



Exercise 2: Image Undistortion

Acquired x-ray images are distorted by the earth magnetic field, artificial magnetic fields, scattering and a convex entrance screen. Therefore a mapping of the 2-D pixel grid from the distorted to an undistorted/corrected image is necessary. N 2-D points are precisely known (from the ideal image): $(x'_1, y'_1), (x'_2, y'_2), \dots, (x'_N, y'_N)$ and N observed points in the distorted image are given: $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$. To correct the image distortion we use a parametric mapping

1 Pre-processing

1. Load one of the images **undistorted.jpg**, **frame32.jpg** or **frame90.jpg**. They show the coronary arteries, sometimes a catheter.
2. Normalize the intensity values. The output values should be in the range of $[0, 1]$.
3. Cut the $(m \times n)$ image so that both dimensions are equal. We want to continue with a quadratic image. You can cut starting from the origin in the upper left. No centered cut-out is necessary.
4. Create a grid to sample the whole image.
5. Create an artificial distortion field which is applied to the ideal image to construct the distorted one.

The distortion field can be created by: $R = d\sqrt{a\left(\frac{X-\frac{n}{2}}{n}\right)^2 + b\left(\frac{Y-\frac{m}{2}}{m}\right)^2}$. X and Y are

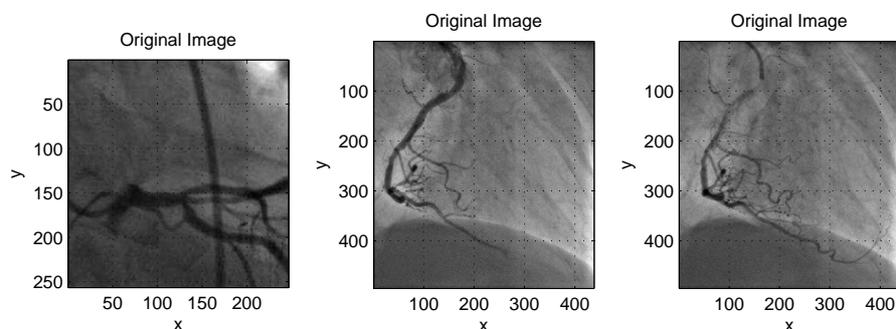


Figure 1: Undistorted images.

the sampling coordinates. Plot the distortion field R .

Find some values for the parameters a, b and d . What is adjusted by these parameters?

The resulting values of R are in the range of $[0, \max R]$. The range should be shifted to $[-\frac{\max R}{2}, \frac{\max R}{2}]$. Plot the shifted artificial distortion field. Draw a line with ImageJ through both distortion fields and push "k". This creates a line plot and you can see the influence of the shift. Now, add the shifted distortion field to the grid to get the sampling grid points for a distorted image: X_d, Y_d .

6. To generate the distorted image, an interpolation method is necessary. We use bilinear interpolation. In CONRAD, the class *InterpolationOperators* contains the method `interpolateLinear(Grid2D image, double x, double y)`, where x and y are the sampling points for the *image*.

2 Image Undistortion - Workflow

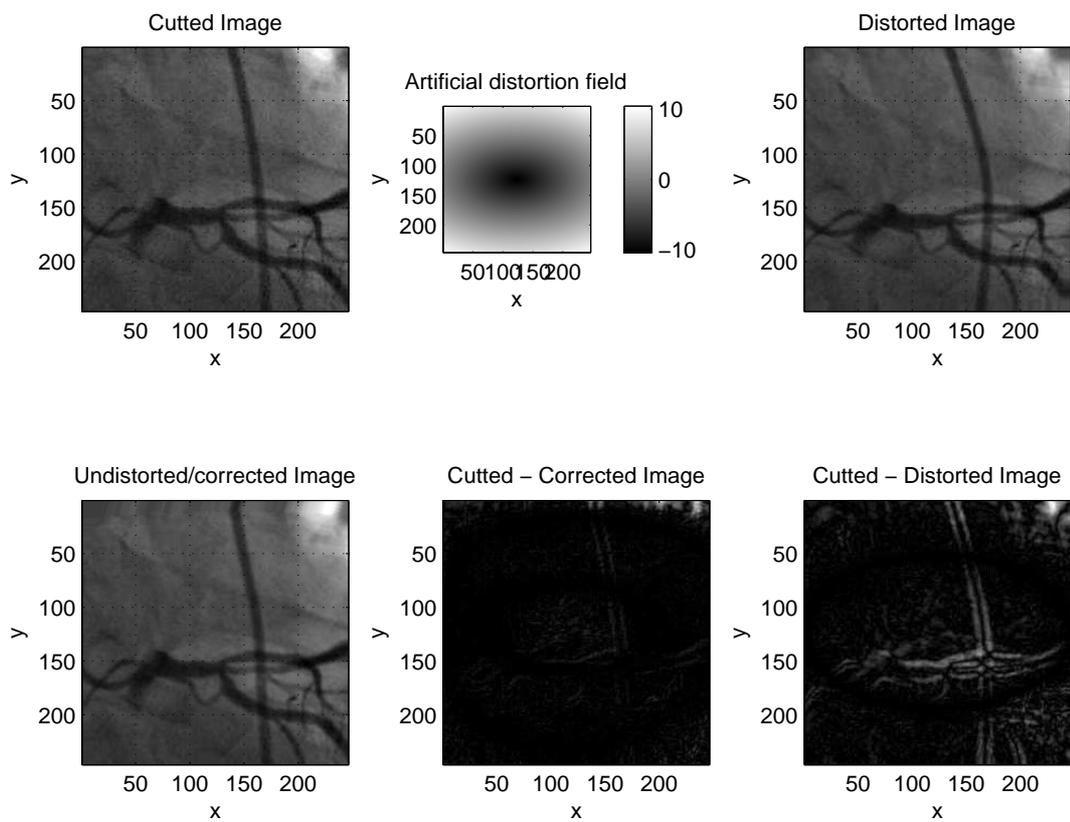
1. In real world we would know the relation between the undistorted and the distorted image by point correspondences of a calibration pattern. Instead, we use our artificial distortion field created in the preprocessing. Therefore, we assume lattice points distributed over the whole image domain, e.g. a 8×8 lattice. For these 64 positions we know the relation between the undistorted (X, Y) and the distorted image (X_d, Y_d) . Following, we can compute the coefficients of the distortion function.

Set the number of lattice points and sample the undistorted and distorted image at those positions.

Attention: We want to sample the distorted image to get the corrected image, but we sampled the ideal image to get the distorted one. Check the direction of the distortion regarding the undistorted and distorted image!

2. The distortion correction is described on (Slide 29). d is the polynomial's degree. What is the maximum number for d ?
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3. Build the matrix A (number of rows: number of correspondences, number of columns: number of coefficients [see Gaußsche Summenformel]). Compute the pseudo-inverse A^\dagger to estimate the coefficients $u_{i,j}$ and $v_{i,j}$.
4. Compute the sampling grid for an undistorted image with formulas (7) and (8) on (Slide 29).
5. Resample the distorted image to obtain the undistorted image. Use also the bilinear interpolation. Plot the result.



Pixel info: (X, Y) Pixel Value

Figure 2: Original image, artificial distortion field and image undistortion result.