Filtering and Edges

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Noise Sources

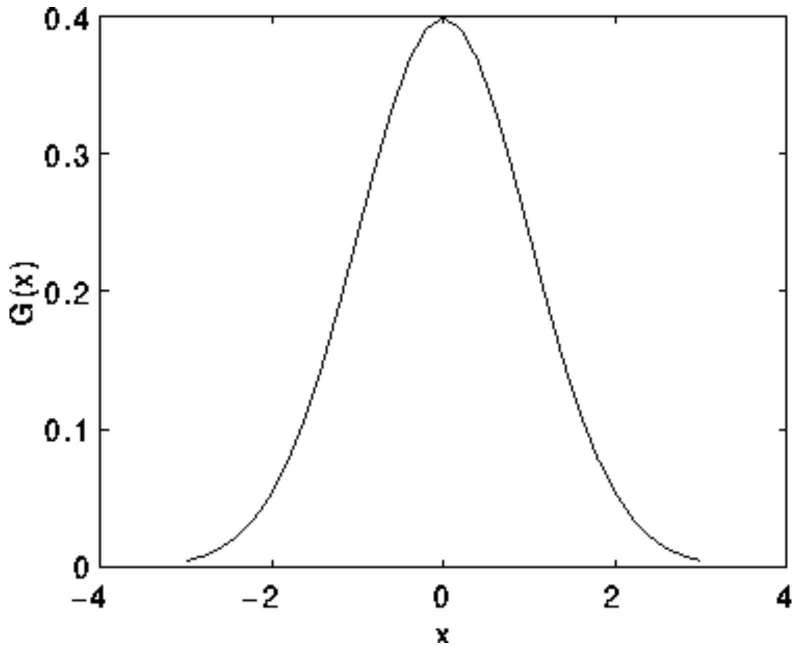
- Photon noise: variation in the #photons falling on a pixel per time interval T.
- Saturation: each pixel can only generate a limited amount of charge.
- Blooming: saturated pixel can overflow to neighboring pixels.
- Thermal noise: heat can free electrons and generate a response when there is none.
- Electronic noise.
- Burned pixels.
- Black is not black.



- Keep in mind: Camera response may not be linear over the number of photons falling on a surface (camera gamma)

Detector Noise

- Source of noise: the discrete nature of radiation, i.e. the fact that each imaging system is recording an image by counting photons.
- Can be modeled as an independent additive noise which can be described by a zero-mean Gaussian.



Salt and Pepper Noise

- A common form of noise is caused by *data drop-out noise.*
- It is also known as commonly referred to as intensity spikes, speckle or salt and pepper noise.
- Sources of error:
  - Errors in the data transmission.
  - Burned pixels: the corrupted pixels are either set to the maximum value (which looks like snow in the image) or are set to zero ("peppered" appearance), or a combination of the two.
  - Single bits are flipped over.
- Isolated/localized noise. It only affects individual pixels

Filtering

- Most of the images we capture are noisy
- Goal:

  Noisy Image_{in} → Filter → Clean Image_{out}

- This notion of filtering is more general and can be used in a wide range of transformations that we may want to apply to images.

  Image_{in} → Filter → Image_{out}

- Mathematically, a filter *H* can be treated as a function on an input image *I*:

$$H(I) = R$$

- Note: We use the terms *filter* and *transformation* interchangeably

Linear Transformation

- A transformation H is **linear** if, for any inputs $I_1(x,y)$ and $I_2(x,y)$ (in our case input images), and for any constant scalar α we have:

$$H(\alpha I_1(x,y)) = \alpha H(I_1(x,y))$$

and

$$H(I_1(x,y) + I_2(x,y)) = H(I_1(x,y)) + H(I_2(x,y))$$

- This means:
  - Multiplication in the input corresponds to multiplication in the output
  - Filtering an additive image is equivalent to filtering each image separately and then adding the results.

Shift-Invariant Transformation

- A transformation *H* is **shift-invariant** if for every pair (x_0, y_0) and for every input image $I(x,y)$, such that

$$H(I(x,y)) = R(x,y)$$

we get

$$H(I(x-x_0, y-y_0)) = R(x-x_0, y-y_0)$$

- This means that the filter *H* does not change as we shift it in the image (as we move it from one position to the next).

Convolution

- If a transformation (or filter) is linear shift-invariant (LSI) then one can apply it in a systematic manner over every pixel in the image.
- **Convolution** is the process through which we apply **linear shift-invariant filters** on an image.

- Convolution is defined as:

$$R(x,y)=\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}H(x-i,y-j)I(i,j)$$

and is denoted as:

$$R=H*I$$

Another Look at Convolution

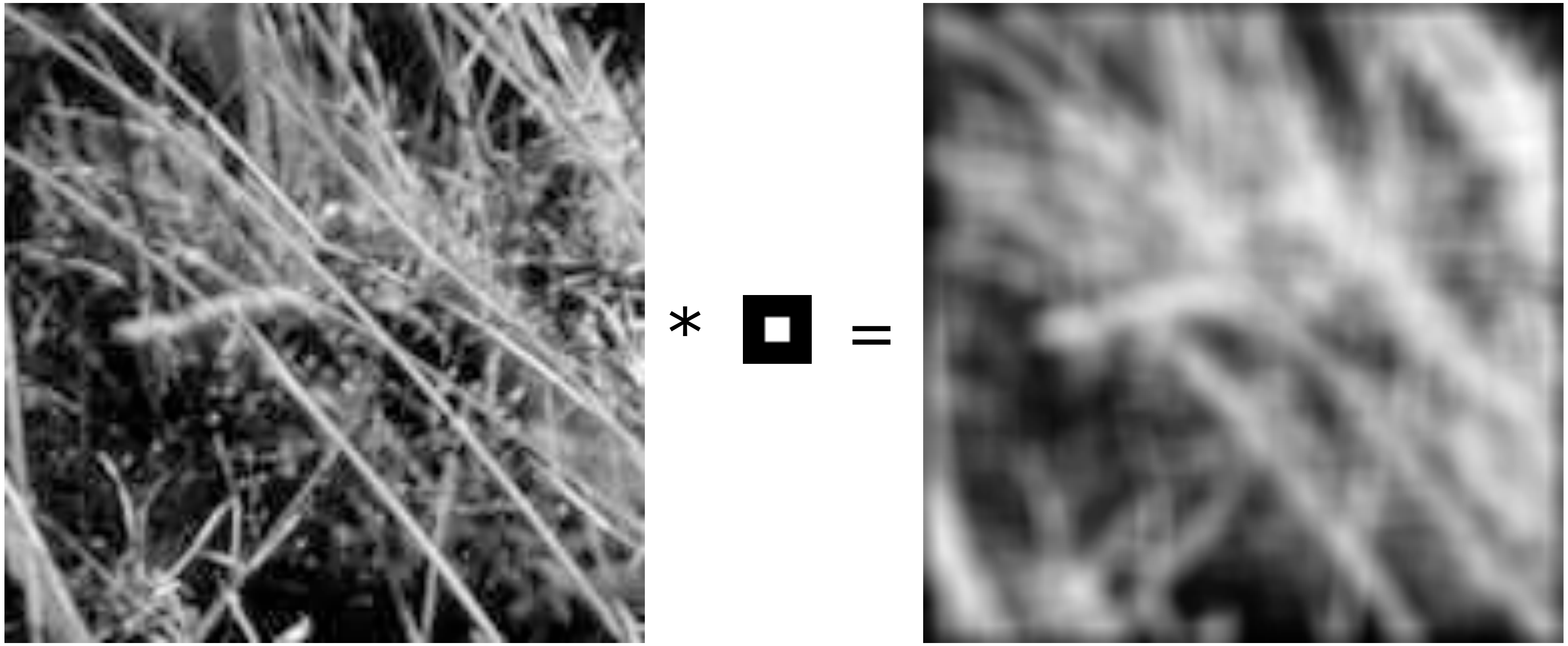
- Filtering often involves replacing the value of a pixel in the input image *F* with the weighted sum of its neighbors.
- Represent these weights as an image, ***H***
- ***H*** is usually called the **kernel**
- The operation for computing this weighted sum is called **convolution.**

$$R = H * I$$

- Convolution is:
  - commutative, $H * I = I * H$
  - associative, $H_1 * (H_2 * I) = (H_1 * H_2) * I$
  - distributive, $(H_1 + H_2) * I = (H_1 * I) + (H_2 * I)$

Smoothing via Simple Averaging

- One of the simplest filters is the mean filter: $H = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$
- In this case, $R(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} I(x-i, y-j) H(i,j)$
- It is used for removing image noise, i.e. for smoothing.

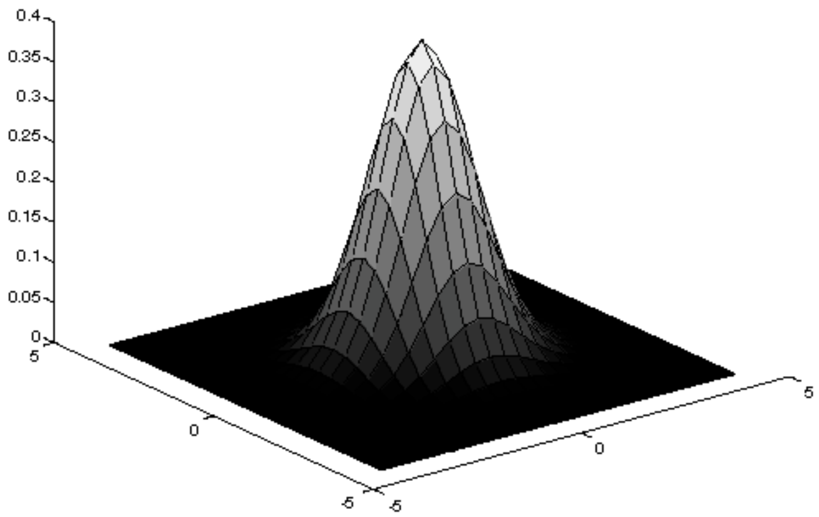


Original image

Image after mean filtering (25x25 kernel)

Gaussian Smoothing

- Idea: Use a weighted average. Pixels closest to the central pixel are more heavily weighted.
- The Gaussian function has exactly that profile.
- Gaussian also better approximates the behavior of a defocused lens.



Isotropic Gaussian Filter

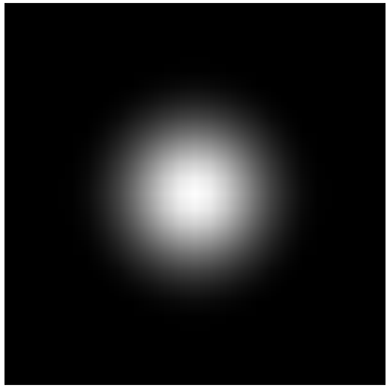
- To build a filter *H*, whose weights resemble the Gaussian distribution, assign the weight values on the matrix *H* according to the Gaussian function:

$$H(i,j) = e^{-(i^2 + j^2)/2\sigma^2}$$

$$H_{Gauss} = \begin{bmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{bmatrix}$$

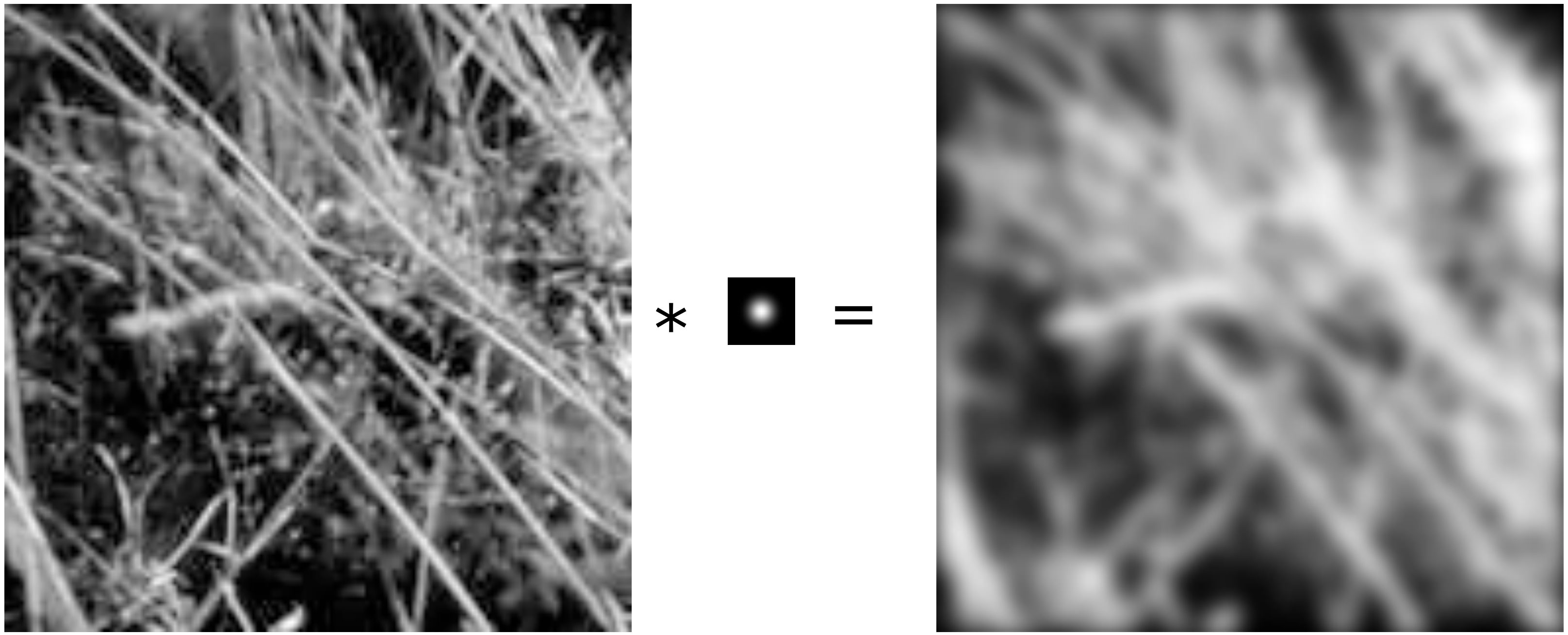
- Small σ, almost no effect, weights at neighboring points are negligible.
- Large σ, blurring, neighbors have almost the same weight as the central pixel.
- Commonly used σ values: Let w be the size of the kernel *H*. Then σ=w/5.

  For example for a 3x3 kernel, σ=3/5=0.6



Gaussian Smoothing Example

- Compared to mean filtering, Gaussian filtering exhibits no "ringing" effect.



Original image

Image after Gaussian filtering (25x25 kernel)

"Ringing" effect

Original image

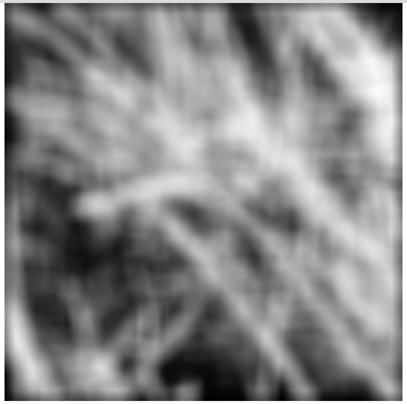


Image after Mean filtering (25x25 kernel)

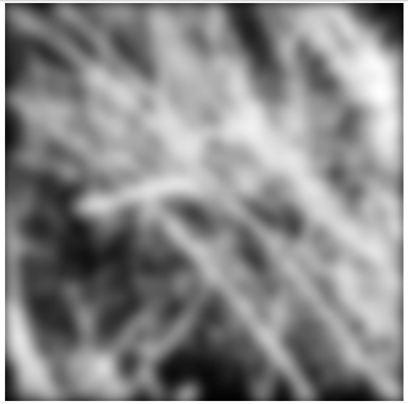
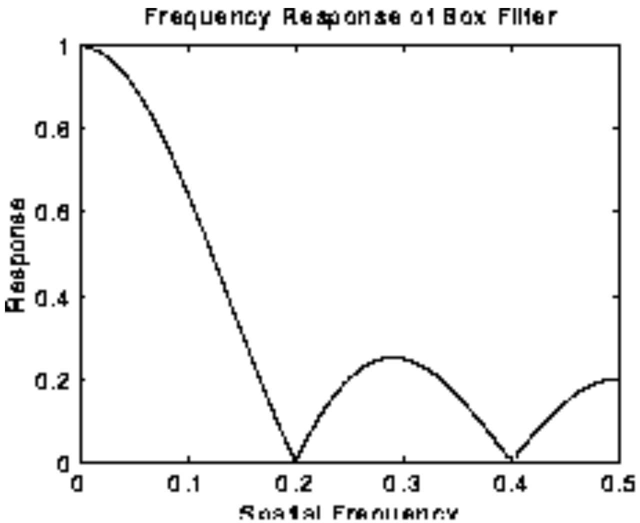
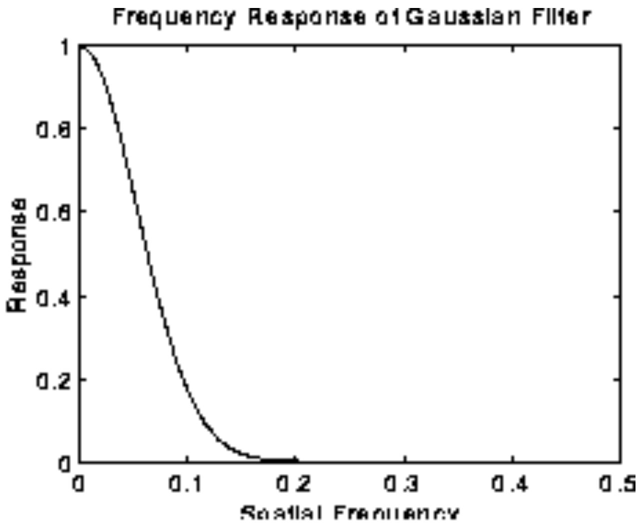


Image after Gaussian filtering (25x25 kernel)

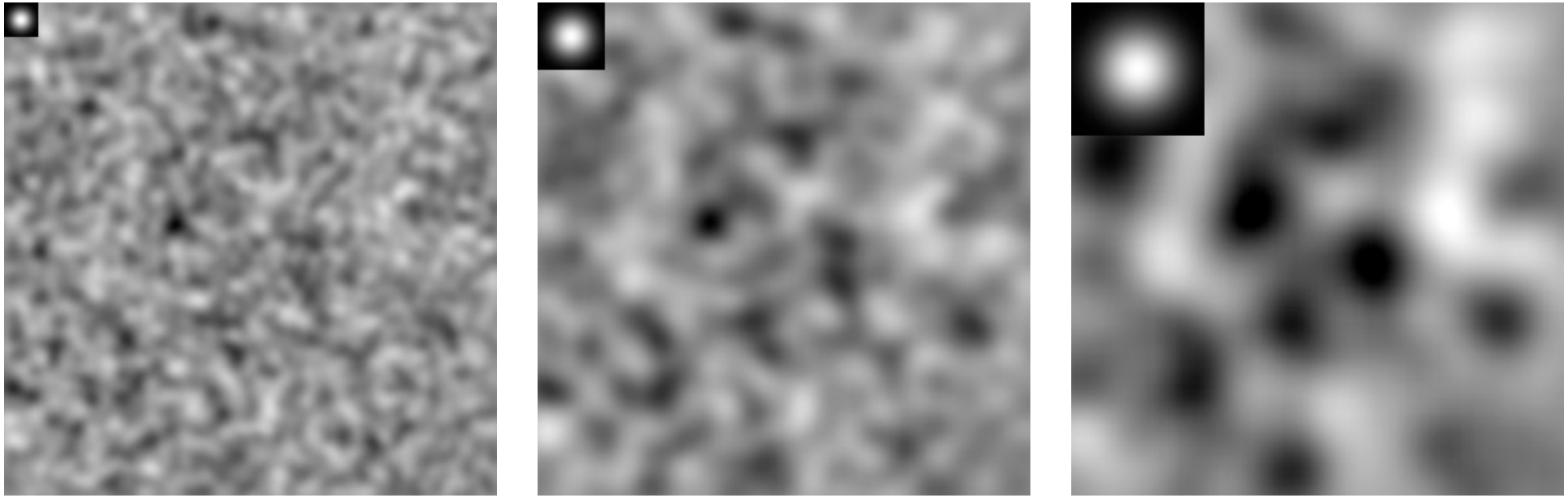




A close look at the frequency response of the two filters show that: compared to Gaussian filtering, median filtering exhibits oscillations

The Effect of σ

- Different σ values affect the amount of blurring, but also emphasize different characteristics of the image.



Non-Linear Smoothing

- The **median** filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings.
- It replaces a pixel value with the median of all pixel values in the neighborhood.
- It is a relatively slow filter, because it involves sorting.
- Can **not** be implemented via convolution.

Smoothing Examples

Original image

Image after 9x9 Mean filtering

Image after 9x9 Gaussian filtering

Mean Filter

Original image

Image after 3x3 Mean filtering

Image after 7x7 Mean filtering

Image after applying 3 times 3x3 Mean filtering

- Mean filtering is sensitive to outliers.
- It typically blurs edges.
- It often causes a ringing effect.

Gaussian Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (1% prob. that a bit is flipped)

Image after 5x5 Gaussian filtering, σ=1.0

Image after 9x9 Gaussian filtering, σ=2.0

- Gaussian filtering works very well for images affected by Gaussian noise
- It is not very effective in removing Salt and Pepper noise.

Median Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (5% prob. that a bit is flipped)

Image after 3x3 Median filtering

Image after 7x7 Median filtering

Image after applying 3 times 3x3 Median filtering

- Median filtering preserves high spatial frequency details.
- It works well when less than half of the pixels in the smoothing window have been affected by noise.
- It is not very effective in removing Gaussian noise.

Image Sources

1. “Image with salt & pepper noise”, Marko Meza.
2. Many of the smoothing and edge detection images are from the slides by D.A. Forsyth, University of California at Urbana-Champaign.