Exercises for
Pattern Analysis
Thomas Köhler
Assignment 9, 02.07.2014

General Information:

Lecture (3 SWS):	Mon 08.15 – 09:45 (H16) and Tue 08.15 – 09.45 (H16)
Exercises (1 SWS):	Wed 12.15 – 13.15 (00.151-113) and Thu 12.30 – 13.30 (00.151-113)
Certificate:	Oral exam at the end of the semester
Contact:	thomas.koehler@fau.de

Markov Random Fields

Exercise 1 Image denoising can be expressed as an optimization over a Markov Random Field (MRF). In this exercise, we consider this task for binary images. Here, $y_i \in \{-1, +1\}$ is the i-th pixel in a noisy binary image and $x_i \in \{-1, +1\}$ is the associated pixel in the denoised image.

(a) In the MRF, pixels y_i and x_i can be modeled as random variables with limited statistical dependency. Draw a MRF to model binary image denoising based on

- the assumption that x_i is only dependent on the associated y_i and
- a 4-neighborhood of the pixels in the denoised image.

(b) Write down a maximum a posteriori (MAP) estimate for the denoised pixels x_i in terms of $p(x_i)$ and $p(y_i|x_i)$.

(c) Write down the general form of the energy function $E(\boldsymbol{x})$ associated with the MAP estimate for binary image denoising, where $\boldsymbol{x} = \begin{pmatrix} x_i & \ldots x_n \end{pmatrix}^\top$. The energy function should be formulated as a sum of a likelihood term $\mathcal{L}(\boldsymbol{x})$ and a prior $\mathcal{R}(\boldsymbol{x})$.

Hint: Similar to the log-likelihood function you can use the logarithm of the a posteriori probability $p(x_i|y_i)$ to formulate $E(\boldsymbol{x})$.

(d) Derive the likelihood term $\mathcal{L}(\boldsymbol{x})$ of the energy function. Therefore, you can assume that x_i is more likely if it has the same sign as y_i.

(e) Derive the prior $\mathcal{R}(\boldsymbol{x})$ of the energy function. Therefore, you can assume that x_i is more likely if the resulting denoised image is spatially smooth in the neighborhood of x_i.

(f) Explain how this concept for binary image denoising can be extended to grayscale images.