Exercises for
Introduction to Pattern Recognition (IntroPR)
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Image Filtering and Thresholds

Exercise 13 Find out what the terms separability and cascading are with respect to filtering methods, and describe how they can help to speed up Gaussian smoothing.

Exercise 14 Compare the time complexity of convolution with a $n \times n$ kernel when using

(a) direct convolution with the 2-D mask,

(b) a separable kernel, and

(c) cascading with a separable kernel.

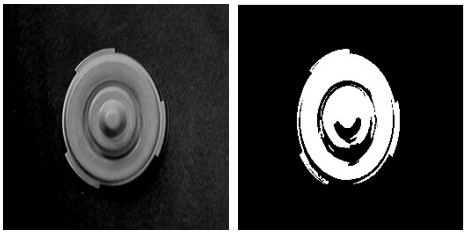
Exercise 15 Prove that convolving a 1-D signal twice with a Gaussian kernel of standard deviation σ is equivalent to convolving the signal with a Gaussian kernel of $\sigma_c = \sqrt{2} \cdot \sigma$, scaled by the area of the Gaussian filter.
Hint: Make use of the identity

$$\int_{-\infty}^{+\infty} e^{-ax^2+bx+c} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} e^{\frac{b^2-4ac}{4a}}$$

Exercise 16 Given images of size $N \times N$ that all show a bright object O in front of a dark background B. The probabilities of the grey values within the two regions are assumed to be normally distributed, with the probability densities p_O and p_B. The density of the grey values $x \in [0, 255]$ can then be characterized by the mixture

$$p(x) = \frac{1}{N^2}(|B|\ p_B(x) + |O| p_O(x)) \ , \tag{1}$$

where $|O|$ und $|B|$ denote the number of pixels of the object and the background.



(a) Draw a sketch that visualizes the densities of object and background pixels.

(b) Derive the rule to determine a threshold value θ that minimizes the number of wrongly assigned object and background pixels.

(c) Compute the threshold for the following example:
$|O| = |B|$, $\mu_B = 50$, $\mu_O = 200$, $\sigma_O^2 = \sigma_B^2 = 50$

(d) What consequences for the threshold has a change in the object size?