Diagnostic Medical Image Processing (DMIP)
WS 2015/16
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Exercise 6: Fan-beam Reconstruction and Short Scan

The parallel-beam geometry was once used in practical scanner, while the diverging-geometry, e.g., fan-beam geometry, is employed exclusively today. In a fan-beam geometry, the so-called short scan that acquires data only over an angle π plus the fan angle is commonly used, aiming to reduce scan time and X-ray dose. The goal for this exercise is to reconstruct short scan data using the fan-beam FBP algorithm with an appropriate redundancy weighting function, i.e., Parker weights.

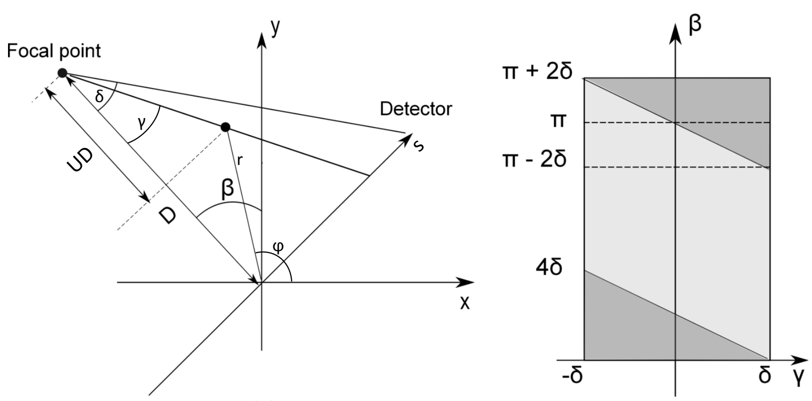
1 Theory

1.1 Fan-beam FBP Algorithm

In a fan-beam geometry, the X-ray source is a single focal point and beams are not parallel but characterized by a fan shape with a opening angle δ. A straightforward substitution of the integral variables from the parallel-beam data $p(s, \theta)$ to the fan-beam data $g(\gamma, \beta)$ leads to the corresponding fan-beam geometry algorithm (see Fig. 1a for the notation):

- *Step 1:* Cosine weighting of projection data to obtain $g_1(s, \beta)$:

$$g_1(s, \beta) = \frac{D}{\sqrt{D^2 + s^2}} g(s, \beta) \ , \tag{1}$$



(a) (b)

Figure 1: (a) Fan-beam imaging geometry, (b) fan-beam sinogram and data redundancy.

- *Step 2*: Perform fan-beam filtering :

$$g_F(s,\beta) = \int_{-\infty}^{\infty} h_R(s-s')\, g_1(s',\beta)\, ds' \,, \tag{2}$$

  where $h_R(s)$ is the filter kernel.

- *Step 3:* Backprojection with a weighting function of object-focal point distance U:

$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{U^2} g_F(s,\beta)\, d\beta \tag{3}$$

$$U = \frac{D + r\sin(\beta-\varphi)}{D} \tag{4}$$

1.2 Short Scan

A short scan measures some redundant rays at the beginning and at the end of data acquisition (see the two dark triangles in Fig. 1b). Corresponding redundant rays can be determined by the relation

$$g(\gamma,\beta) = g(-\gamma,\beta+\pi+2\gamma) \,. \tag{5}$$

The commonly used weighting function for redundancies is the Parker weighting function, in which the projection rays measured twice are normalized to unity while guaranteeing smooth transitions between non-redundant and redundant data. The weighting can be defined as:

$$\omega(\gamma,\beta) = \begin{cases} \sin^2\left(\frac{\pi}{4}\frac{\beta}{\delta-\gamma}\right), & 0 \le \beta \le 2\delta - 2\gamma \\ 1, & 2\delta-2\gamma \le \beta \le \pi - 2\gamma \\ \sin^2\left(\frac{\pi}{4}\frac{\pi+2\delta-\beta}{\delta+\gamma}\right), & \pi - 2\gamma \le \beta \le \pi + 2\delta \end{cases}$$

2 Implementation tasks

We will implement a fan-beam based FBP algorithm to reconstruct different provided sinograms.
Complete the gaps in the provided CONRAD class that are marked with "**TODO**"

1. Line 64ff: Initialize the parameters. Compute the fan angle and the short scan range.
2. Line 91ff: Compute direction and position of the detector border at a rotation angle β
3. Line 105 & 109: Transform the pixel coordinates to world coordinates.
4. Line 114: Compute the intersection point of a ray with the detector.
5. Line 139ff: Compute the distance weights for this point at the given rotation angle β.
6. Line 175ff: Complete the cosine-weights computation

7. Line 224,235: Complete the parker-weights implementation

Some useful CONRAD classes and methods:

- **Math**: PI, sin(), cos(), atan(), atan2()
- **NumericPointWiseOperators**: multiplyBy()
- **PointND**: getAbstractVector()
- **StraightLine**: intersect()