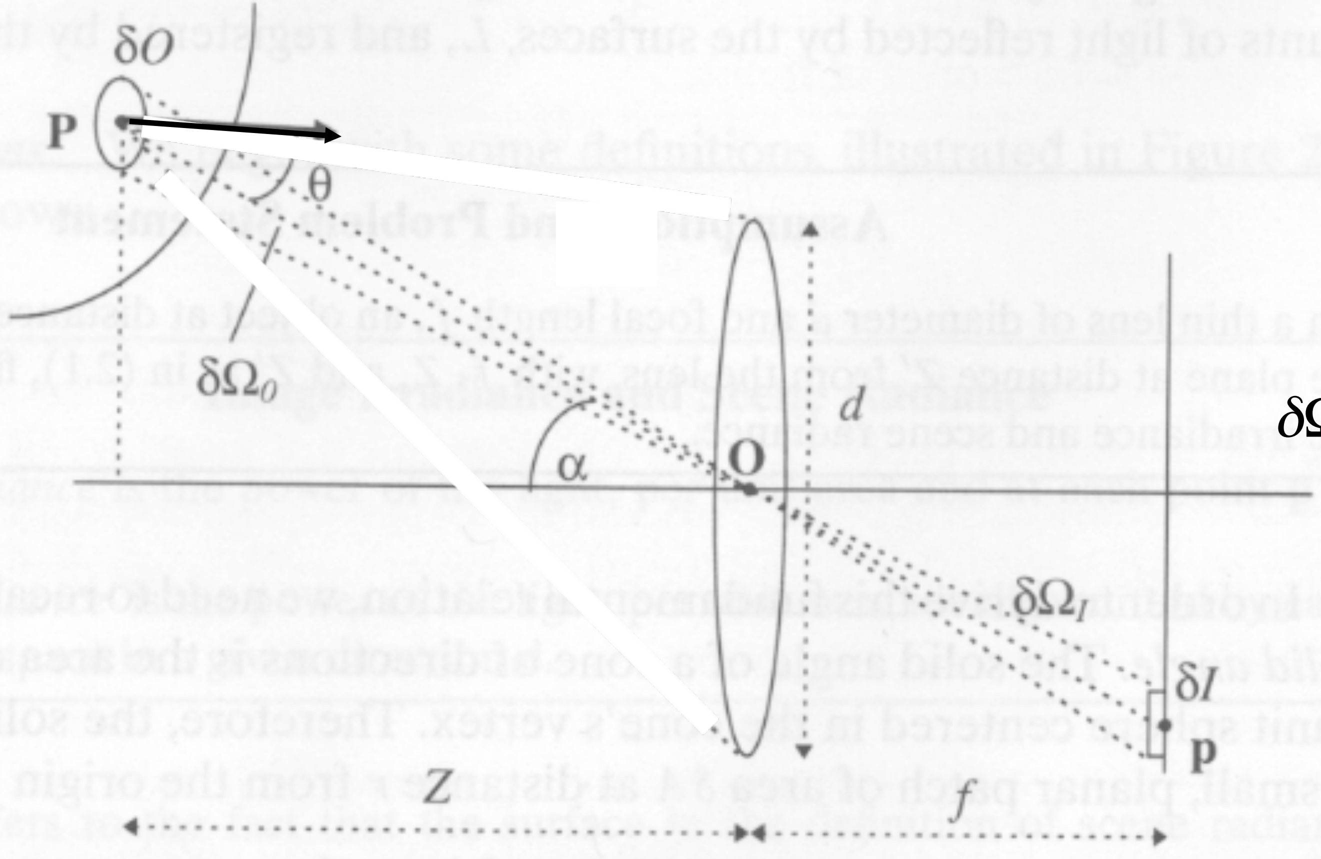


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_i = \frac{\delta I \cos\alpha}{r^2} = \frac{\delta I \cos\alpha}{\left(f / \cos\alpha \right)^2}$$

Similar Solid Angles

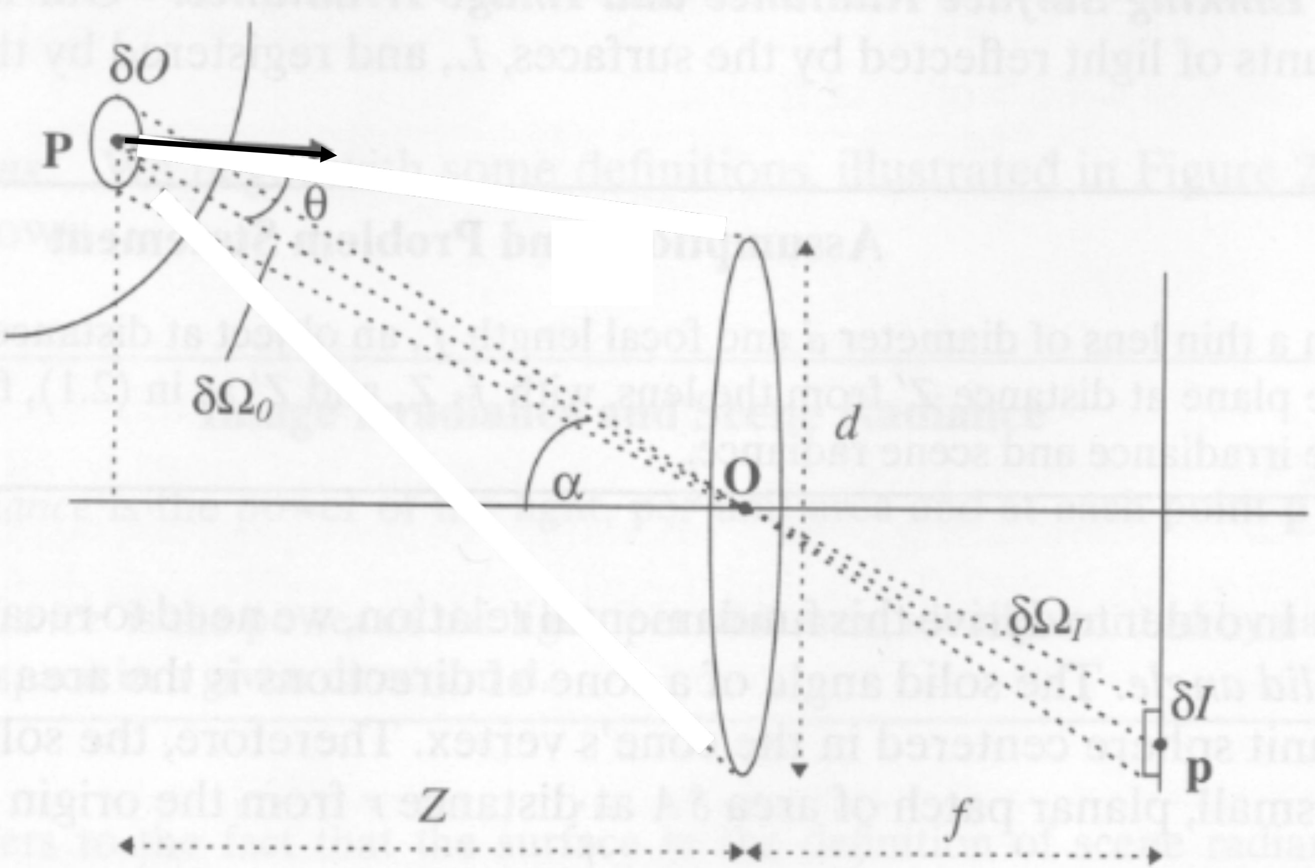


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_o = \frac{\delta O \cos\vartheta}{\left(Z / \cos\alpha \right)^2}$$

$$\delta\Omega_i = \frac{\delta I \cos\alpha}{\left(f / \cos\alpha \right)^2}$$

$$\delta\Omega_o = \delta\Omega_i \Rightarrow$$

$$\Rightarrow \frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\vartheta} \left(\frac{Z}{f} \right)^2$$

Solid Angle Subtended by the Lens at P

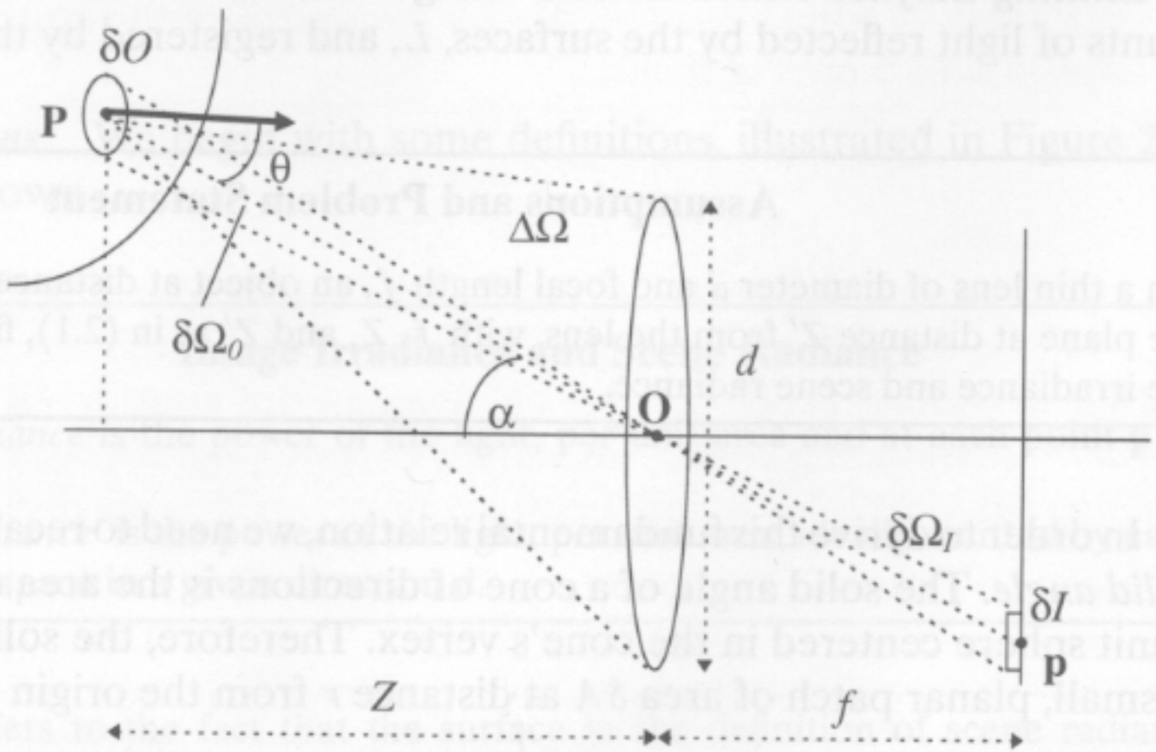


Figure 2.7 Radiometry of the image formation process.

$$\Delta\Omega = \frac{\frac{\pi}{4} d^2 \cos\alpha}{R^2} = $$

$$= \frac{\pi}{4} \frac{d^2 \cos\alpha}{\left(\frac{Z}{\cos\alpha}\right)^2} = $$

$$= \frac{\pi}{4} \frac{d^2}{Z^2} \cos^3\alpha$$

Radiance Incident on the Lens

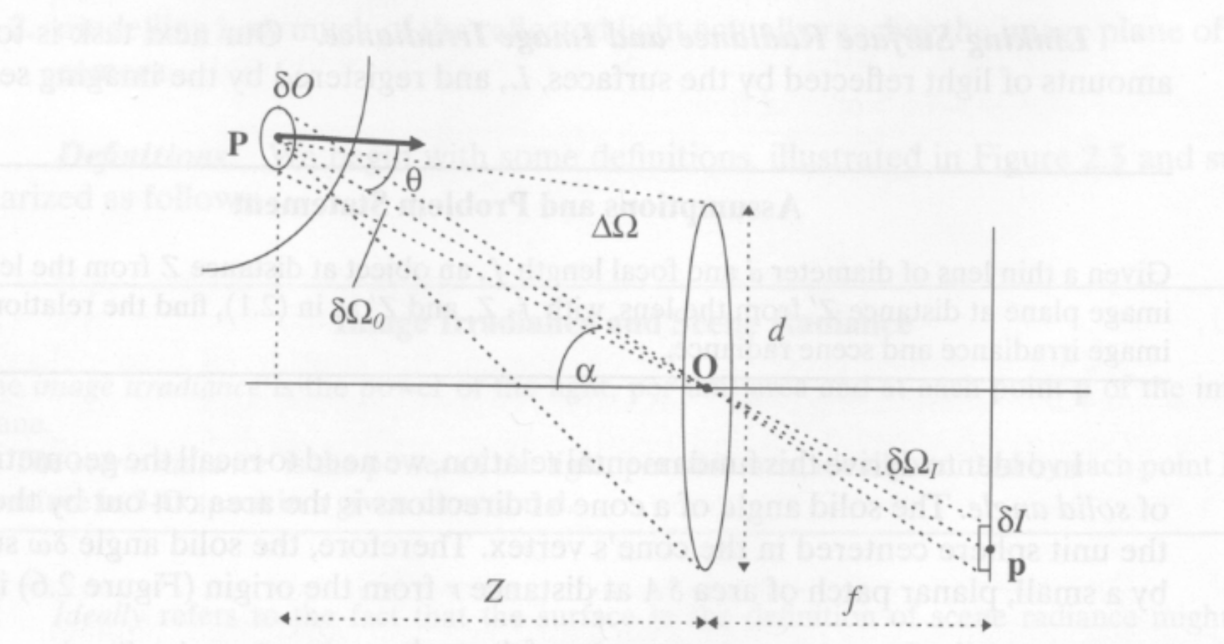


Figure 2.7 Radiometry of the image formation process.

The scene radiance incident on the lens from a scene point P is:

$$L = \frac{\delta P}{\Delta\Omega \delta O \cos\vartheta} = \frac{\delta P}{\frac{\pi}{4}\frac{d^2}{Z^2}\cos^3\alpha \delta O \cos\vartheta}$$

Powerloss Transfer through the Lens

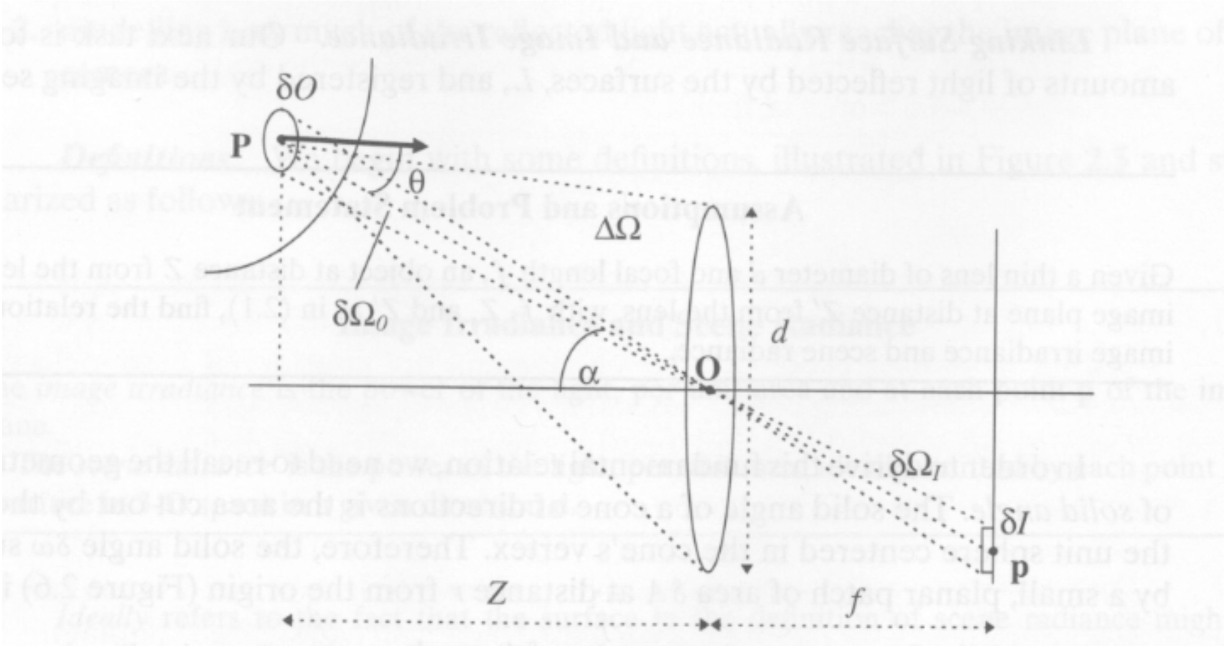


Figure 2.7 Radiometry of the image formation process.

In a perfect lens, all the light power incident on it, is focused on point p.

$$\delta P = L \frac{\pi}{4} \frac{d^2}{Z^2} \cos^3 \alpha \delta O \cos \vartheta$$

Irradiance Incident on Image Point p

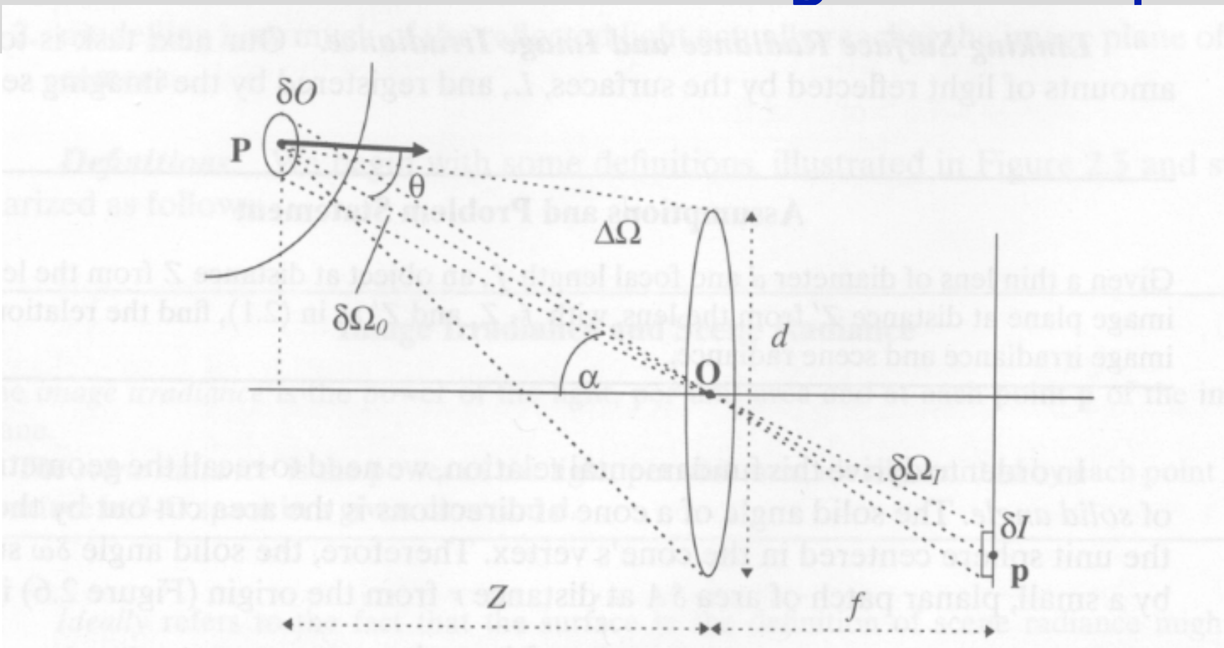


Figure 2.7 Radiometry of the image formation process.

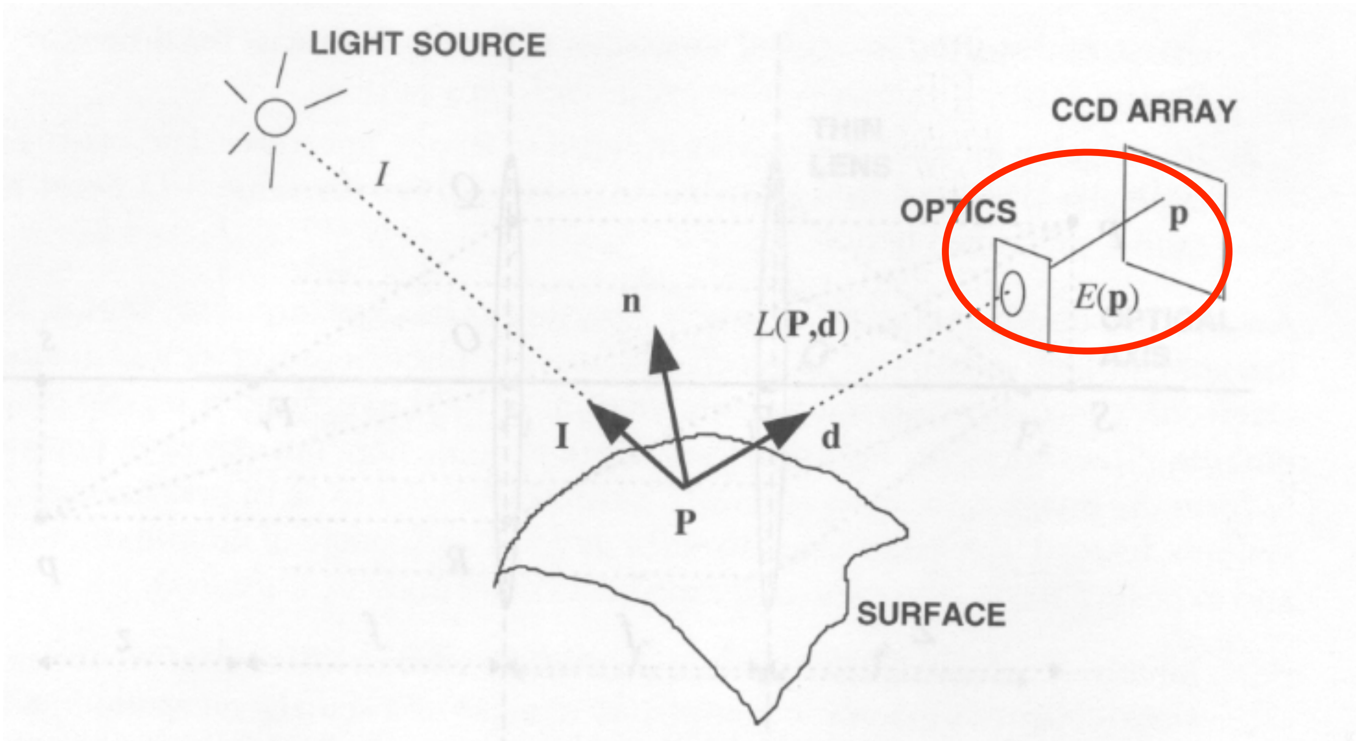
Thus the incident irradiance at point p is:

$$E = \frac{\delta P}{\delta I} = \frac{L \frac{\pi}{4} \frac{d^2}{Z^2} \delta O \cos\vartheta \cos^3\alpha}{\delta I}$$

Recall that: $$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\vartheta}\left(\frac{Z}{f}\right)^2$$

$$\Rightarrow E = L \frac{\pi}{4} \frac{d^2}{f^2} \cos^4\alpha$$

Light-surface-camera



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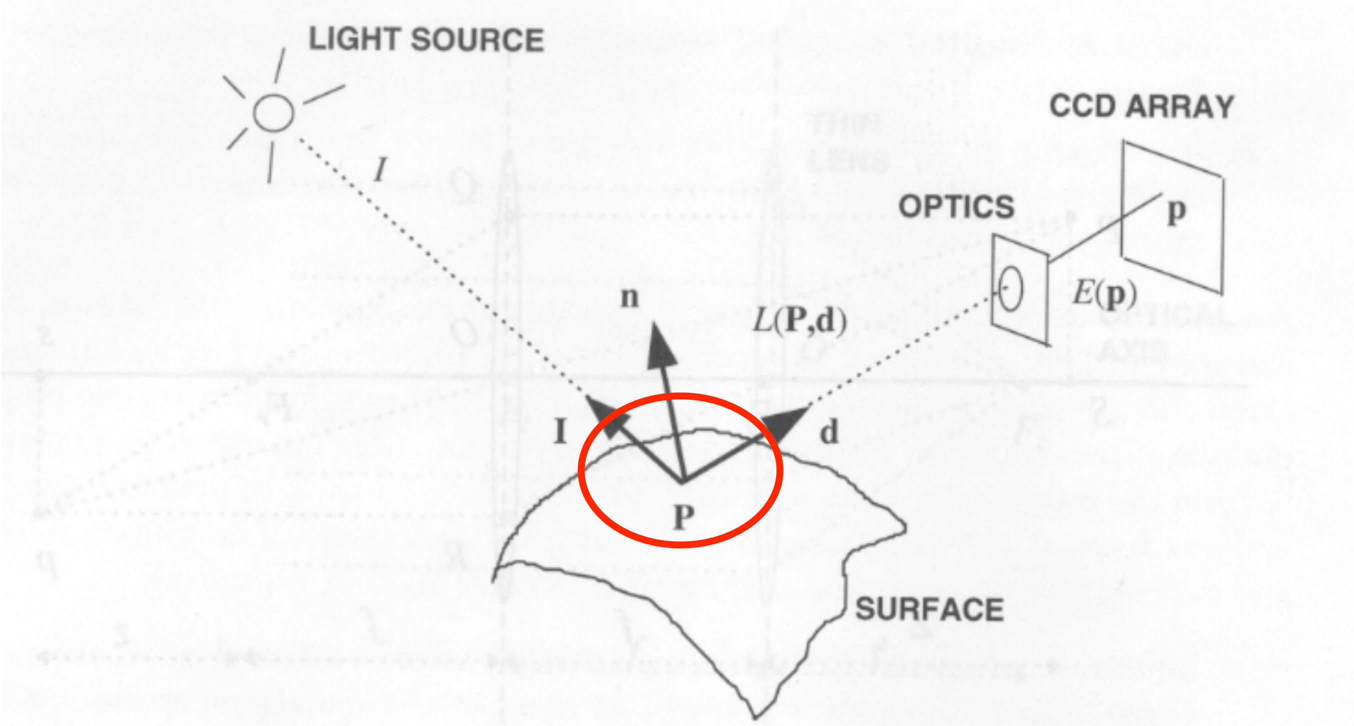

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Light and Surfaces

- Surfaces can absorb, transmit (transparent objects), scatter, reemit, or reflect the light incident on them.
- The **Bidirectional Reflectance Distribution Function** (BRDF) describes the relationship between the light falling on a surface patch and the light leaving that surface patch.
- BRDF is the defined as the ratio of the radiance in the outgoing direction to the incident irradiance.

$$BRDF(\vartheta_r,\phi_r,\vartheta_i,\phi_i) = f(\vartheta_r,\phi_r,\vartheta_i,\phi_i) = \frac{dL_r(\vartheta_r,\phi_r)}{dE_i(\vartheta_i,\phi_i)}$$

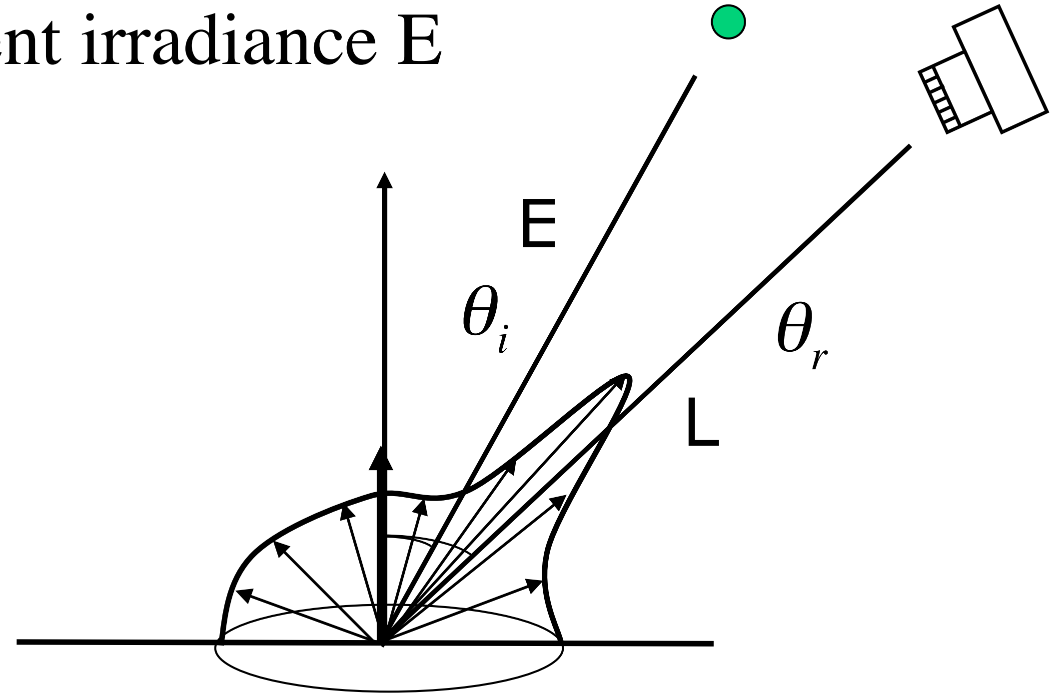
BRDF

- BRDF is measured in $1/sr$.
- It is the ratio of the power of light leaving an infinitesimal surface patch centered around ***P*** in the direction (ϑ_r, ϕ_r) over the the power of light falling on that surface patch from the direction (ϑ_i, ϕ_i).
- Helmholz reciprocity principle: The BRDF is symmetric in the ingoing and outgoing directions.
- BRDF assumptions:
  - No wavelength shifting (i.e. no fluorescence)
  - Surfaces are not generating light
  - Departing radiance is only due to incident irradiance.

BRDF Figure

Bidirectional Reflectance Distribution Function (BRDF):

$$f(\theta_r, \phi_r; \theta_i, \phi_i) = \frac{\text{reflected radiance L}}{\text{incident irradiance E}}$$



Slide courtesy of Frank Dellaert, http://www.cc.gatech.edu/~dellaert/vision/slides/04-Radiometry.ppt

BRDF (continued)

- If we know the BRDF of a surface and the incident irradiance from a specific direction (ϑ_i, ϕ_i), then we know the radiance leaving the surface:

$$dL_r(\vartheta_r, \phi_r) = f(\vartheta_r, \phi_r, \vartheta_i, \phi_i) dE_i(\vartheta_i, \phi_i)$$

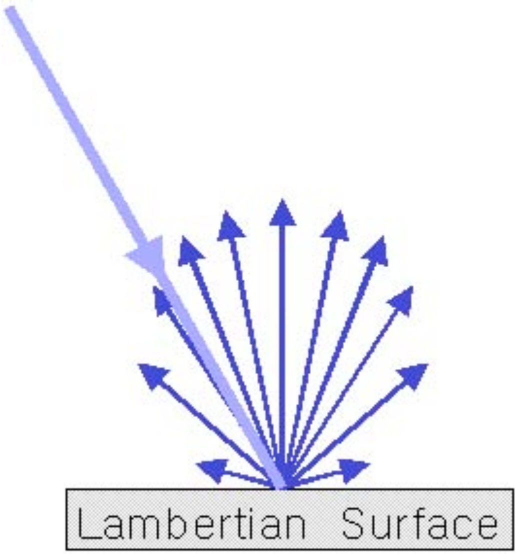
- We can measure the power of light leaving a surface patch and travelling towards our camera caused by **all** the light falling on that surface patch by integrating over the entire hemisphere of incident illumination:

$$dL_r(\vartheta_r, \phi_r) = \int_0^{2\pi} \int_0^{\pi/2} f(\vartheta_r, \phi_r, \vartheta_i, \phi_i) dE_i(\vartheta_i, \phi_i) \sin\vartheta_i \, d\vartheta_i \, d\phi_i$$

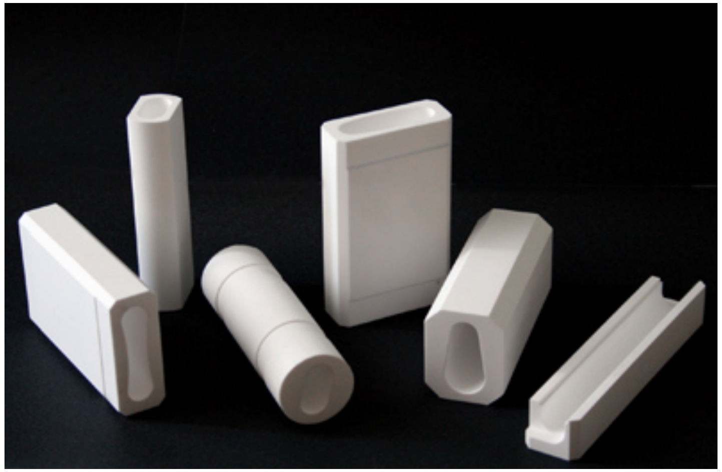
Perfectly Diffuse Surfaces

- For perfectly diffuse surfaces (Lambertian surfaces, perfectly matte surfaces), the BRDF is independent from the outgoing direction:

$$dL_{r,Lambertian} = 1/\pi$$



Graph courtesy of T. Cummings, http://laser.physics.sunysb.edu/~thomas/report2/reflection.html



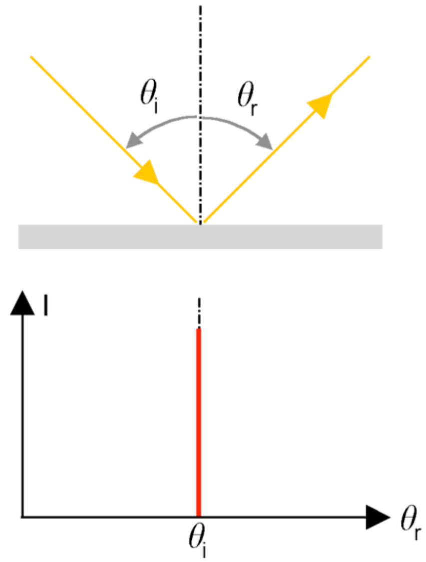
A picture of objects made of Spectralon, a material that is 99% Lambertian. Image courtesy of Labsphere.

Perfectly Specular Surfaces

- For perfectly specular surfaces (i.e. mirror-like surfaces), all the exiting radiance occurs in only direction, obtained by reflecting the incoming direction about the surface normal. Everywhere else it is zero:

$$dL_{r,Specular} = \frac{\delta(\vartheta_r - \vartheta_i)\delta(\phi_r - \phi_i - \pi)}{\sin\vartheta_i \cos\vartheta_i}$$

"Cloud Gate" sculpture by Anish Kapoor. Picture courtesy of Outokumpu, http://www.outokumpu.com

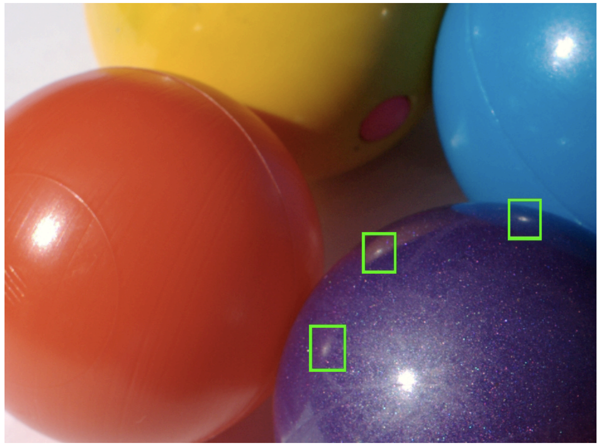


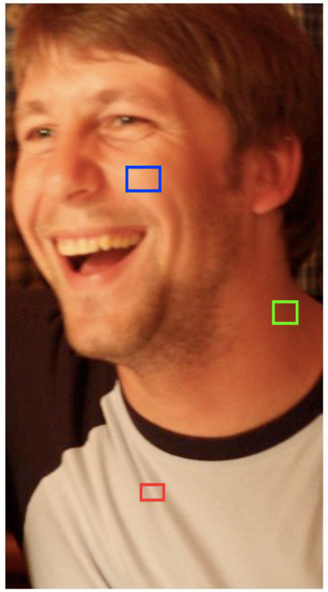
Graph courtesy of wikipedia

Mixed Surfaces

- Most surfaces exhibit a mixture of diffuse and specular behavior:

$$dL_{r,Mixed} = ndL_{r,Lambertian} + (1-n)dL_{r,Specular}$$





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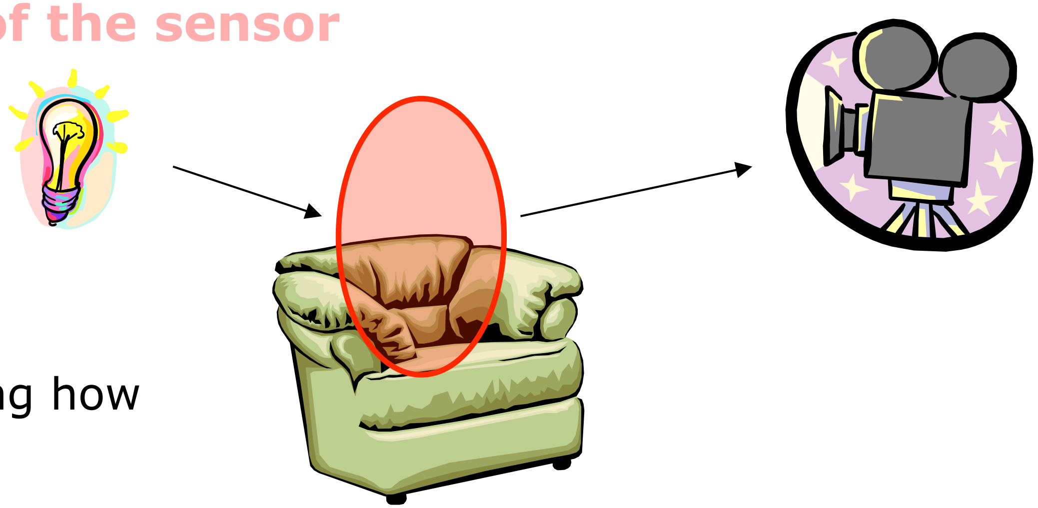
    - **Optical properties of the sensor**
  - Illumination conditions



Note: At this point we are ingnoring how the sensor itself records the data.

Camera Coordinate System

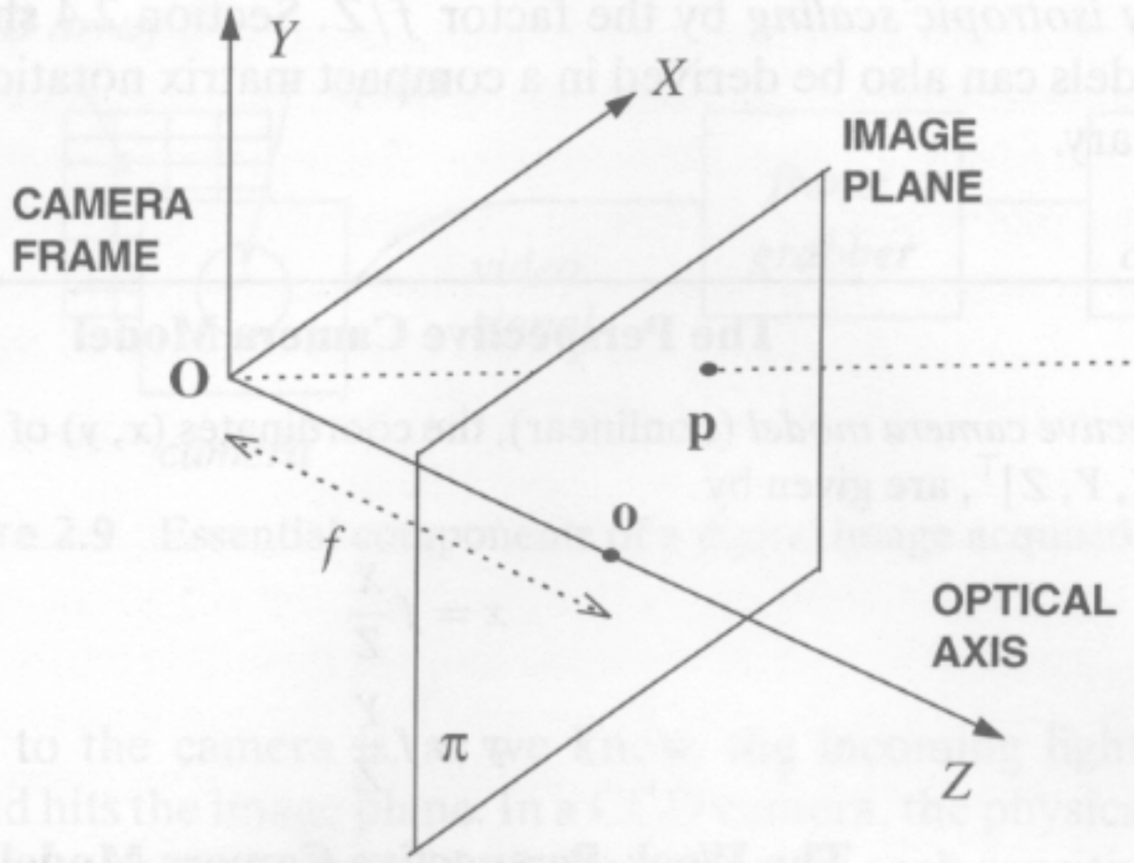


Figure 2.8 The perspective camera model.

Camera Coordinate Frame

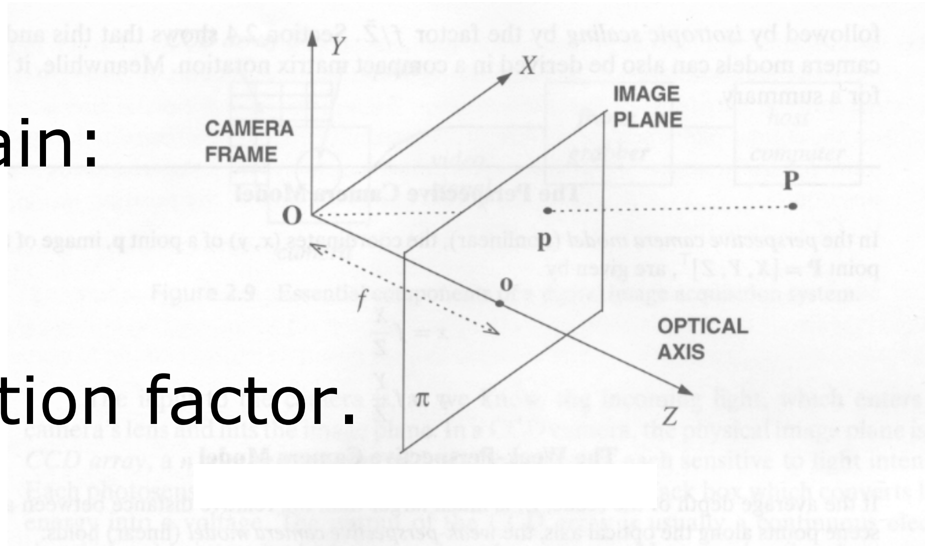
- Right-handed coordinate system.
- Origin, O: center of projection (COP)
- z-axis: perpendicular to the image plane, passes through the COP, points towards the scene, same as the optic axis.
- The intersection of the z-axis with the image plane is called the **principal point** or **image center**.
- x-axis: horizontal axis, parallel to the image plane
- y-axis: vertical axis, pointing upwards, parallel to the image plane.

Perspective Projection

- The pinhole camera model implies perspective projection, i.e. all projection rays pass through a single point (COP).
- Let the image point p=(x,y,z) be the projection of the scene point P=(X,Y,Z).
- The image plane is positioned at distance f from the origin.
- From similar triangles we obtain:

$$x = \left(\frac{f}{Z}\right) X = -mX \qquad y = \left(\frac{f}{Z}\right) Y = -mY$$

where $m = -\frac{f}{Z}$ is the magnification factor.



Comments on Perspective Projection

- The perspective projection eqs. $x = \left(\frac{f}{Z}\right)X$ and $y = \left(\frac{f}{Z}\right)Y$ express how objects that are farther away appear smaller.
- If we know the characteristics of the camera (its position, orientation and focal length) we can compute the exact position where a scene point *P* will appear on the image plane.
- Limitation: Non-linear model. The factor $\frac{1}{Z}$ does not preserve distances between points.
- The same scene length *L* will map to different image lengths *l*, depending on its distance to the camera.

Weak Perspective Projection

- When the range of depth values in the scene is small relative to the average distance from the scene to the camera, we assume that the magnification factor *m* is constant.

$$m = -\frac{f}{\bar{Z}}$$

  where $\bar{Z}$ is the average *Z* value over all the points in the scene.

- The weak perspective projection equations are:

$$x = \left(\frac{f}{\bar{Z}}\right)X \qquad y = \left(\frac{f}{\bar{Z}}\right)Y$$

Orthographic Projection

- Assume that the scene and the camera (relative to the scene) are fixed.
- One can then normalize the image coordinates so that:

$$x = X \qquad y = Y \qquad m = -1$$

- Under orthographic projection, there is no longer a Center of Projection. All projection rays are parallel to each other and perpendicular to the image plane.

Perspective vs. Orthographic Projection

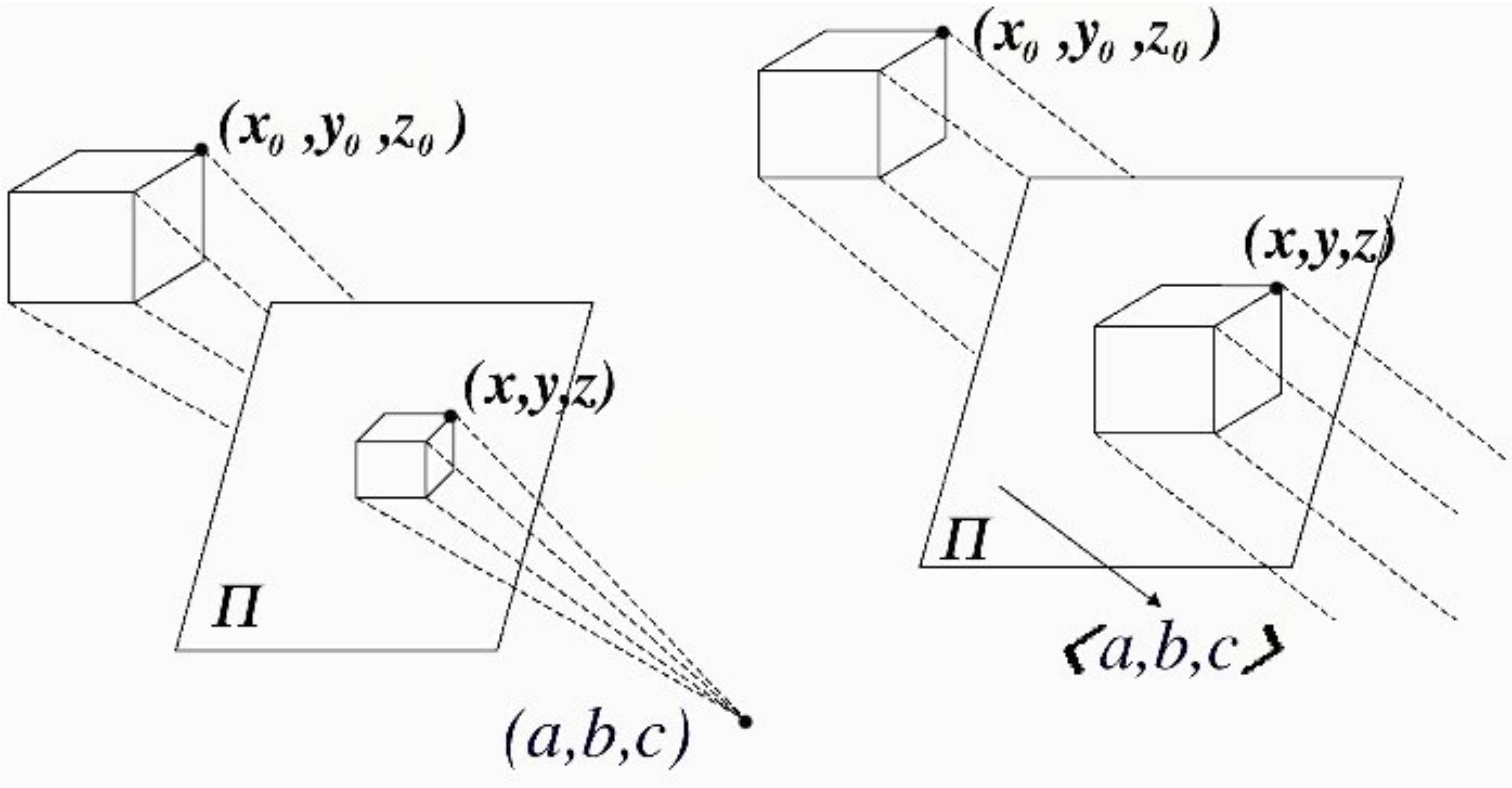


Figure courtesy of MathDL http://mathdl.maa.org/images/cms_upload/

Camera Position in the World

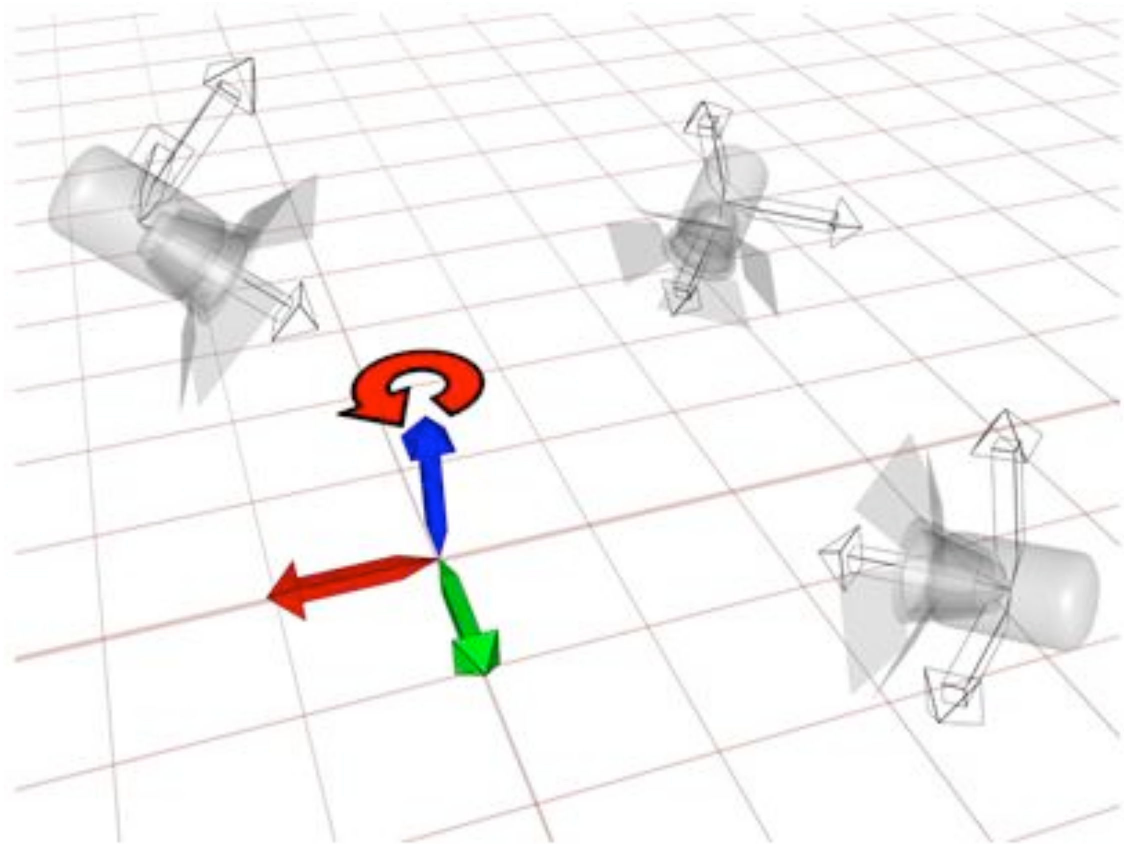


Image courtesy of AutoDesk 3DS Max 9 Reference http://www.kxcad.net/autodesk/3ds_max/Autodesk_3ds_Max_9_Reference/use_transform_coordinate_center.html

Many Camera Positions in One World

Image courtesy of J.-B. Hayet http://www.cimat.mx/~jbhayet/icons/slam.jpg

Extrinsic Camera Parameters

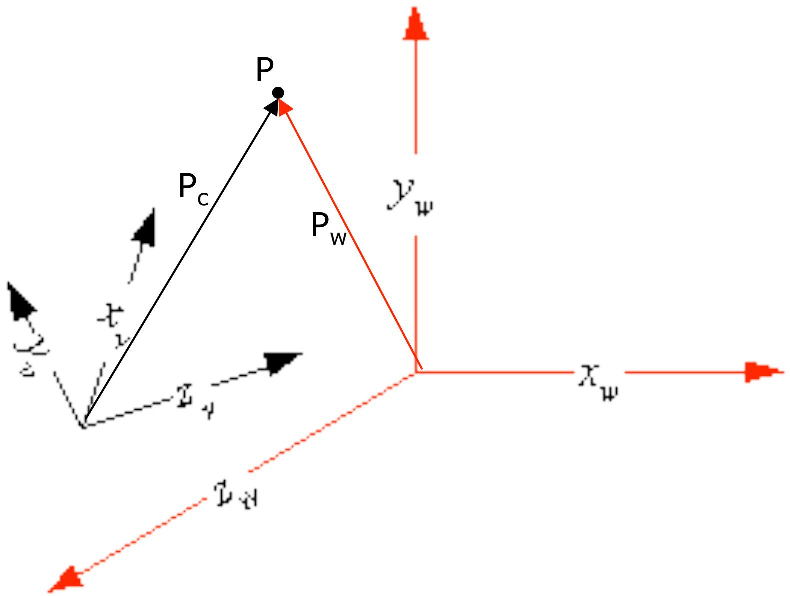


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- Extrinsic parameters: A set of geometric parameters that uniquely identify the transformation between the unknown camera frame and a known reference frame (the world reference frame).

Extrinsic Camera Parameters

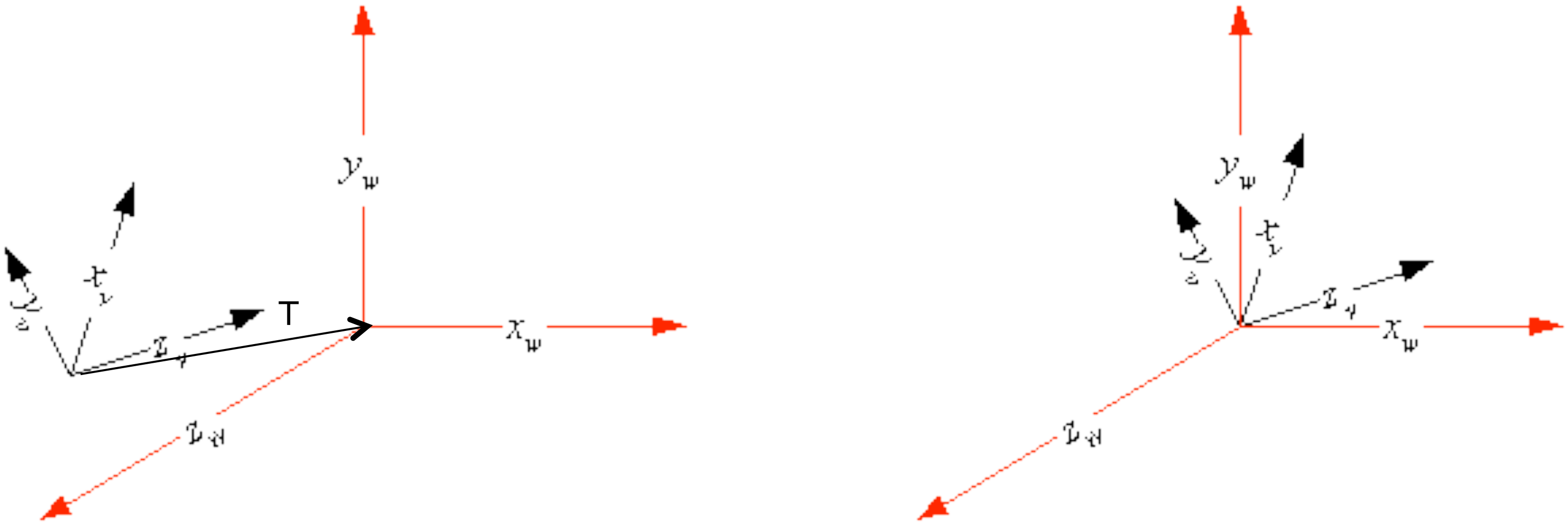


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- T: a 3D translation vector describing the relative position of the 2 origins (camera and world origins)

Extrinsic Camera Parameters

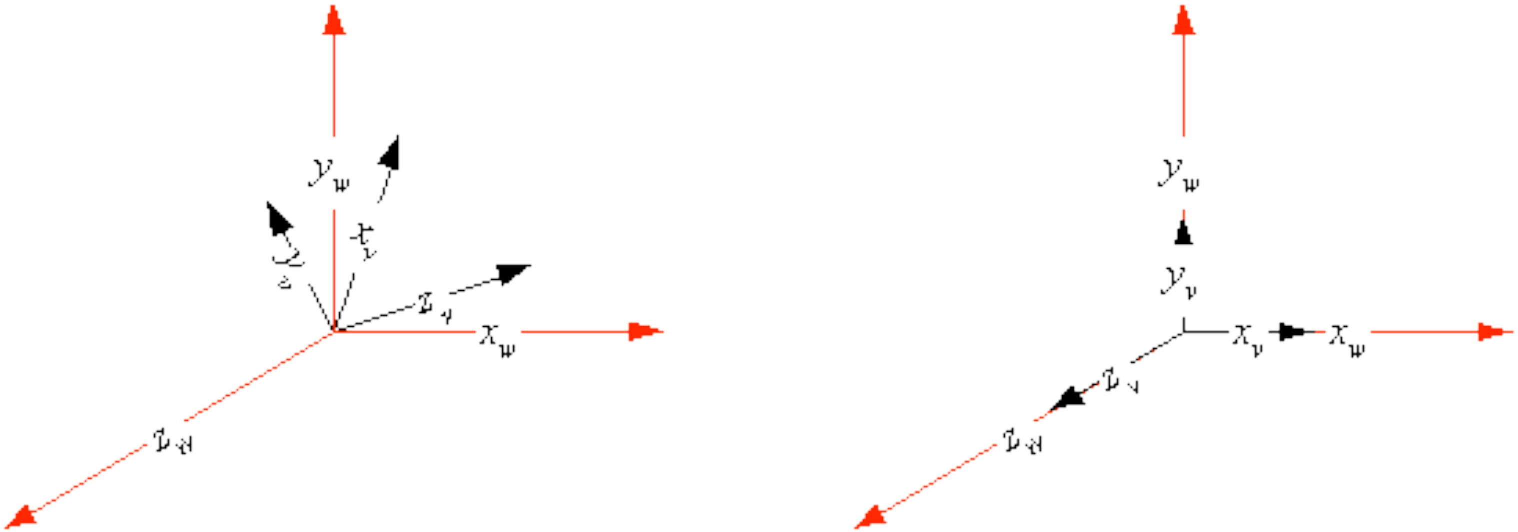


Figure courtesy of Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- R: a 3x3 rotation matrix that aligns the axes of the two frames (camera and world frame)

Extrinsic Camera Parameters

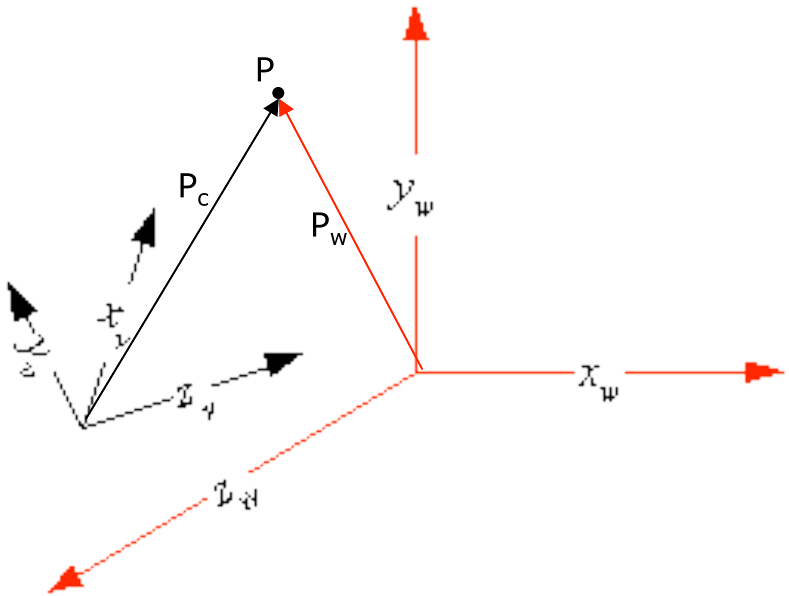


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- The relationship between the coordinates of a point P in world P_w and camera P_c frames is:

$$P_c = R(P_w - T)$$

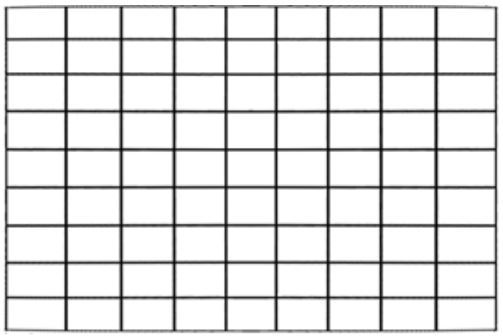
Intrinsic Camera Parameters

- Intrinsic parameters: A set of geometric parameters that link the pixel coordinates of an image point to the corresponding coordinates in the camera reference frame.
- (x_{im}, y_{im}) : image reference frame, i.e. pixel coordinates.
- (o_x, o_y) : pixel coordinates of the image center, i.e. where the optic axis intersects the image plane.
- (s_x, s_y) : effective size of pixel in mm.

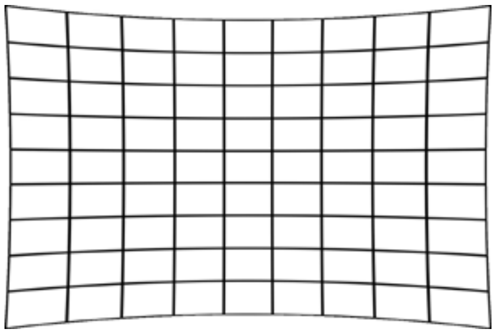
$$x_c = -(x_{im} - o_x)s_x \qquad y_c = -(y_{im} - o_y)s_y$$

Radial Distortion

- A pin-hole image of a square grid.



- A lens-system image of the same square grid



- The amount of distortion depends on the distance between the principal point and the pixel of interest.
- Let (x_d, y_d) be the coordinates of the distorted point in the image coordinate system.

Radial Distortion Correction


before


after

Image courtesy of VIPBase

- The undistorted (corrected) image coordinates are:

$$x = x_d(1 + k_1 r^2 + k_2 r^4) \qquad y = y_d(1 + k_1 r^2 + k_2 r^4) \qquad r^2 = x_d^2 + y_d^2$$

- Typically $k_2 << k_1$, so we often set $k_2 = 0$