Diagnostic Medical Image Processing (DMIP)
WS 2014/15
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**Exercise 7: Rigid Registration**

# 1  Theory

## 1.1  Rigid registration in 2-D

Rigid registration only allows rotations and translations. That means that the objects maintain their shape and size. In 2-D rigid transformations can be described as

$$\mathbf{p}_k = \mathbf{R}\mathbf{q}_k + \mathbf{t} \tag{1}$$

where $\mathbf{R}$ is the rotation matrix

$$\mathbf{R} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \tag{2}$$

$\mathbf{t}$ the translation vector $\mathbf{t} = (\mathbf{t}_1, \mathbf{t}_2)$ and $(\mathbf{p}_k, \mathbf{q}_k) \in \mathbb{R}^2$ is the $k-$th pair of corresponding image points. The corresponding optimization problem is

$$\arg\min_{\varphi, \mathbf{t}_1, \mathbf{t}_2} \sum_{k=1}^{N} \left\| \mathbf{p}_k - \mathbf{R}\mathbf{q}_k - \mathbf{t} \right\|^2 \tag{3}$$

To avoid solving this nonlinear problem, one can make use of complex numbers.

## 1.2  Distance measures

With distance measures (aka similarity measures) you can calculate the similarity between objects. Let $F$ be the reference image and $M$ be the moving image.

### 1.2.1  Sum of Squared Differences - SSD

The Sum of Squared Differences is defined as

$$\mathcal{D}_{SSD} = \sum_{i,j} (F_{ij} - M_{ij})^2 \tag{4}$$

SSD is based on the assumption that the intensities of the same tissue within 2 datasets are equal.

### 1.2.2 Mutual Information - MI

The Mutual Information is defined as

$$\mathcal{D}_{MI} = H(F) + H(M) - H(F, M) \tag{5}$$

where $H(F), H(M)$ are the entropies of the images $F$ and $M$ and $H(F, G)$ is the entropy in the bivariate case (joint entropy). MI evaluates how much information is shared in both pictures.

## 2 Rotation and Translation

We want to rotate and translate an image:

1. Generate a phantom image.

2. Transform the image as follows:

   (a) Rotate the image by 45?
   (b) Translate the image with $\mathbf{t} = (20, 1)$.

## 3 Rigid Registration

### 3.1 Rigid Registration with SSD

We now want to register two images:

1. Apply the filter to Image1

2. Transform Image1 to Image2 with the initial information

3. Apply the optimization using fminsearch

4. Apply the Sum of Squared Differences as distance measure

### 3.2 Rigid Registration with Mutual Information

1. Calculate the joint histogram and the marginal entropies

2. Define the mutual information