

Convolution identities for periodic functions and sequences

from functions

$$f = f(t)$$

$$f(t) = \sum_{n \in \mathbb{Z}} \widehat{\mathbf{f}}[n] e^{2\pi i n t}$$

$$(f \cdot g)(t) = f(t) \cdot g(t)$$

$$(f \star g)(t) = \int_0^1 f(x) g(t - x) dx$$

to functions

$$\widehat{a} = \widehat{a}(\omega)$$

$$\widehat{a}(\omega) = \sum_{n \in \mathbb{Z}} \mathbf{a}[n] e^{in\omega}$$

$$\widehat{(a \cdot b)} = \widehat{a} \star \widehat{b}$$

$$\widehat{(a \star b)} = \widehat{a} \cdot \widehat{b}$$

to sequences

$$\widehat{\mathbf{f}} = (\widehat{\mathbf{f}}[n])_{n \in \mathbb{Z}}$$

$$\widehat{\mathbf{f}}[n] = \int_0^1 f(t) e^{-2\pi i n t} dt$$

$$\widehat{(f \cdot g)} = \widehat{\mathbf{f}} \star \widehat{\mathbf{g}}$$

$$\widehat{(f \star g)} = \widehat{\mathbf{f}} \cdot \widehat{\mathbf{g}}$$

from sequences

$$\mathbf{a} = (\mathbf{a}[n])_{n \in \mathbb{Z}}$$

$$\mathbf{a}[n] = \frac{1}{2\pi} \int_0^{2\pi} \widehat{a}(\omega) e^{-in\omega} d\omega$$

$$(\mathbf{a} \cdot \mathbf{b})[n] = \mathbf{a}[n] \cdot \mathbf{b}[n]$$

$$(\mathbf{a} \star \mathbf{b})[n] = \sum_{k \in \mathbb{Z}} \mathbf{a}[k] \cdot \mathbf{b}[n - k]$$