Image Formation

Lens Optics, Photometry, Geometric Optics

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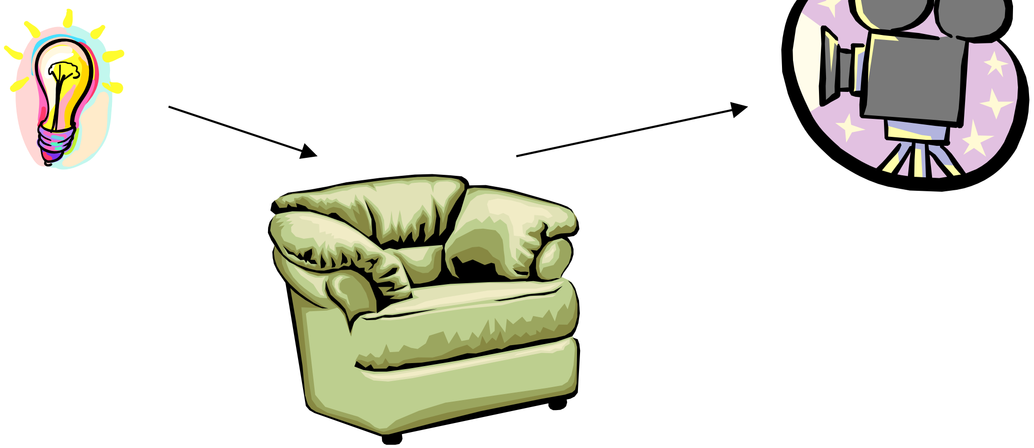
Image Formation

- There are three major components that determine the appearance of an image
  - Geometry
    - Geometry of the scene
    - Geometry of the projection to the camera
  - Optical properties
    - Optical properties of the materials in the scene
    - Optical properties of the sensor
  - Illumination conditions



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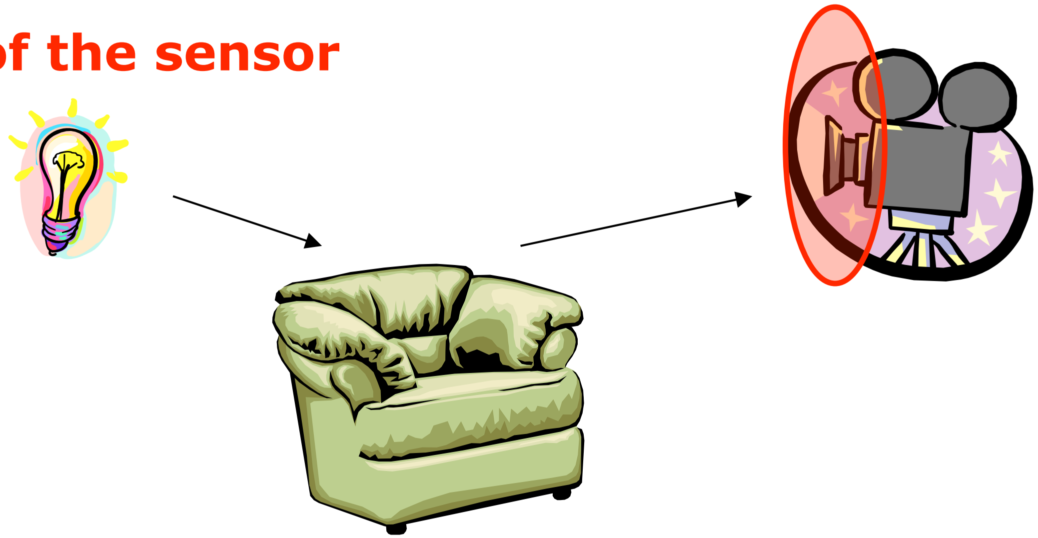


Pinhole Camera

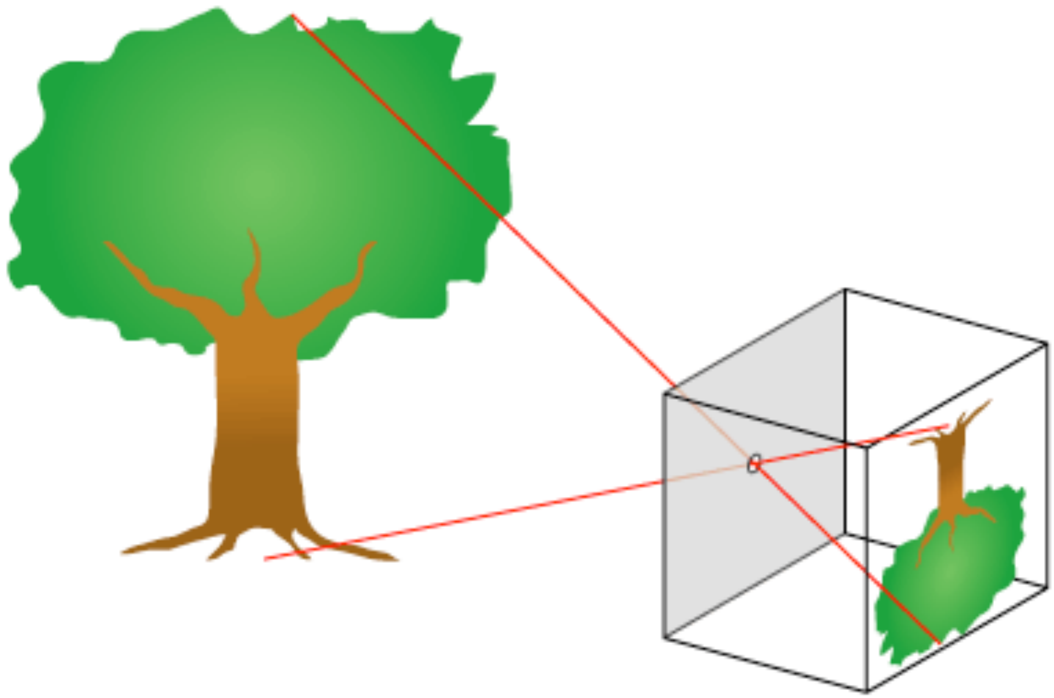


Image courtesy of wikipedia, http://upload.wikimedia.org/wikipedia/commons/8/81/Pinhole-camera.png

Pinhole + Lens

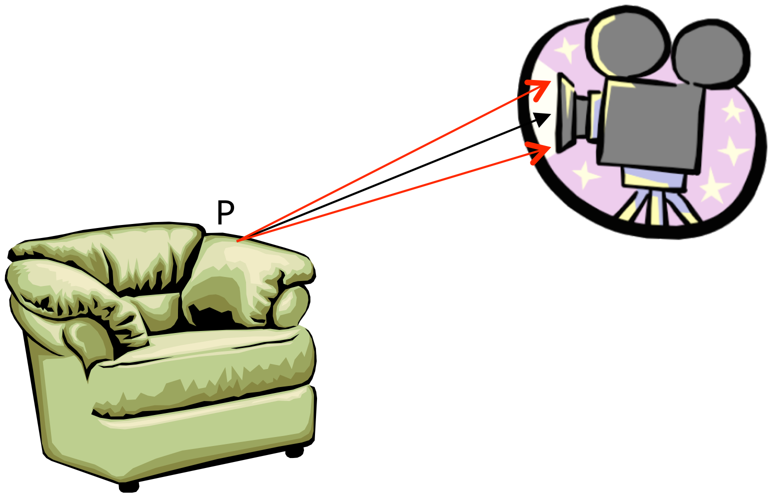
- Pinhole cameras can produce very crisp images of stationary scenes.
- They require long exposure time, since all the incoming light has to go through a single hole.
- Solution: Open up the hole ⇒ blurred images
- Solution: Use a lens to focus the light rays

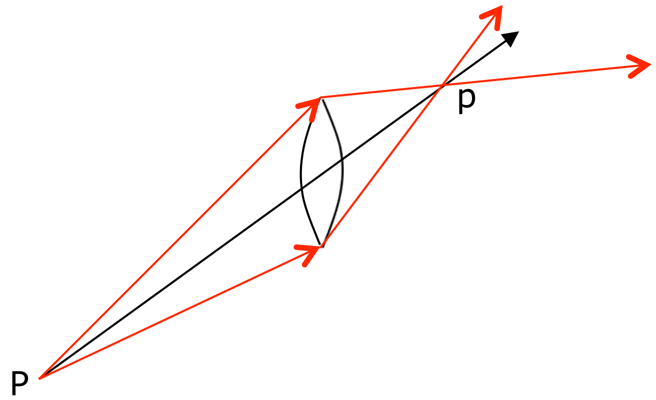
  ⇒ crisp, bright image obtained at shorter exposure times

Lens

From a point P on the scene, a cone of rays reaches the lens and then converges to a point p on the other side of the lens.

Where does the point p lie?





Thin (Convex-Convex) Lens

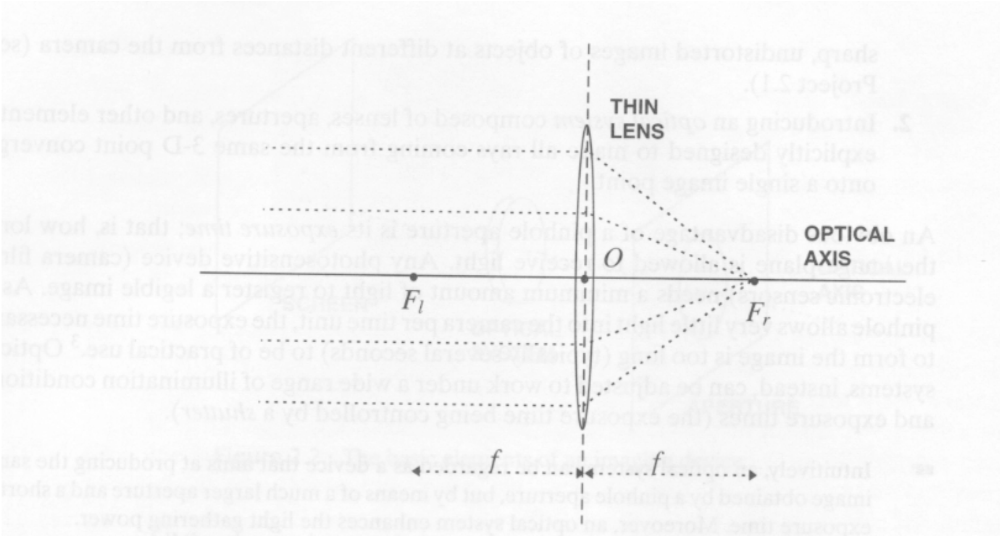


Figure 2.3 Geometric optics of a thin lens (a perpendicular view to the plane approximating the lens).

Center of projection (COP), O: center of lens

Optic axis: axis perpendicular to the lens that passes through the COP

Focus of the lens, F_r: point on the optic axis where all the parallel rays incident on the lens converge.

Focal length, f: the perpendicular distance between the lens and the focus point of the lens F_r

Imaging with Thin Lens

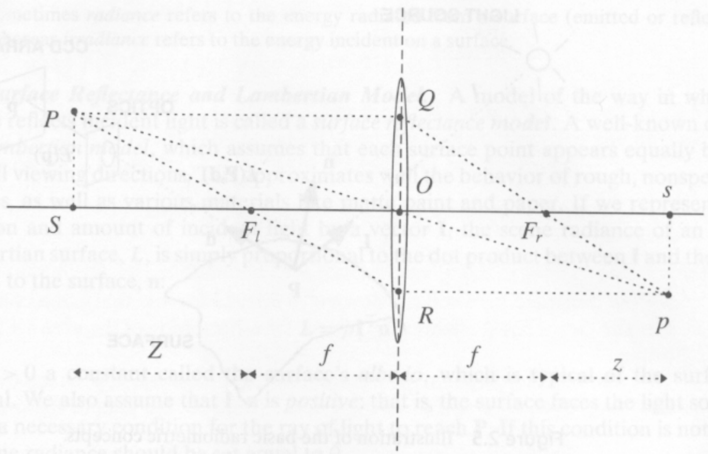


Figure 2.4 Imaging by a thin lens. Notice that, in general, a real lens has two different focal lengths, because the curvatures of its two surfaces may be different. The situation depicted here is a special case, but it is sufficient for our purposes. See the Further Readings at the end of this chapter for more on optics.

From the similar triangles $P\ F_l\ S$ and $R\ F_l\ O$ => $\frac{Z}{f} = \frac{\overline{PS}}{\overline{OR}}$

and the similar triangles $p\ F_r\ s$ and $Q\ F_r\ O$ => $\frac{z}{f} = \frac{\overline{sp}}{\overline{QO}}$

But $\overline{PS} = \overline{QO}$ and $\overline{OR} = \overline{sp}$ => $Zz = f^2$

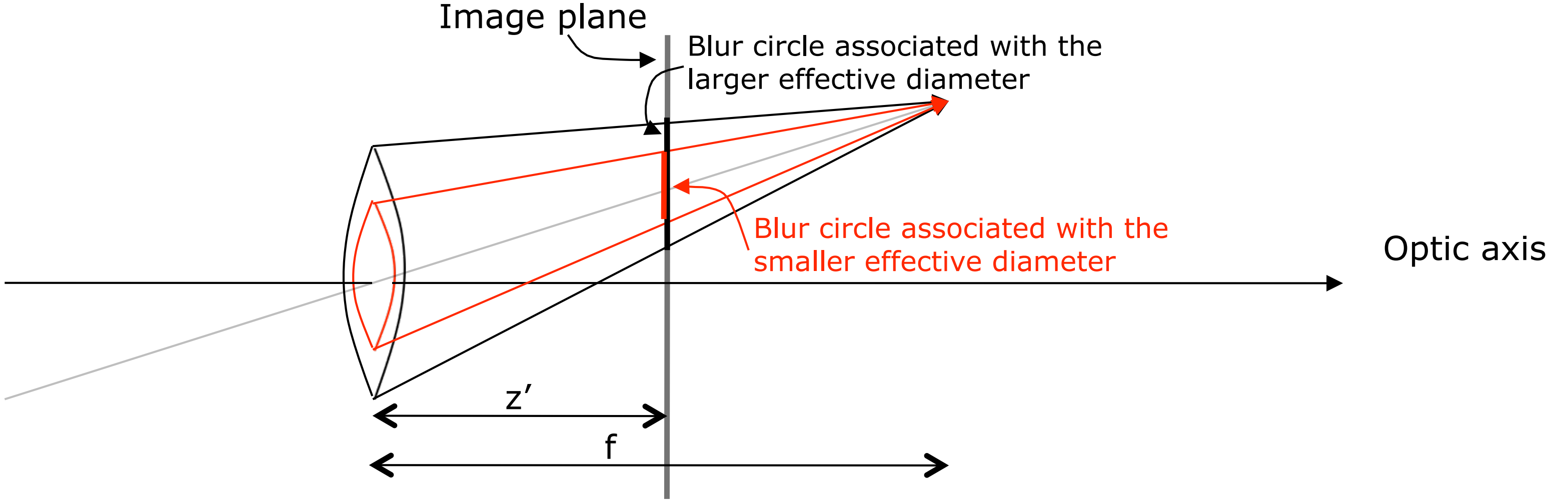
Let $\hat{Z} = Z + f$ and $\hat{z} = z + f$. Then $\boxed{\frac{1}{f} = \frac{1}{\hat{z}} + \frac{1}{\hat{Z}}}$ ⇐ Thin Lens Law

More on Thin Lenses

- As $\hat{Z} \rightarrow \infty \Rightarrow \hat{z} \rightarrow f$
- We assume that $\hat{z} = f$
- Effective diameter of a lens, *d*: diameter of the lens reachable by light rays (an adjustable iris changes the effective diameter of a lens).
- Field of view, *w*: the half-angle subtended by the effective lens diameter, *d*, on the focus, *F*:

$$w = \tan^{-1} \frac{d}{2f}$$

Image Blur



- Depending on the distance *Z* of a point *P* in the scene, the focus point *p* may not lie on the image plane.
- Thus, a blur circle instead of a point is formed on the image plane.
- The diameter, *b*, of the blur circle depends on the effective lens diameter, *d*. $b = \frac{d}{f}(f - z')$

Depth of Field

- Depth of field: range of depth values that appear in-focus in an image.
- F-stop number: the ratio of the focal length over the effective diameter f/d
- Recall that the radius of the blur disk, *b* is given by

$$b = \frac{d}{f}(f - z')$$

- Dilemma:
  - Large F-stop (i.e. small effective diameter)

    => crisp but dim image (weak signal)
  - Small F-stop (i.e. large effective diameter)

    => more blurred regions but bright image (strong signal)

Image Formation

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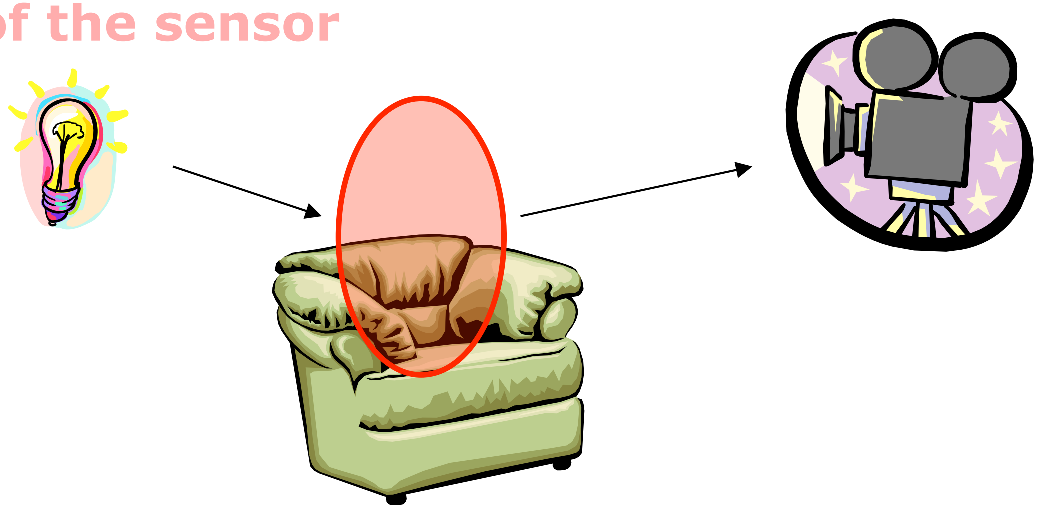
    - **Optical properties of the sensor**
  - Illumination conditions



Light-surface-camera

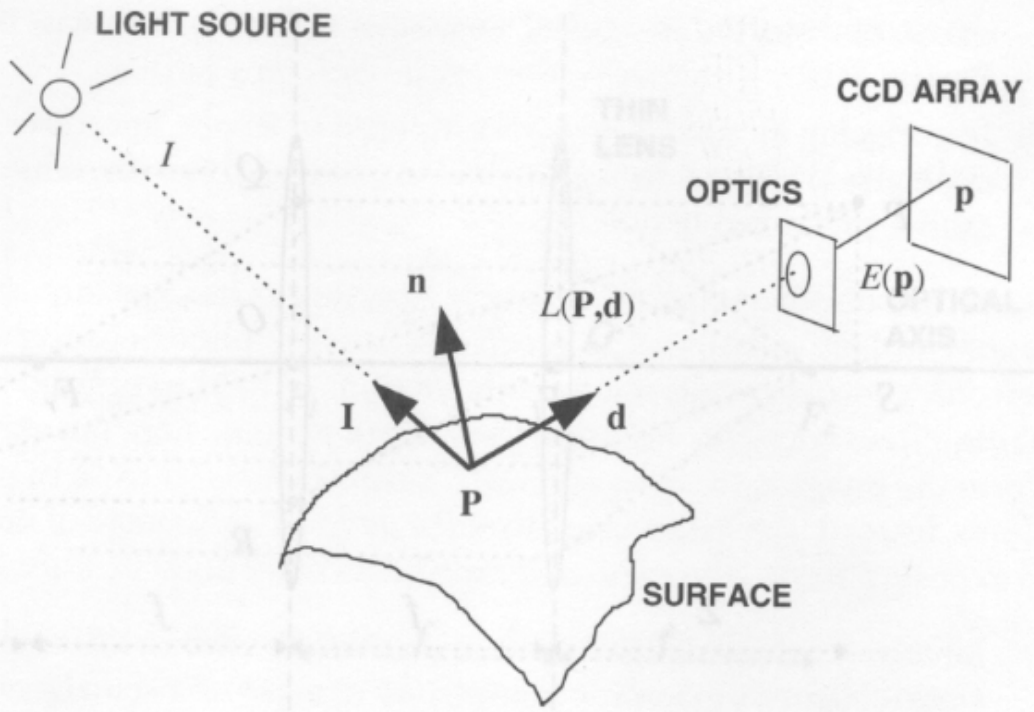


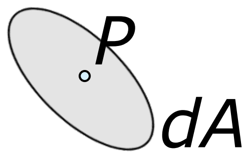
Figure 2.5 Illustration of the basic radiometric concepts.

$$I(p) \propto E(p) \propto L(P)$$

where $I(p)$ is the camera response at pixel p, $E(p)$ is the amount of light that falls on pixel ***p***, and $L(P)$ is related to how bright is the scene point ***P*** that corresponds to pixel ***p***.

Radiometry

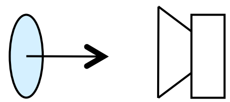
- Informal terms like "brightness" and "amount of light" need to be accurately defined.
- Light is a form of Electromagnetic Energy.
- Radiometry is a field of Physics that measures light. We will use metrics and measurement units established in radiometry.
- In all the radiometric discussions we will always consider an infinitesimal area patch dA centered around a point ***P***.



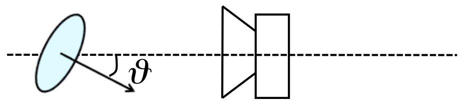
$$dA \rightarrow 0$$

Foreshortening

- A patch dA can have different orientations with respect to the image plane.
  - Parallel



  - Tilted

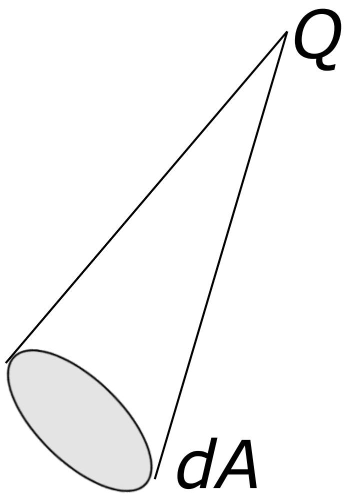


Patch appears smaller to the viewer, i.e. **foreshortened**

- Foreshortening affects
  - How bright a patch appears to the viewer, $E(p)$
  - How much light falls on a patch and hence how bright it is, $L(P)$
- A patch with area dA whose surface normal forms an angle ϑ with the viewing direction (optic axis) has a **foreshortened area** of $dA\cos\vartheta$

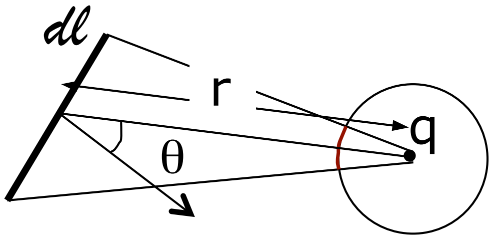
3D Angle (Solid Angle)

- So far we have talked about the patch dA.
- The patch dA is illuminated by a point light source, positioned at Q.
- We need to consider the light that travels through the cone with its tip at Q and its base at dA.



2D Angle

- The angle **subtended** by the line dl at a point q.



- Derivation:
  - Draw a unit circle centered at q
  - Project dl on the perimeter of that circle.
  - The length of that projection is the angle in radians. It depends on:
    - The angle ϑ between the normal to line dl and the radius that reaches the center of dl.
    - The distance r between the center of the line dl and the center of the circle q.

$$d\varphi = \frac{dl \cos\vartheta}{r}$$

Solid Angle

- The solid angle **subtended** by the patch dA at a point q.

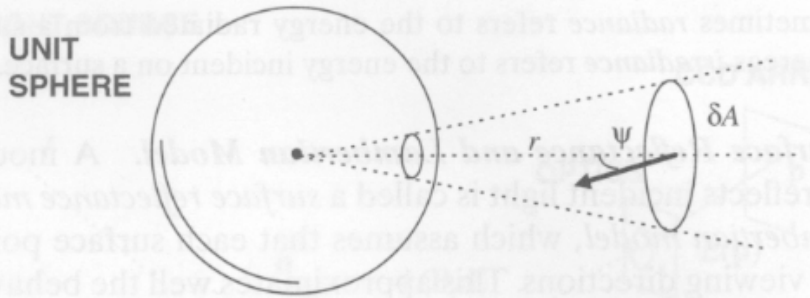


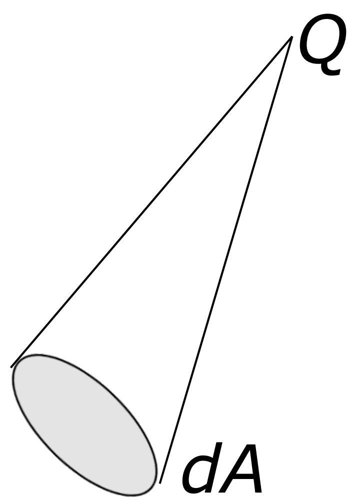
Figure 2.6 The definition of solid angle.

- Derivation:
  - Draw a unit sphere centered at q
  - Project dA on the perimeter of that sphere.
  - The area of that projection is the angle in steradians. It depends on:
    - The angle ψ between the normal to the patch dA and the radius that reaches the center of dA.
    - The distance r between the center of the patch dA and the center of the circle q.

$$d\omega = \frac{dA\cos\psi}{r^2}$$

Light Measurement

- Light: Electromagnetic energy, Q
- Flux: Power carried by the EM radiation, $P = \frac{dQ}{dt}$
- Intensity: Power of light traveling in a specific direction $I = \frac{dP}{d\omega}$

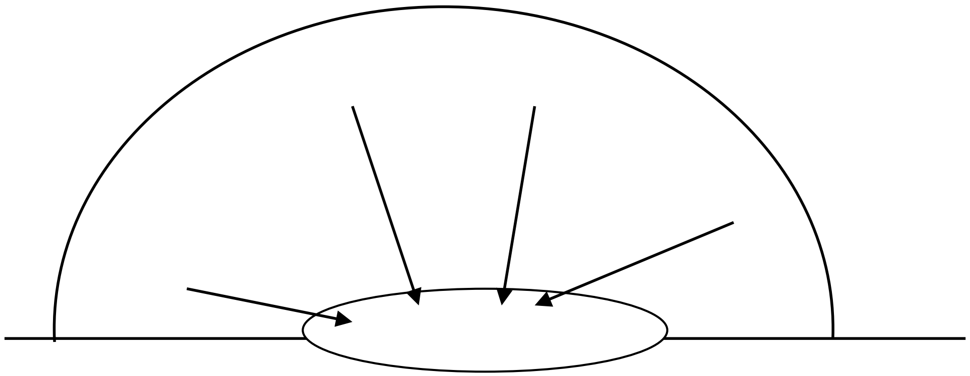


Irradiance

- Irradiance: power of light falling on a surface patch

$$E = \frac{dP}{dA}$$

- Measured in W/m^2
- It's a measure of concentration of power.
- It is independent of direction (direction is **irr**elevant)

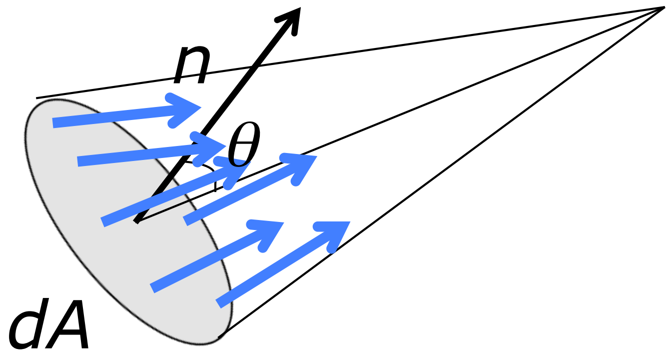


Radiance

- Radiance: power of light falling on a surface patch from a specific direction

$$L = \frac{d^2 P}{d\omega dA \cos\vartheta}$$

- Measured in W/sr*m^2
- It is a measure of the distribution of light in space.
- It is directional light



Light-surface-camera

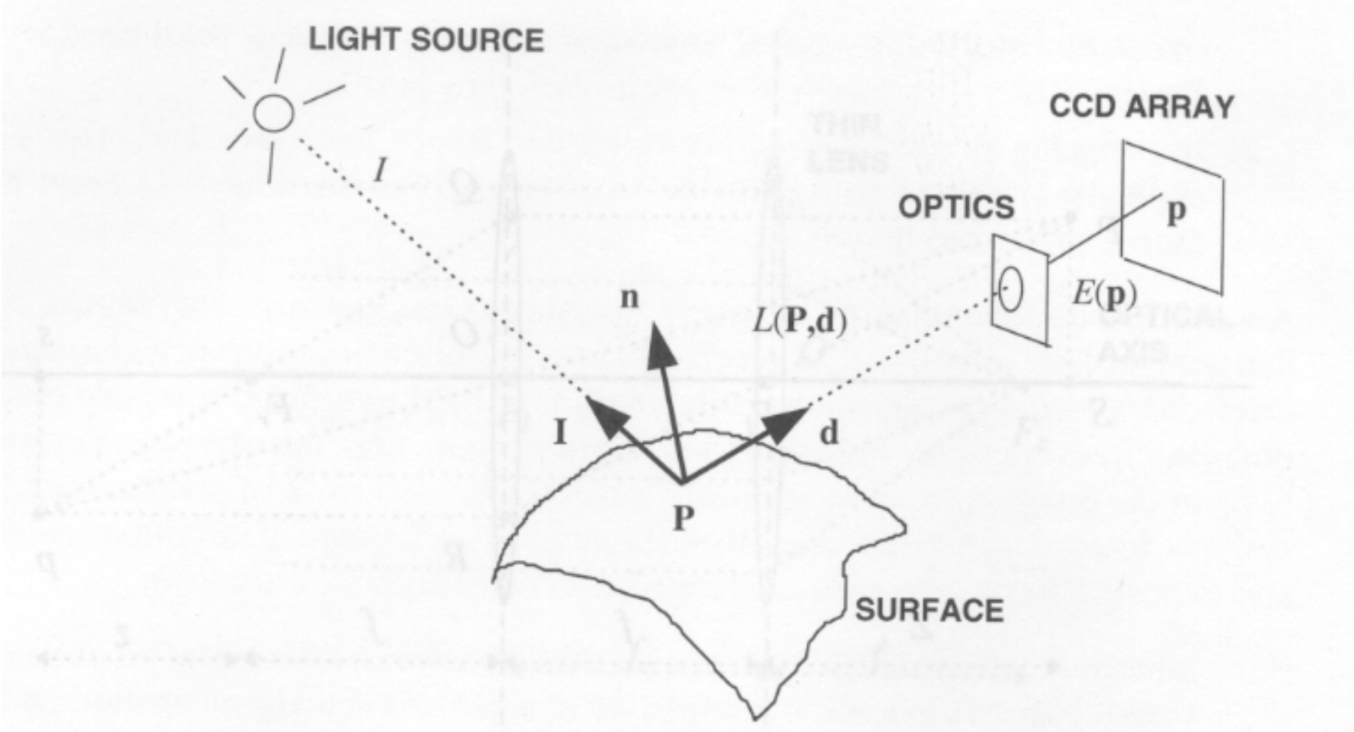


Figure 2.5 Illustration of the basic radiometric concepts.

$$I(p) \propto E(p) \overset{?}{\propto} L(P)$$

Solid Angle Subtended by Scene Patch

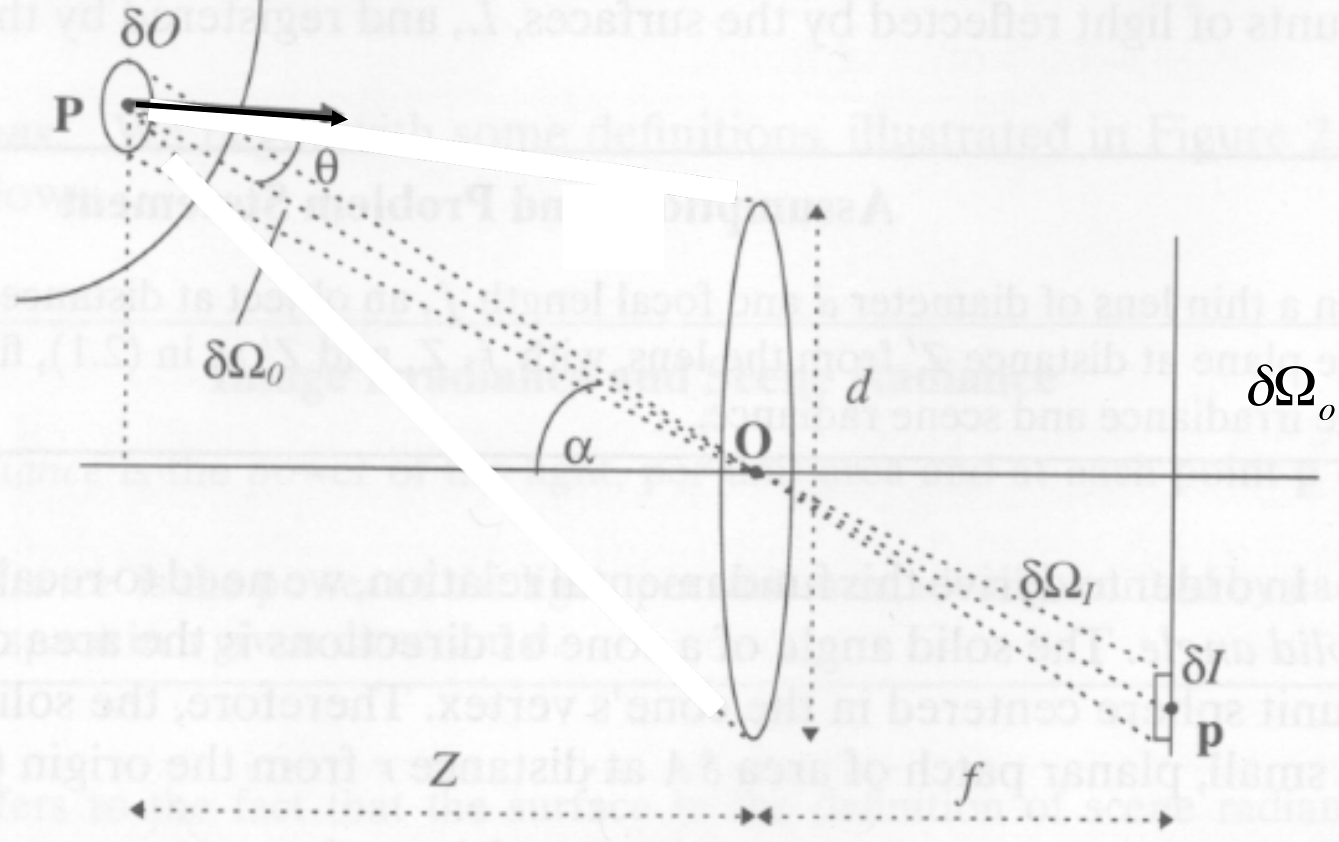


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_o = \frac{\delta O \cos\vartheta}{R^2} = \frac{\delta O \cos\vartheta}{\left(Z / \cos\alpha \right)^2}$$

Solid Angle Subtended by Image Patch

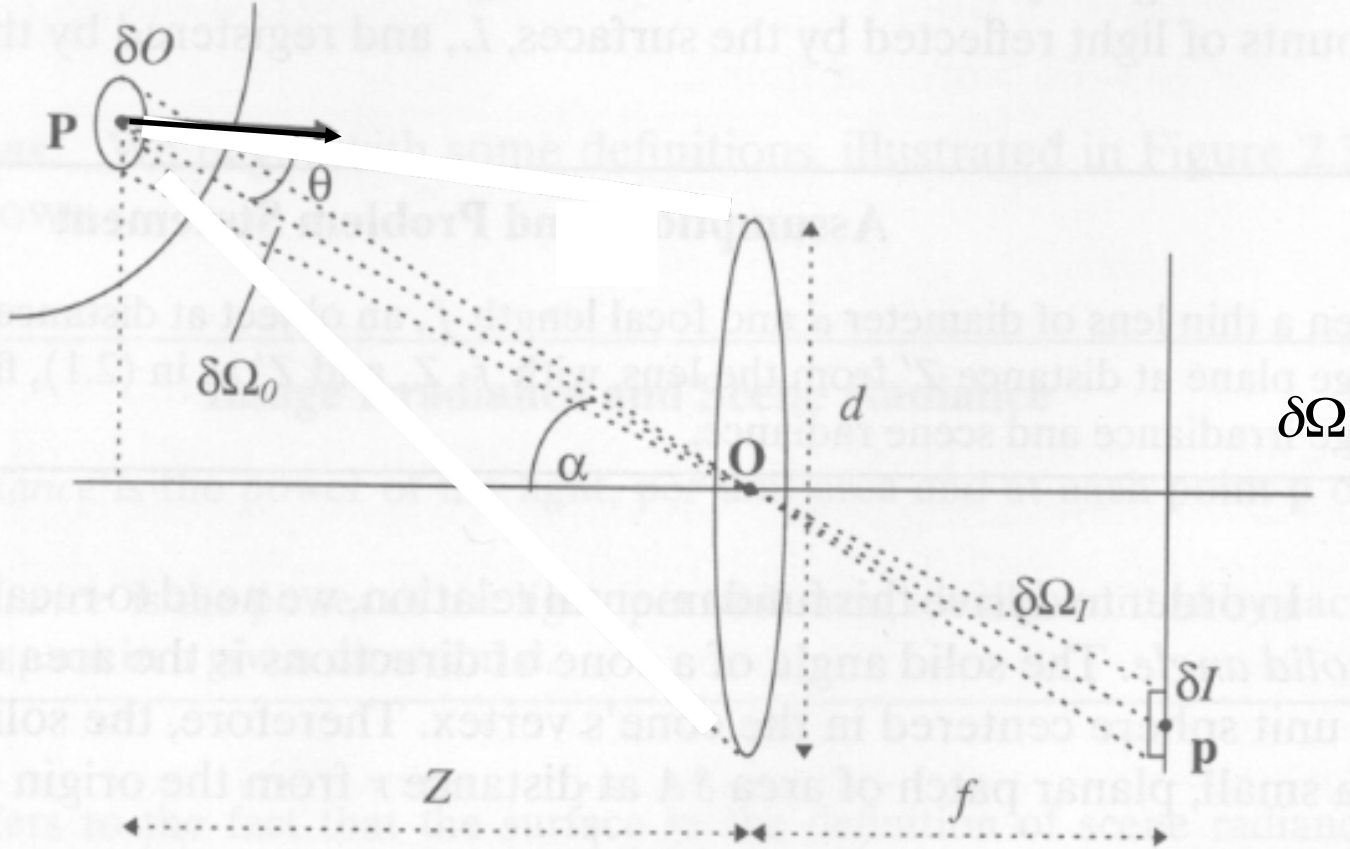


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_i = \frac{\delta I \cos\alpha}{r^2} = \frac{\delta I \cos\alpha}{\left(f/\cos\alpha\right)^2}$$

Similar Solid Angles

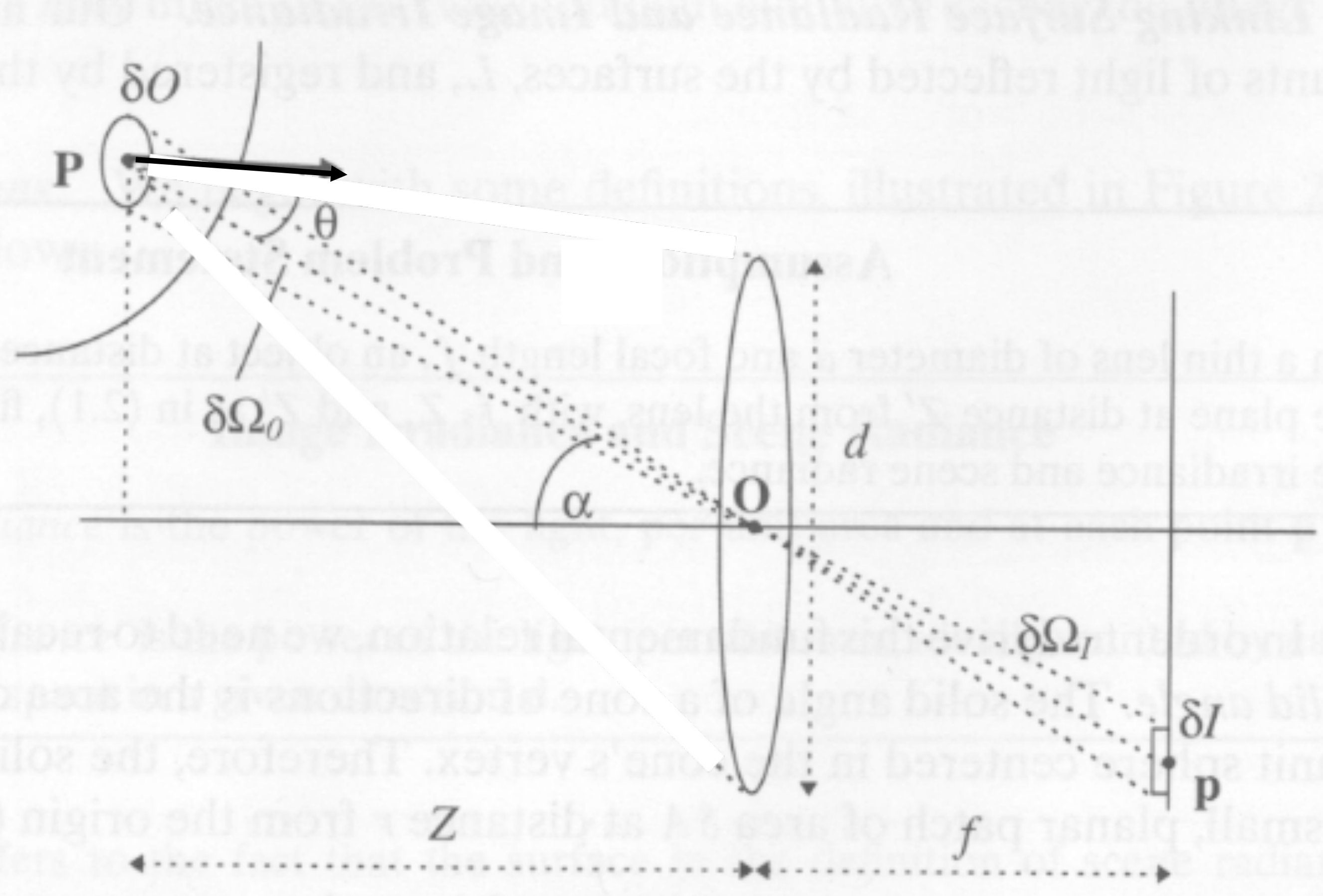


Figure 2.7 Radiometry of the image formation process.

$$\delta\Omega_o = \frac{\delta O \cos\vartheta}{\left(Z / \cos\alpha\right)^2}$$

$$\delta\Omega_i = \frac{\delta I \cos\alpha}{\left(f / \cos\alpha\right)^2}$$

$$\delta\Omega_o = \delta\Omega_i \Rightarrow$$

$$\Rightarrow \frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\vartheta}\left(\frac{Z}{f}\right)^2$$

Solid Angle Subtended by the Lens at P

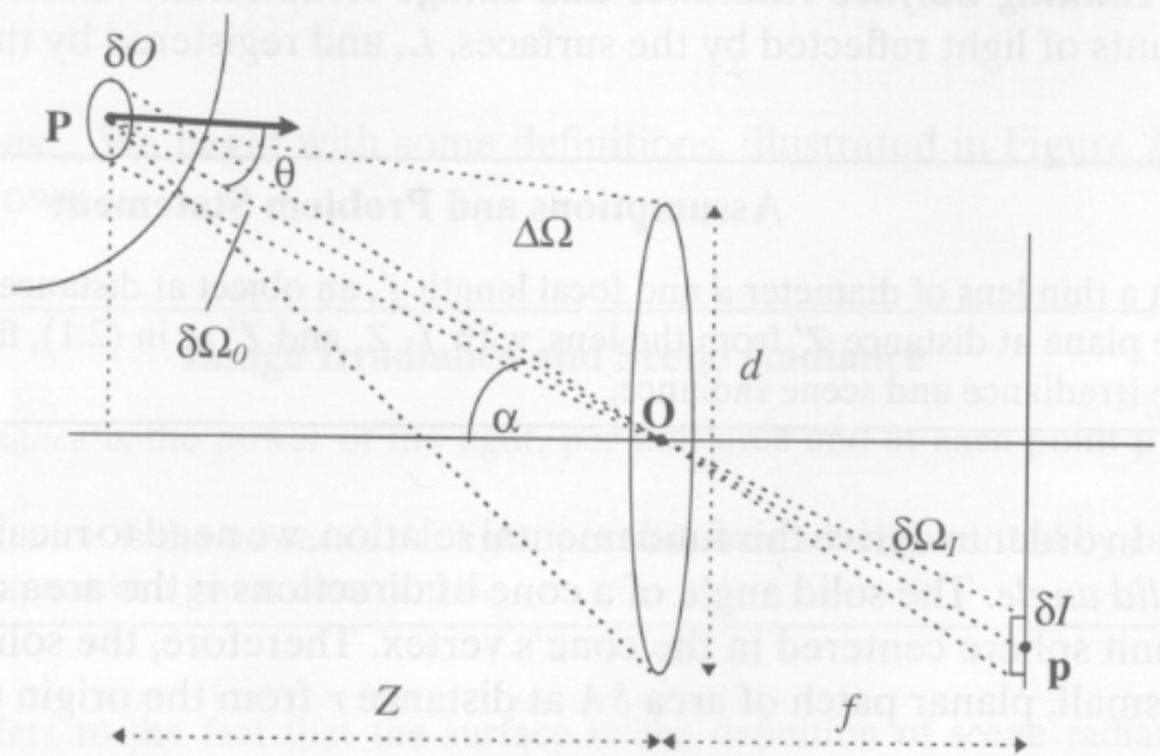


Figure 2.7 Radiometry of the image formation process.

$$\Delta\Omega = \frac{\frac{\pi}{4} d^2 \cos\alpha}{R^2} = \frac{\pi}{4}\frac{d^2\cos\alpha}{\left(\frac{Z}{\cos\alpha}\right)^2} = \frac{\pi}{4}\frac{d^2}{Z^2}\cos^3\alpha$$

Radiance Incident on the Lens

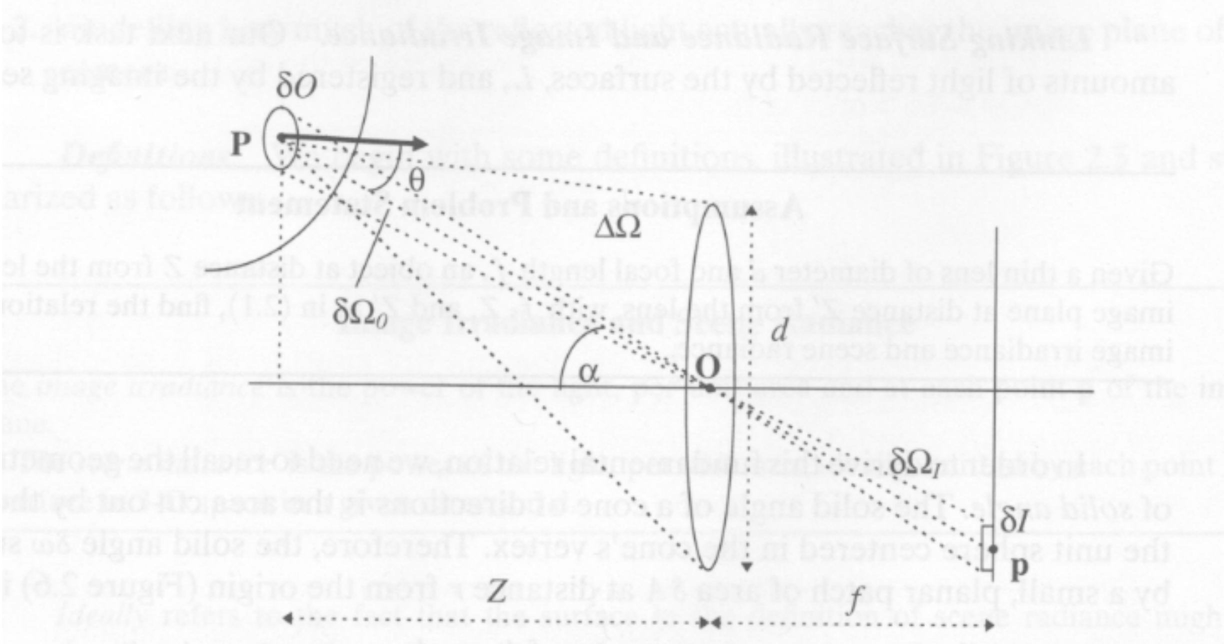


Figure 2.7 Radiometry of the image formation process.

The scene radiance incident on the lens from a scene point P is:

$$L = \frac{\delta P}{\Delta\Omega\,\delta O \cos\vartheta} = \frac{\delta P}{\frac{\pi}{4}\frac{d^2}{Z^2}\cos^3\alpha\,\delta O\cos\vartheta}$$

Powerloss Transfer through the Lens

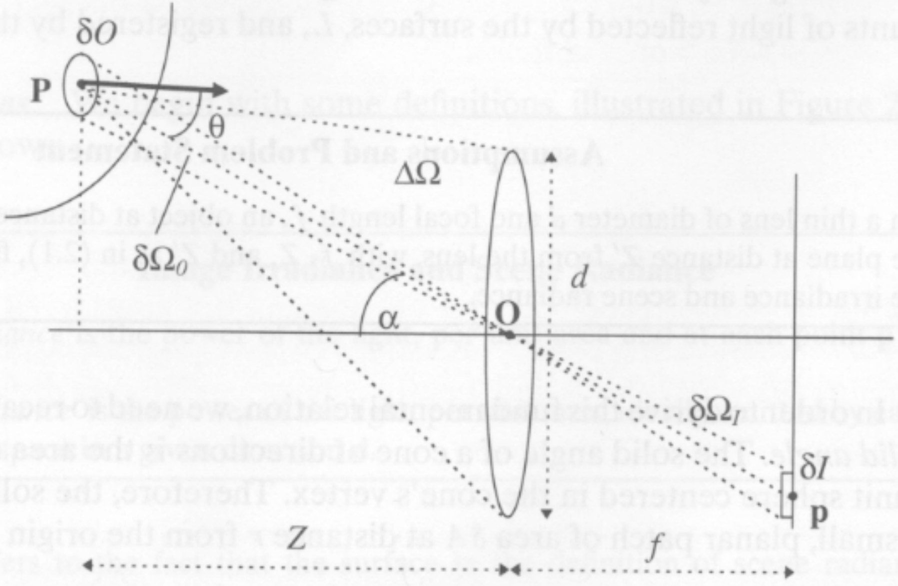


Figure 2.7 Radiometry of the image formation process.

In a perfect lens, all the light power incident on it, is focused on point p.

$$\delta P = L \frac{\pi}{4} \frac{d^2}{Z^2} \cos^3 \alpha \delta O \cos \vartheta$$

Irradiance Incident on Image Point p

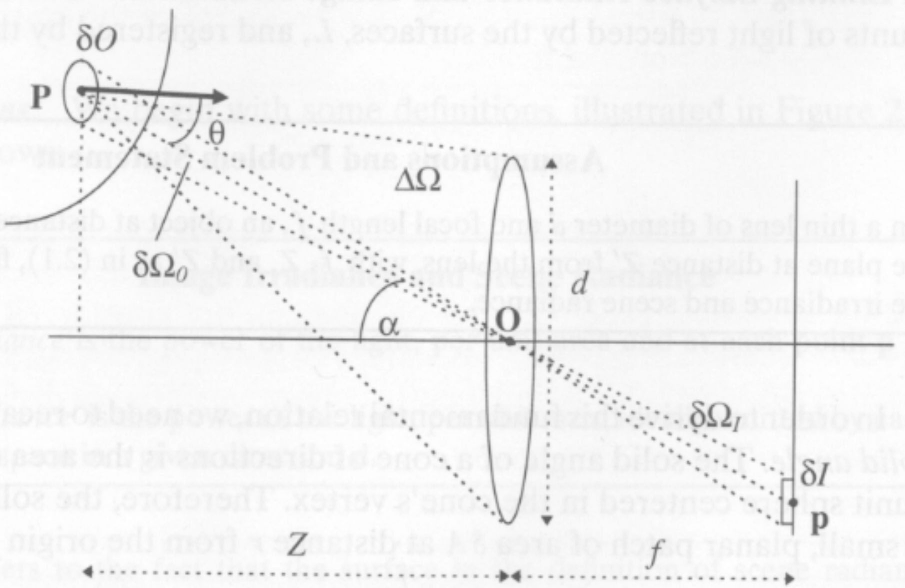


Figure 2.7 Radiometry of the image formation process.

Thus the incident irradiance at point p is:

$$E = \frac{\delta P}{\delta I} = \frac{L \frac{\pi}{4} \frac{d^2}{Z^2} \delta O \cos\vartheta \cos^3\alpha}{\delta I}$$

Recall that:

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\vartheta}\left(\frac{Z}{f}\right)^2$$

$$\Rightarrow E = L \frac{\pi}{4} \frac{d^2}{f^2} \cos^4\alpha$$

Light and Surfaces

- Surfaces can absorb, transmit (transparent objects), scatter, reemit, or reflect the light incident on them.
- The **Bidirectional Reflectance Distribution Function** (BRDF) describes the relationship between the light falling on a surface patch and the light leaving that surface patch.
- BRDF is the defined as the ratio of the radiance in the outgoing direction to the incident irradiance.

$$BRDF(\vartheta_r,\phi_r,\vartheta_i,\phi_i) = f(\vartheta_r,\phi_r,\vartheta_i,\phi_i) = \frac{dL_r(\vartheta_r,\phi_r)}{dE_i(\vartheta_i,\phi_i)}$$

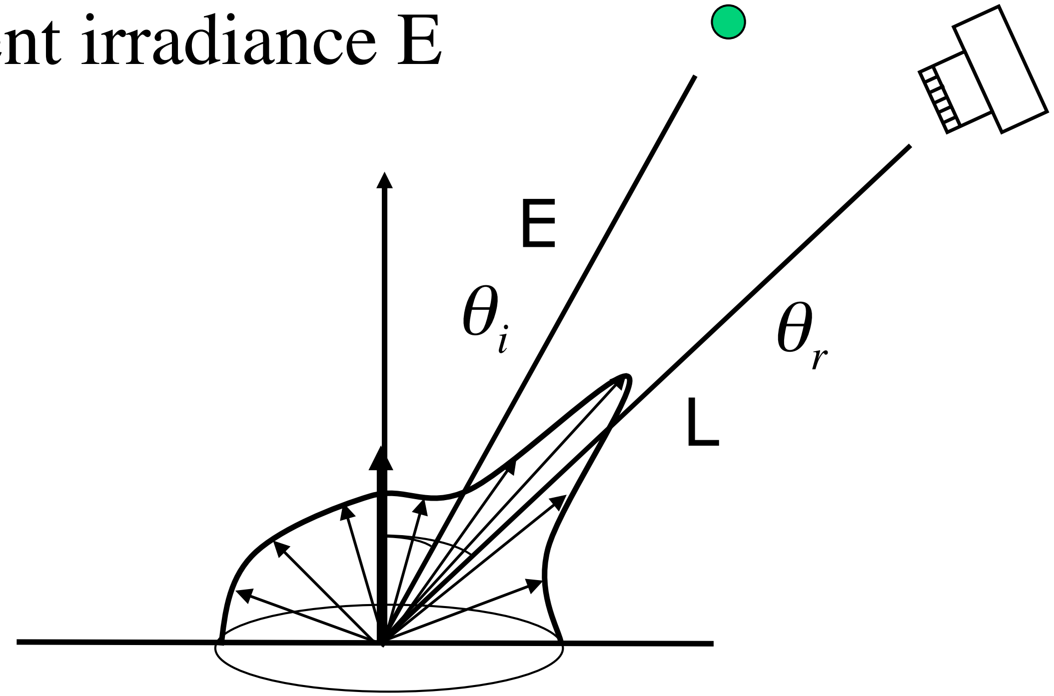
BRDF

- BRDF is measured in $\frac{1}{sr}$.
- It is the ratio of the power of light leaving an infinitesimal surface patch centered around ***P*** in the direction (ϑ_r, ϕ_r) over the the power of light falling on that surface patch from the direction (ϑ_i, ϕ_i).
- Helmholz reciprocity principle: The BRDF is symmetric in the ingoing and outgoing directions.
- BRDF assumptions:
  - No wavelength shifting (i.e. no fluorescence)
  - Surfaces are not generating light
  - Departing radiance is only due to incident irradiance.

BRDF Figure

Bidirectional Reflectance Distribution Function (BRDF):

$$f(\theta_r,\phi_r;\theta_i,\phi_i) = \frac{\text{reflected radiance L}}{\text{incident irradiance E}}$$



Slide courtesy of Frank Dellaert, http://www.cc.gatech.edu/~dellaert/vision/slides/04-Radiometry.ppt

BRDF (continued)

- If we know the BRDF of a surface and the incident irradiance from a specific direction (ϑ_i, ϕ_i), then we know the radiance leaving the surface:

$$dL_r(\vartheta_r, \phi_r) = f(\vartheta_r, \phi_r, \vartheta_i, \phi_i) dE_i(\vartheta_i, \phi_i)$$

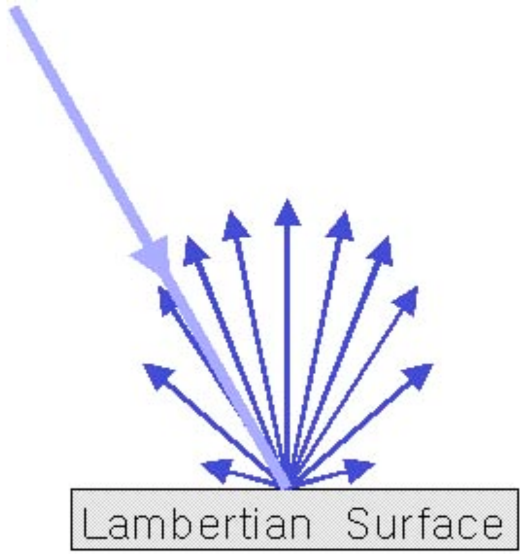
- We can measure the power of light leaving a surface patch and travelling towards our camera caused by **all** the light falling on that surface patch by integrating over the entire hemisphere of incident illumination:

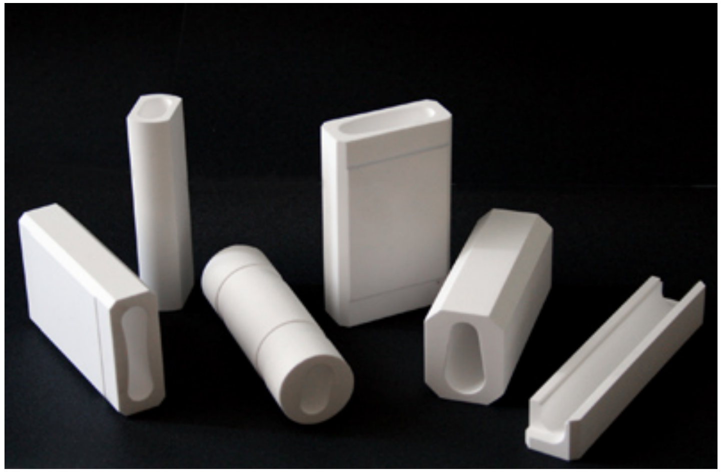
$$dL_r(\vartheta_r, \phi_r) = \int_0^{2\pi} \int_0^{\pi/2} f(\vartheta_r, \phi_r, \vartheta_i, \phi_i) dE_i(\vartheta_i, \phi_i) \sin\vartheta_i \, d\vartheta_i \, d\phi_i$$

Perfectly Diffuse Surfaces

- For perfectly diffuse surfaces (Lambertian surfaces, perfectly matte surfaces), the BRDF is independent from the outgoing direction:

$$dL_{r,Lambertian} = 1/\pi$$





Graph courtesy of T. Cummings, http://laser.physics.sunysb.edu/~thomas/report2/reflection.html

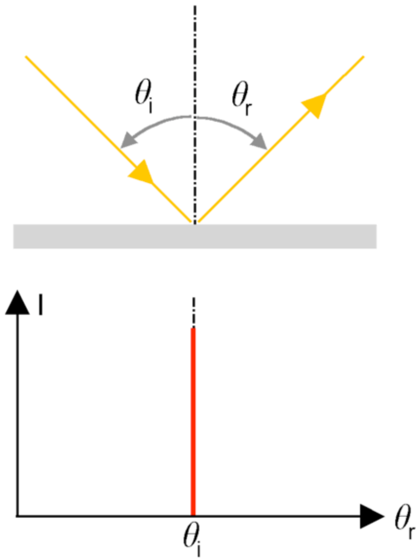
A picture of objects made of Spectralon, a material that is 99% Lambertian. Image courtesy of Labsphere.

Perfectly Specular Surfaces

- For perfectly specular surfaces (i.e. mirror-like surfaces), all the exiting radiance occurs in only direction, obtained by reflecting the incoming direction about the surface normal. Everywhere else it is zero:

$$dL_{r,Specular} = \frac{\delta(\vartheta_r - \vartheta_i)\delta(\phi_r - \phi_i - \pi)}{\sin\vartheta_i \cos\vartheta_i}$$

"Cloud Gate" sculpture by Anish Kapoor. Picture courtesy of Outokumpu, http://www.outokumpu.com

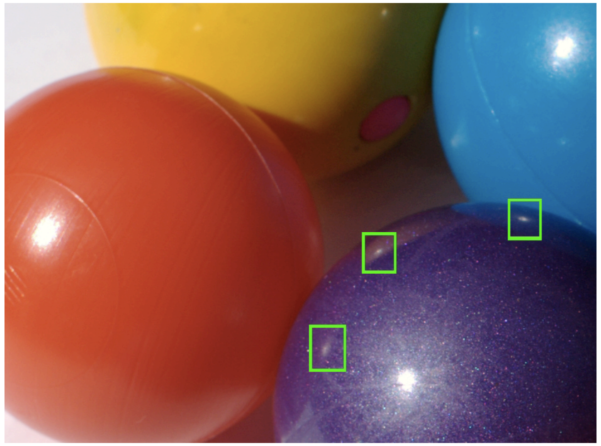


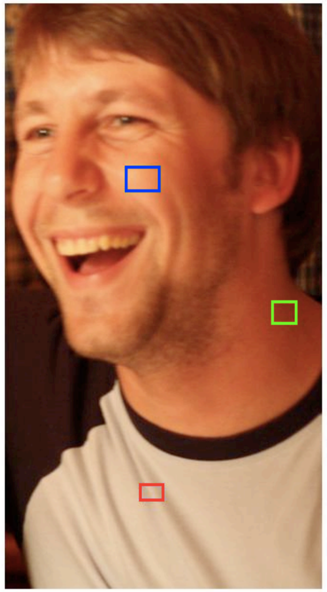
Graph courtesy of wikipedia

Mixed Surfaces

- Most surfaces exhibit a mixture of diffuse and specular behavior:

$$dL_{r,Mixed} = n\,dL_{r,Lambertian} + (1-n)\,dL_{r,Specular}$$





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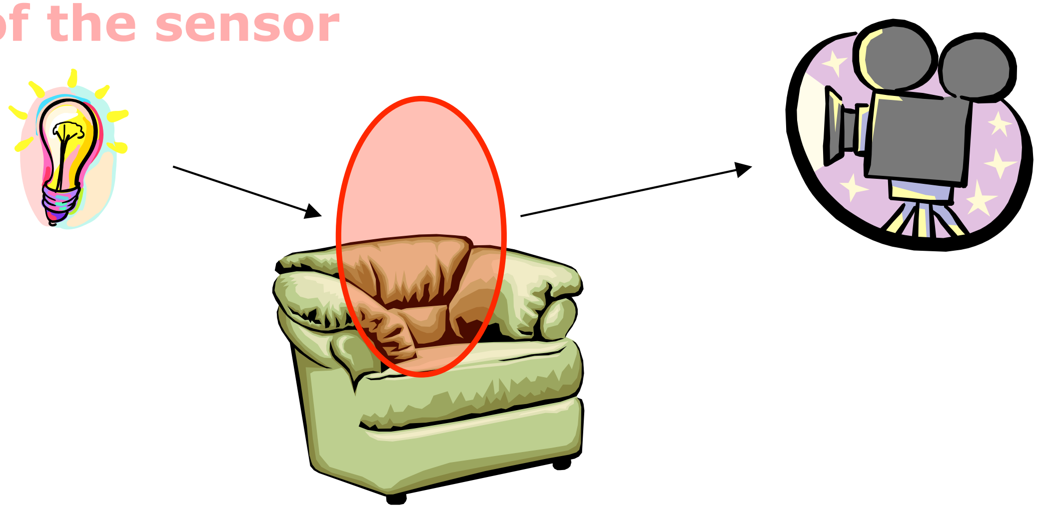


Camera Coordinate System

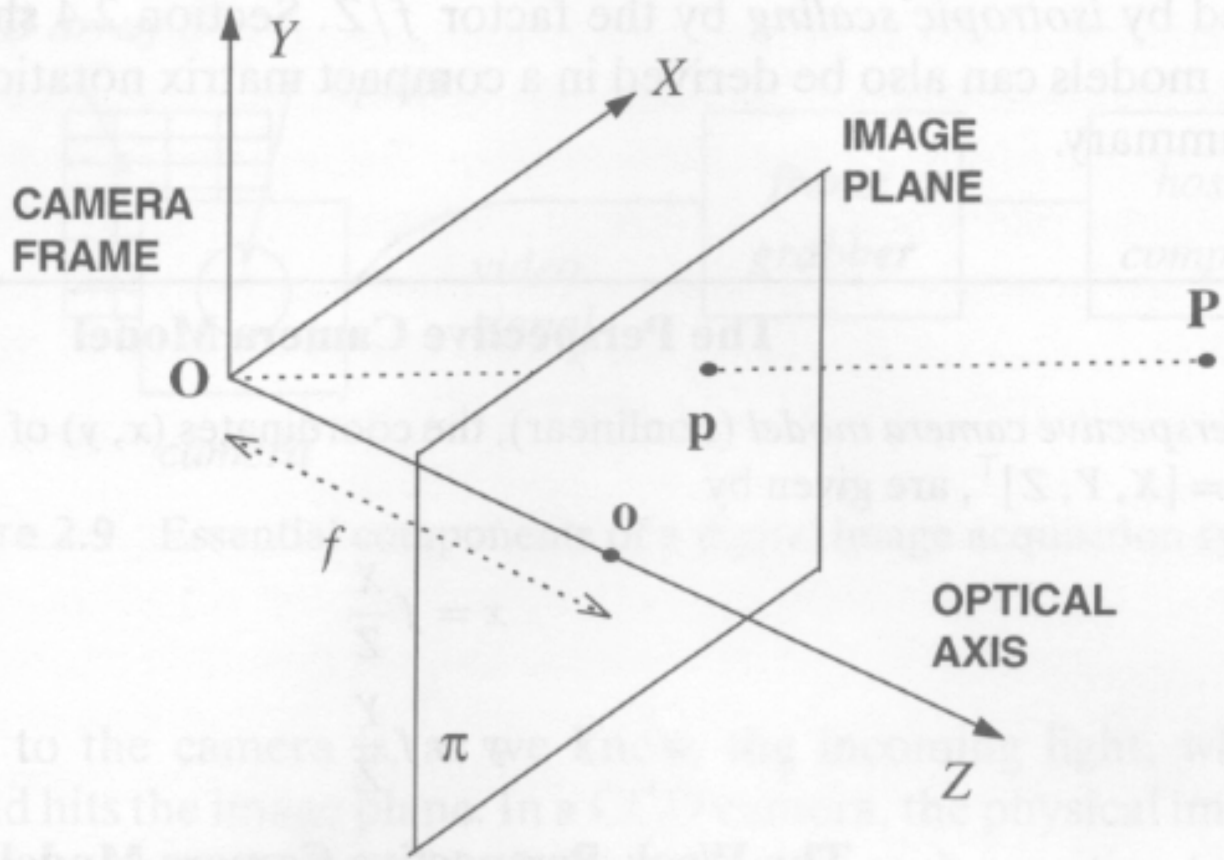


Figure 2.8 The perspective camera model.

Camera Coordinate Frame

- Right-handed coordinate system.
- Origin, O: center of projection (COP)
- z-axis: perpendicular to the image plane, passes through the COP, points towards the scene, same as the optic axis.
- The intersection of the z-axis with the image plane is called the **principal point** or **image center**.
- x-axis: horizontal axis, parallel to the image plane
- y-axis: vertical axis, pointing upwards, parallel to the image plane.

Perspective Projection

- The pinhole camera model implies perspective projection, i.e. all projection rays pass through a single point (COP).
- Let the image point p=(x,y,z) be the projection of the scene point P=(X,Y,Z).
- The image plane is positioned at distance f from the origin.
- From similar triangles we obtain:

$$x = \left(\frac{f}{Z}\right)X = -mX \qquad y = \left(\frac{f}{Z}\right)Y = -mY$$

where $m = -\frac{f}{Z}$ is the magnification factor.

Comments on Perspective Projection

- The perspective projection eqs. $x = \left(\frac{f}{Z}\right)X$ and $y = \left(\frac{f}{Z}\right)Y$ express how objects that are farther away appear smaller.
- If we know the characteristics of the camera (its position, orientation and focal length) we can compute the exact position where a scene point P will appear on the image plane.
- Limitation: Non-linear model. The factor $\frac{1}{Z}$ does not preserve distances between points.
- The same scene length *L* will map to different image lengths *l*, depending on its distance to the camera.

Weak Perspective Projection

- When the range of depth values in the scene is small relative to the average distance from the scene to the camera, we assume that the magnification factor *m* is constant.

$$m = -\frac{f}{\overline{Z}}$$

  where $\overline{Z}$ is the average *Z* value over all the points in the scene.

- The weak perspective projection equations are:

$$x = \left(\frac{f}{\overline{Z}}\right) X \qquad y = \left(\frac{f}{\overline{Z}}\right) Y$$

Orthographic Projection

- Assume that the scene and the camera (relative to the scene) are fixed.
- One can then normalize the image coordinates so that:

$$x = X \qquad y = Y \qquad m = -1$$

- Under orthographic projection, there is no longer a Center of Projection. All projection rays are parallel to each other and perpendicular to the image plane.

Perspective vs. Orthographic Projection

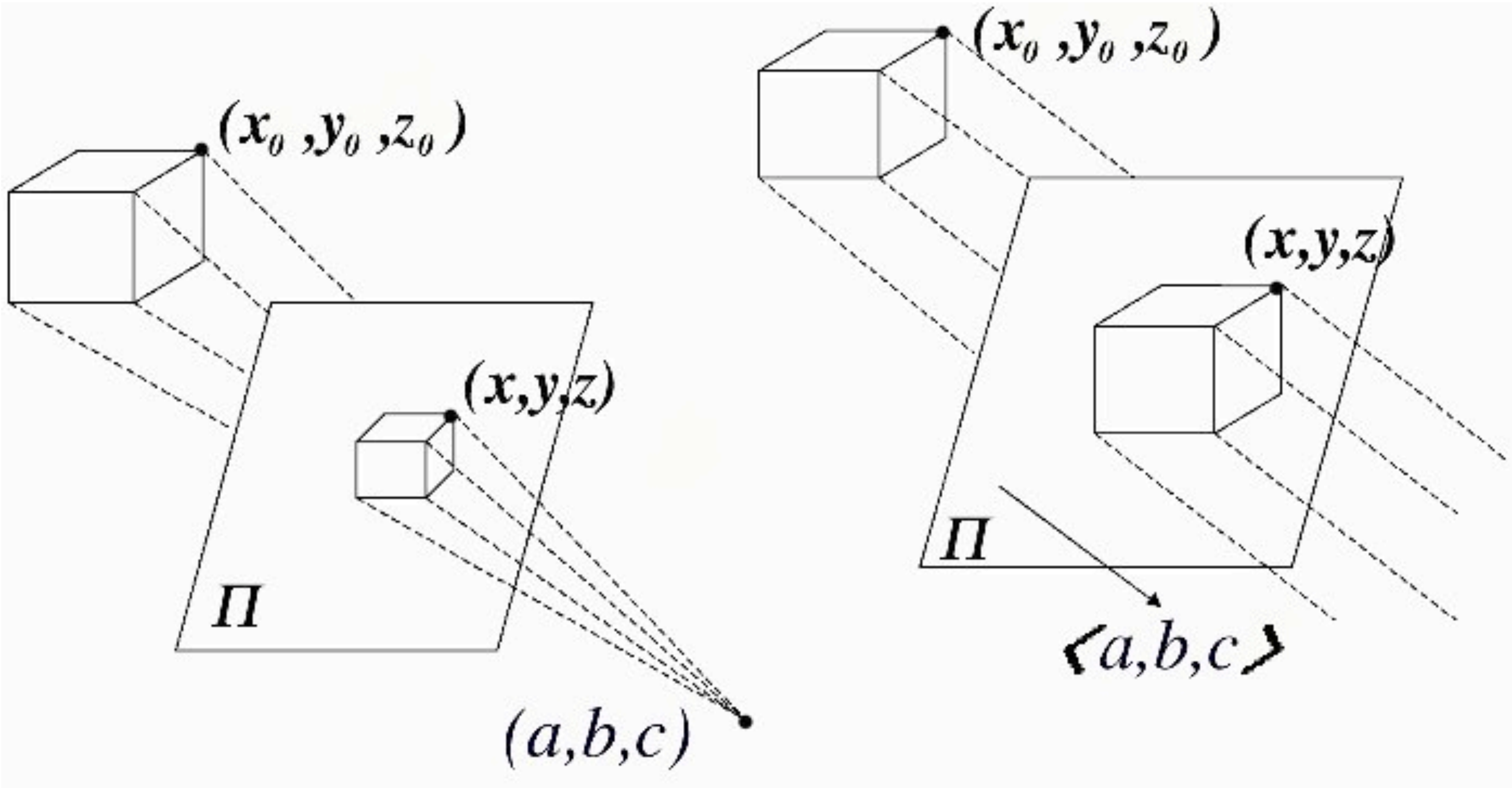


Figure courtesy of MathDL http://mathdl.maa.org/images/cms_upload/

Camera Position in the World

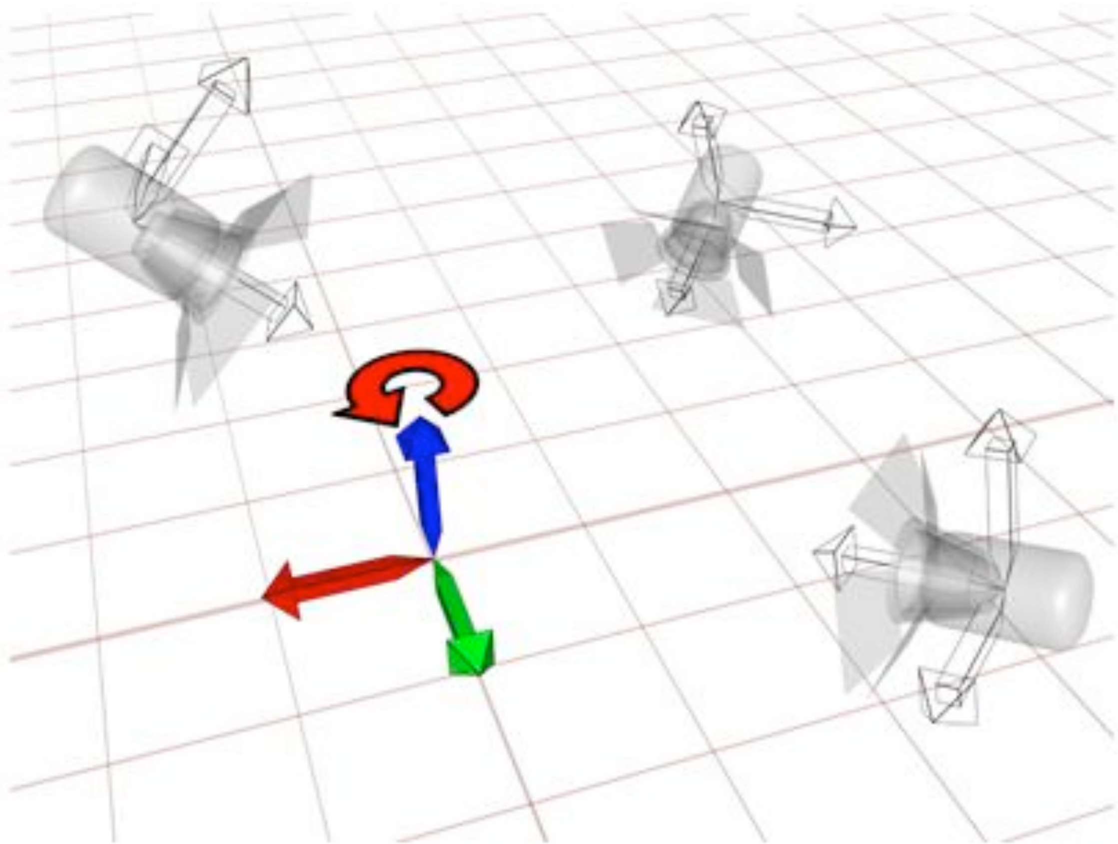


Image courtesy of AutoDesk 3DS Max 9 Reference http://www.kxcad.net/autodesk/3ds_max/Autodesk_3ds_Max_9_Reference/use_transform_coordinate_center.html

Extrinsic Camera Parameters

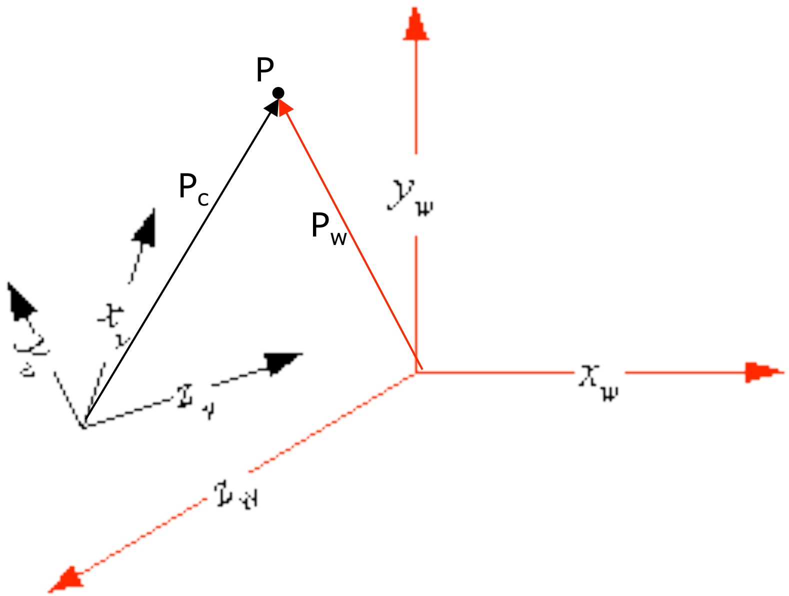


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- Extrinsic parameters: A set of geometric parameters that uniquely identify the transformation between the unknown camera frame and a known reference frame (the world reference frame).

Extrinsic Camera Parameters

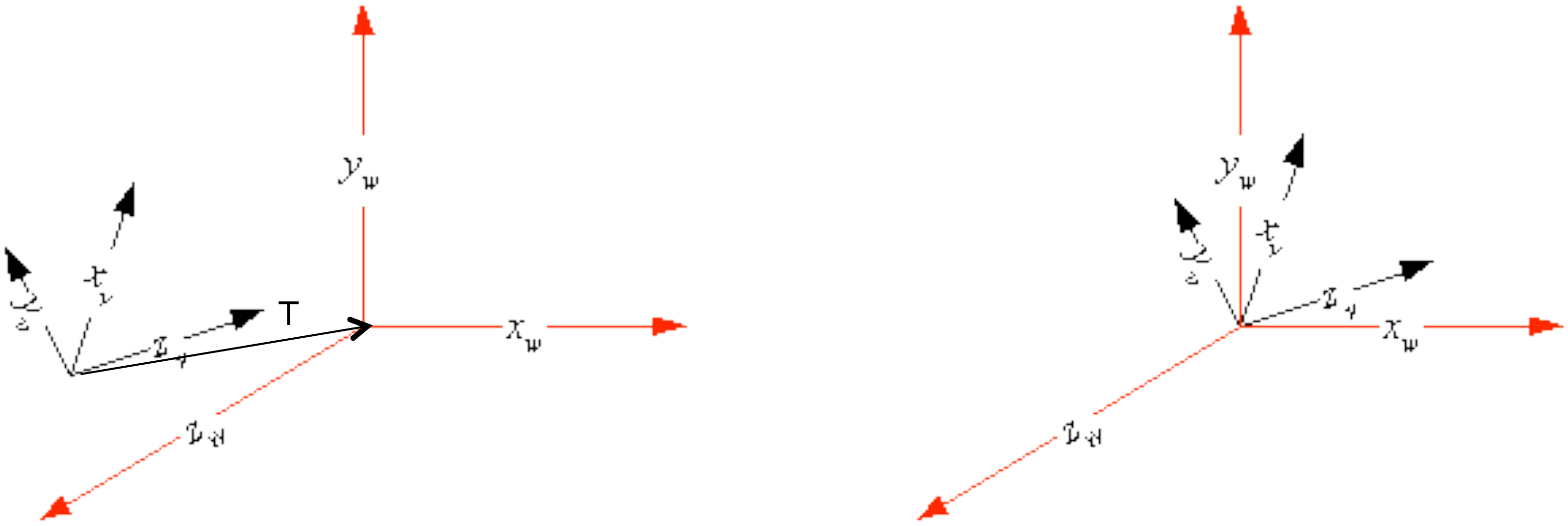


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- T: a 3D translation vector describing the relative position of the 2 origins (camera and world origins)

Extrinsic Camera Parameters

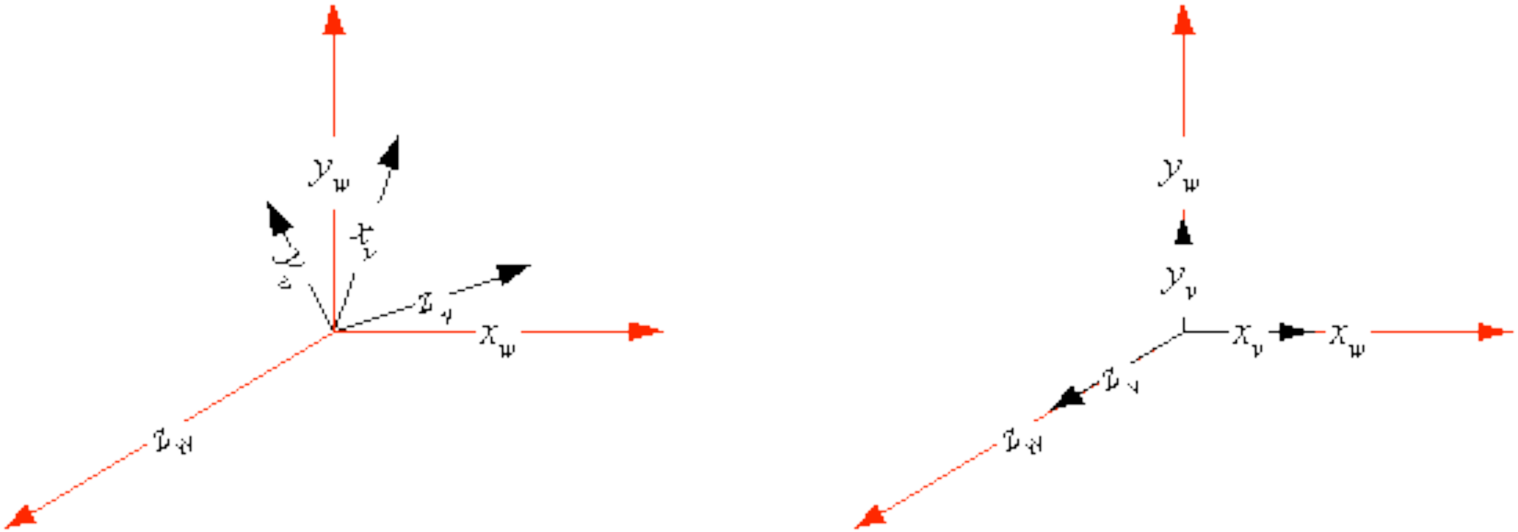


Figure courtesy of Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- R: a 3x3 rotation matrix that aligns the axes of the two frames (camera and world frame)

Extrinsic Camera Parameters

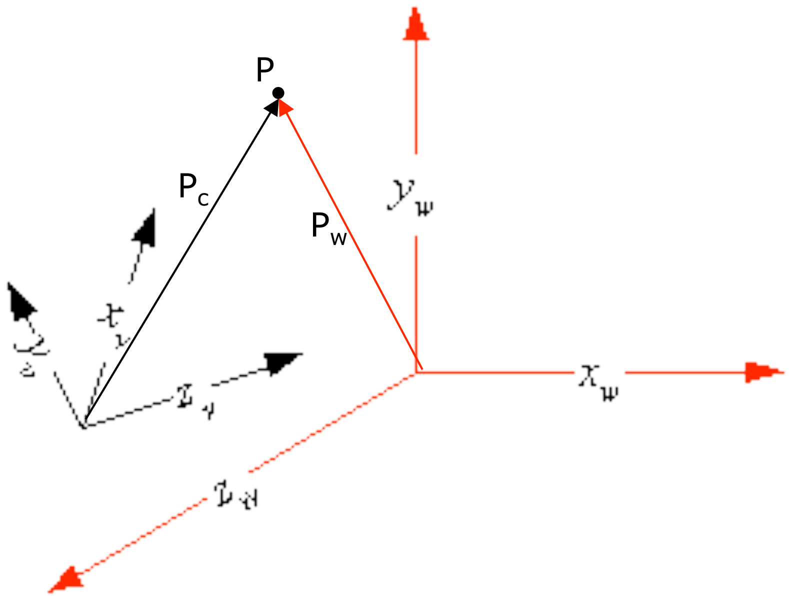


Figure adapted from Dr. Rheingans' Computer Graphics notes http://www.cs.umbc.edu/~rheingan/435/pages/res/gen-8.Viewing-single-page-0.html

- The relationship between the coordinates of a point P in world P_w and camera P_c frames is:

$$P_c = R(P_w - T)$$

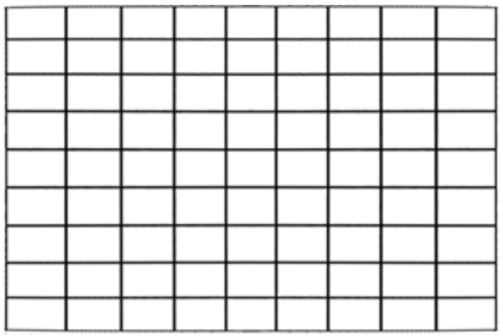
Intrinsic Camera Parameters

- Intrinsic parameters: A set of geometric parameters that link the pixel coordinates of an image point to the corresponding coordinates in the camera reference frame.
- (x_{im}, y_{im}) : image reference frame, i.e. pixel coordinates.
- (o_x, o_y) : pixel coordinates of the image center, i.e. where the optic axis intersects the image plane.
- (s_x, s_y) : effective size of pixel in mm.

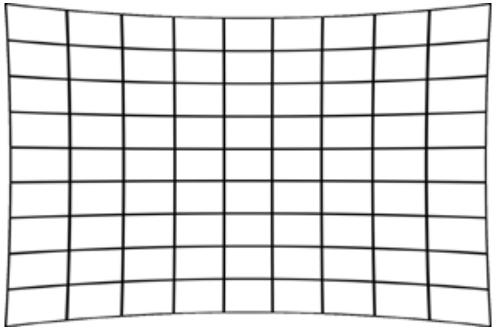
$$x_c = -(x_{im} - o_x)s_x \qquad y_c = -(y_{im} - o_y)s_y$$

Radial Distortion

- A pin-hole image of a square grid.



- A lens-system image of the same square grid



- The amount of distortion depends on the distance between the principal point and the pixel of interest.
- Let (x_d, y_d) be the coordinates of the distorted point in the image coordinate system.

Radial Distortion Correction

Image courtesy of VIPBase

- The undistorted (corrected) image coordinates are:

$$x = x_d(1 + k_1 r^2 + k_2 r^4) \qquad y = y_d(1 + k_1 r^2 + k_2 r^4) \qquad r^2 = x_d^2 + y_d^2$$

- Typically $k_2 << k_1$, so we often set $k_2 = 0$