



General Information:

Lecture (3 SWS): Thu 14.15 – 15.45 (H16) and Tue 12.15 – 13.45 (H16)
Exercises (1 SWS): Mo 12.15 – 13.45 (02.134-113) and Tue 12.15 – 13.45 (E1.12)
Certificate: Oral exam at the end of the semester
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Probability Density Estimation

Exercise 1 Probability density estimation based on the Parzen Window approach can be utilized for the Bayes classifier to determine a decision boundary. Let us assume we have n training samples available whereas n_i samples belong to class y_i .

- Write down the class conditional probability density $p(\mathbf{x}|y_i)$ for class y_i in terms of Parzen window estimation.
- Approximate the prior $P(y_i)$ probability for class y_i .
- Write down the decision rule for the Bayes classifier using the approximated class conditional density as well as the estimated prior.

Exercise 2 Let $p(x)$ be an exponential density with parameter $\lambda > 0$:

$$p(x) = \begin{cases} \lambda \exp(-\lambda x), & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

- Use the window function

$$\varphi(x) = \begin{cases} \exp(-x), & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

to approximate $p(x)$ and calculate the mean of the Parzen window estimation $\bar{p}(x)$.

Hint: $\bar{p}(x)$ is a probability density function since the samples used for Parzen window estimation are drawn out of an exponential distribution. This density is equal to the expected value of all suitable Parzen window estimates.

- Calculate the expected value $\bar{\mu}$ of the mean density $\bar{p}(x)$ and prove that $\bar{\mu}$ converges to the expected value $\mu = 1/\lambda$ if the kernel width h approaches infinity.

Exercise 3 Some image processing techniques, e.g. registration, utilize the *mutual information* of two signals. Familiarize yourself with the concept mutual information. How does it relate to statistical dependency? How can Parzen Windows help when using mutual information?