Structure Tensor

As explained in the lecture

$$J = \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}$$

J has always rank 1

$$\lambda_1 \gg \lambda_2 = 0$$

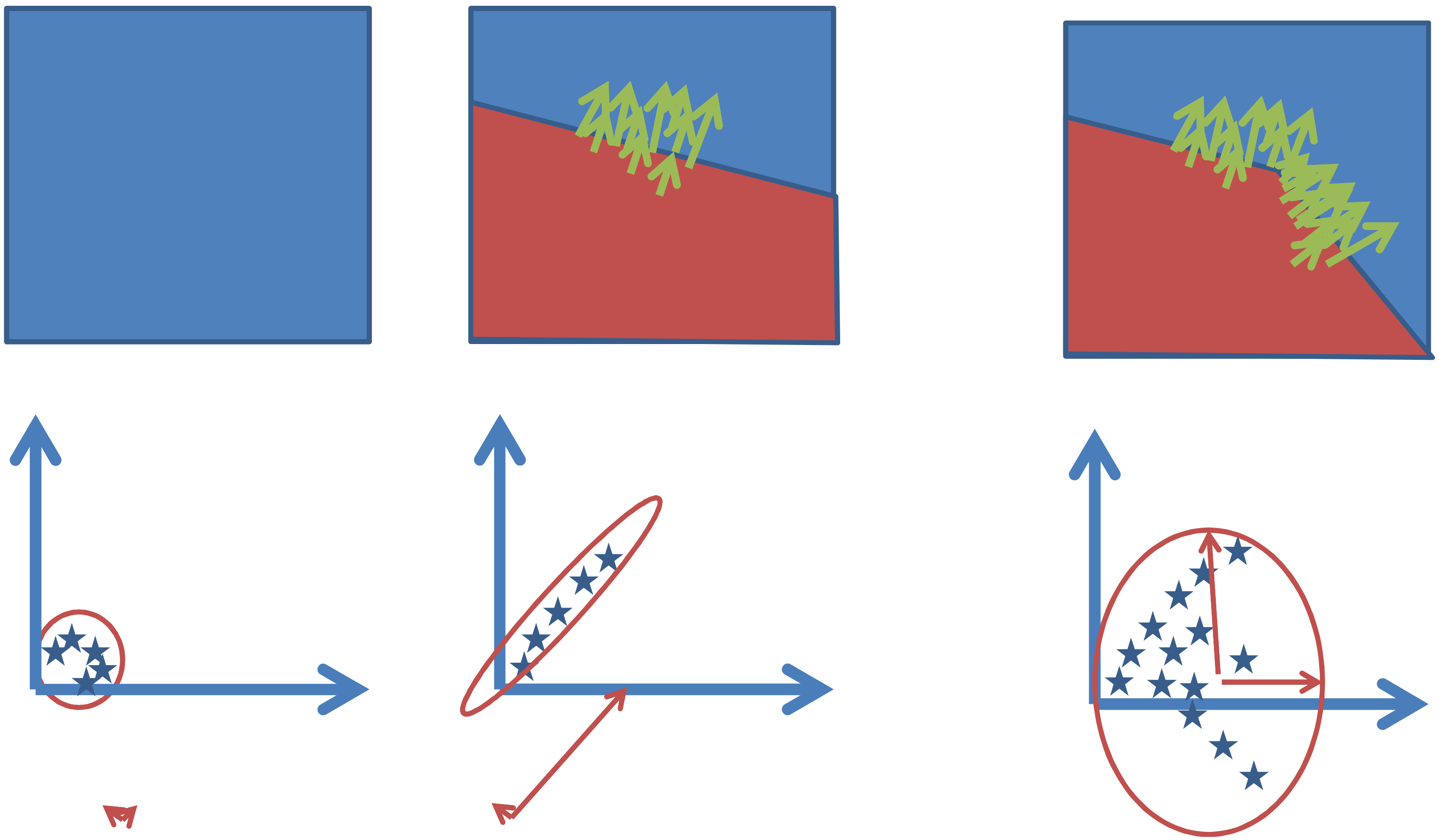
$\Rightarrow$We need a spatial averaging

$$J_\varrho = K_\varrho \star J$$

where $K_\varrho = \frac{1}{\varrho^2 2\pi} \exp\left(-\frac{x^2+y^2}{2\varrho^2}\right)$

Similar to covariance matrix

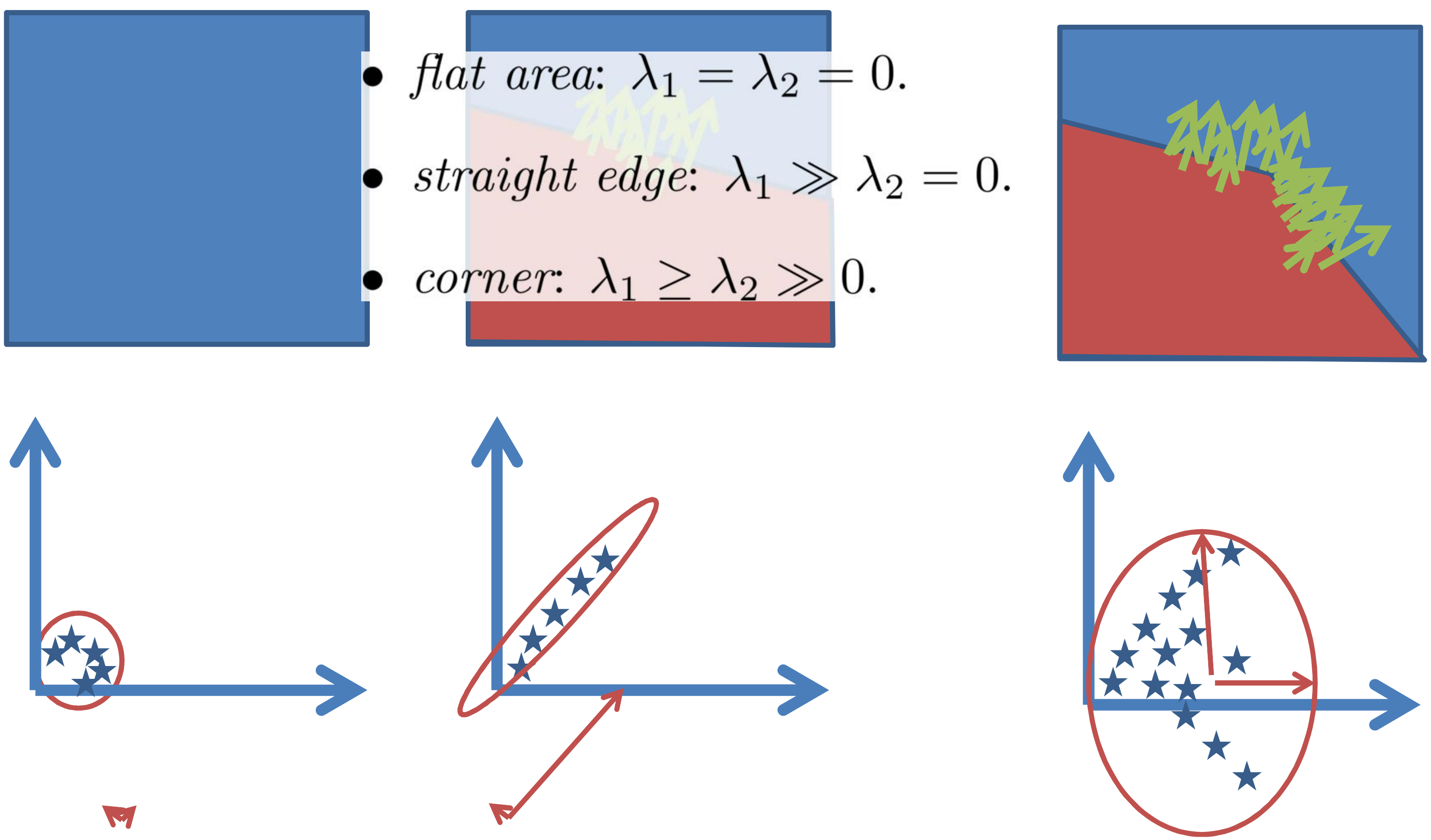
Variations of derivatives in a neighborhood



Similar to covariance matrix

Variations of derivatives in a neighborhood

- *flat area*: $\lambda_1 = \lambda_2 = 0.$
- *straight edge*: $\lambda_1 \gg \lambda_2 = 0.$
- *corner*: $\lambda_1 \geq \lambda_2 \gg 0.$



How to compute the structure tensor in practice

- Computation of f_x and f_y

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = \nabla f$$

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Rule of thumb:
Always prefer the computation of derivatives in continuous space to differentiation in discrete domain.

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DoGMask

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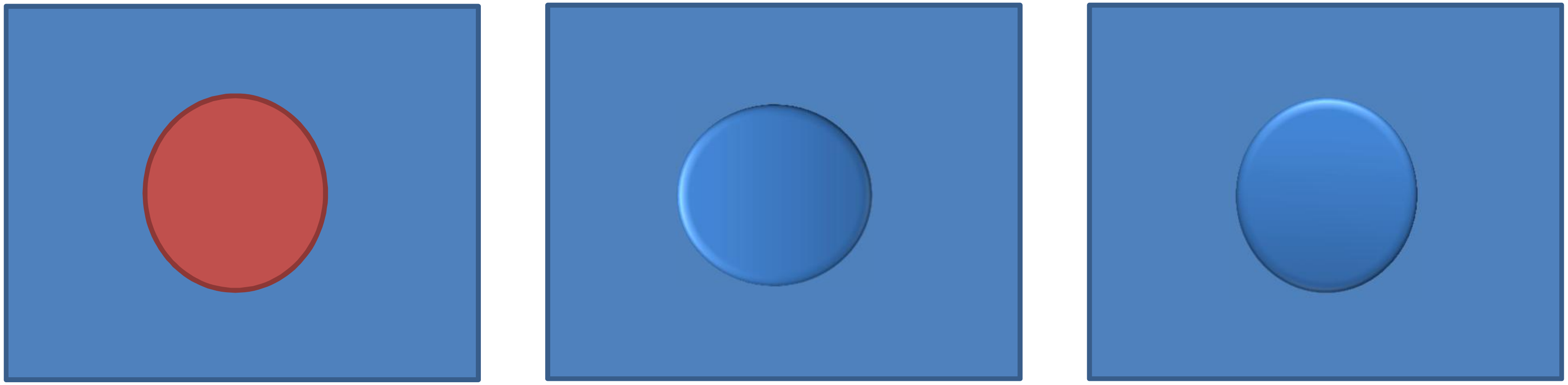
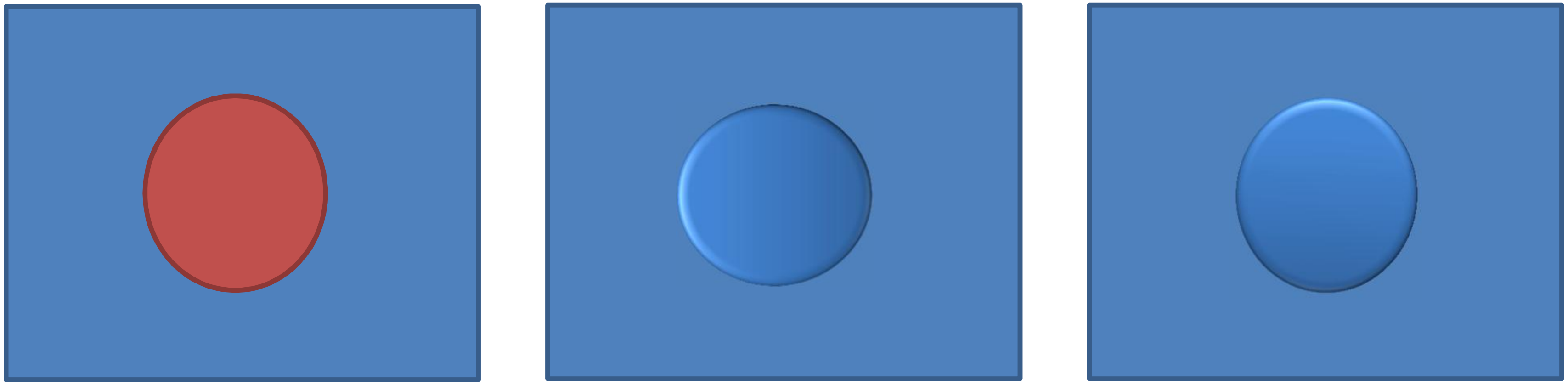


f $\quad f_x$ $\quad f_y$

So lets compute $f_x^2, f_x f_y, f_y^2$ by pixel wise multiplication!



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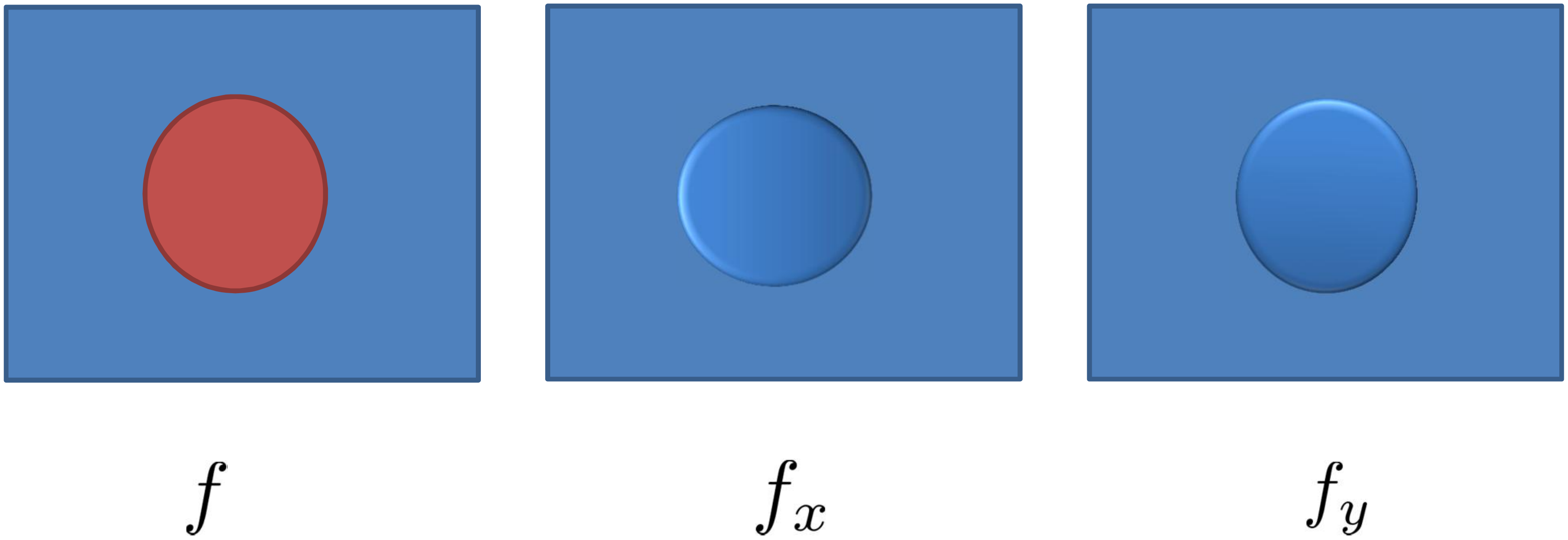

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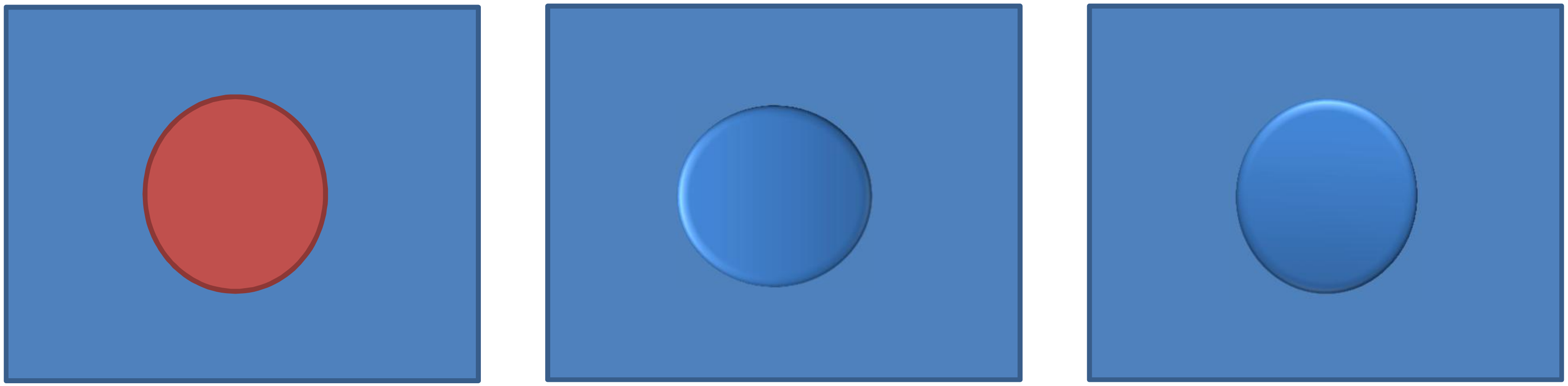

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=> Convolve Jxx, Jxy, Jyy with gaussian filter to obtain Qxx, Qxy, Qyy