IMIP – Exercise

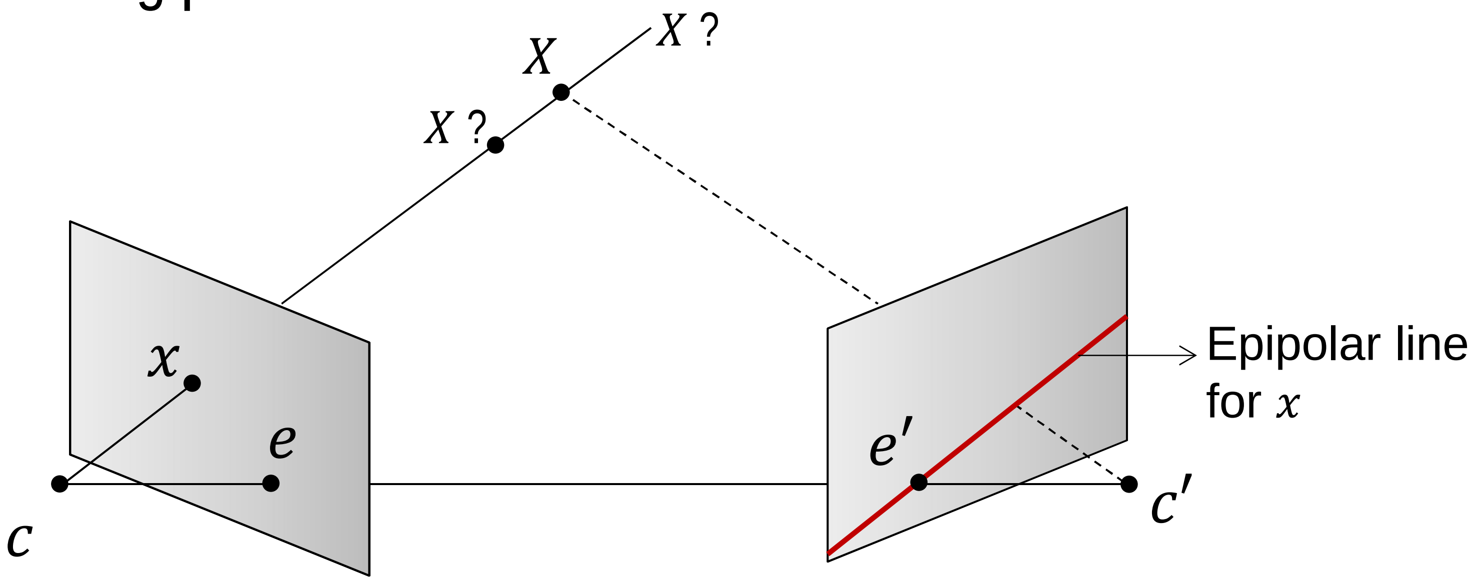
Epipolar Geometry & Fundamental Matrix &
Normalized Eight Point Algorithm

Yan Xia, Matthias Hoffmann
Pattern Recognition Lab (CS 5)

FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Epipolar Geometry

Problem: Given an image point in the first view, where is the corresponding point in the second view?



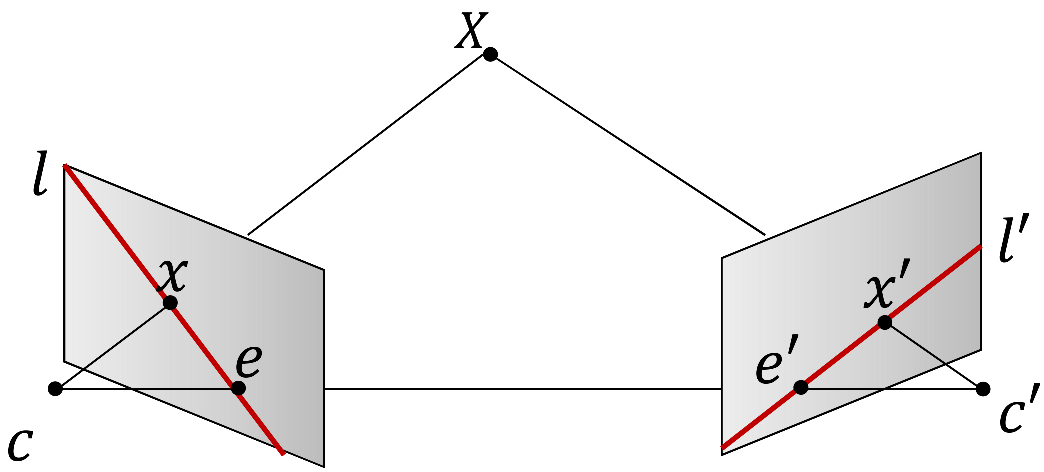
- A point in one image "generates" a line in the other image
- This line is known as an epipolar line, and the geometry which gives rise to it is known as epipolar geometry

Epipolar line

Epipolar constraint

- Reduces correspondence problem to 1D search along an epipolar line

Fundamental Matrix



- If x and x’ are corresponding image points, then the fundamental matrix F is defined by

$$\boldsymbol{x}'^T \boldsymbol{F} \boldsymbol{x} = 0.$$

(see lecture slides 11, 26)

- Epipolar lines:
  - $\boldsymbol{l}' = \boldsymbol{F}\boldsymbol{x}$ is the epipolar line corresponding to $\boldsymbol{x}$
  - $\boldsymbol{l} = \boldsymbol{F}^T \boldsymbol{x}'$ is the epipolar line corresponding to $\boldsymbol{x}'$

Computing Fundamental Matrix

- The basic relation is

$$\boldsymbol{x}'^T \boldsymbol{F} \boldsymbol{x} = 0.$$

- The equation for a pair of points, e.g., $(x, y, 1)$ and $(x', y', 1)$:

$$x'xf_{11} + x'yf_{12} + x'f_{13} + y'xf_{21} + y'yf_{22} + y'f_{23} + xf_{31} + yf_{32} + f_{33} = 0$$

Gives an equation

$$(x'x,\ x'y,\ x',\ y'x,\ y'y,\ y',\ x,\ y,\ 1)\,\mathbf{f} = 0$$

where

$$\mathbf{f} = (f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33})^T$$

holds the entries of the fundamental matrix.

Computing Fundamental Matrix

- For n corresponding points

$$\mathbf{Af} = \underbrace{\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix}}_{\text{Measurement matrix}} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = 0$$

Linear Solution

- We have a homogeneous set of equations $\mathbf{A}\mathbf{f} = 0$
- f can be determined only up to a scale, so there are 8 unknowns, and at least 8 point correspondences are needed
  → hence the name "8 point algorithm"
- The least square solution is the singular vector corresponding the smallest singular values of A
  - Take SVD: $A = UDV^T$
  - Solution is last column of V

Constraint Enforcement

(Enforce the rank-2 constraint) → Why? See slide 15

If f is computed linearly from 8 or more correspondences, singularity condition ($\det(\mathbf{F}) = \mathbf{0}$) does not hold.

SVD Method

(i) SVD: $\mathbf{F} = UDV^{\mathrm{T}}$

(ii) U and V are orthogonal, D = diag(r,s,t)

(iii) $r \geq s \geq t$

(iv) Set F' = U diag(r,s,0) V^{T}

(v) Resulting F' is singular

Problem: 8 point algorithm performs badly in presence of noise

Normalization of data (balancing)

(i) Translate so centroid is at origin

(ii) Scale so that RMS distance of points from origin is $\sqrt{2}$

(i) Translate so centroid is at origin

We first compute the centroid of the set of all points:

$$t_x = \frac{1}{N}\sum_{k=1}^{N} p^i_{x,k}, \qquad t_y = \frac{1}{N}\sum_{k=1}^{N} p^i_{y,k},$$

where $t = (t_x, t_y)$ the centroid of the k=1,..,N points p^i_k

Then, we can center the points by

$$mc = p^i_k - t$$

(ii) Scaling

The distance of each new computed point to the origin (0,0)

$$dc = \sqrt{\sum (mc)^2}$$

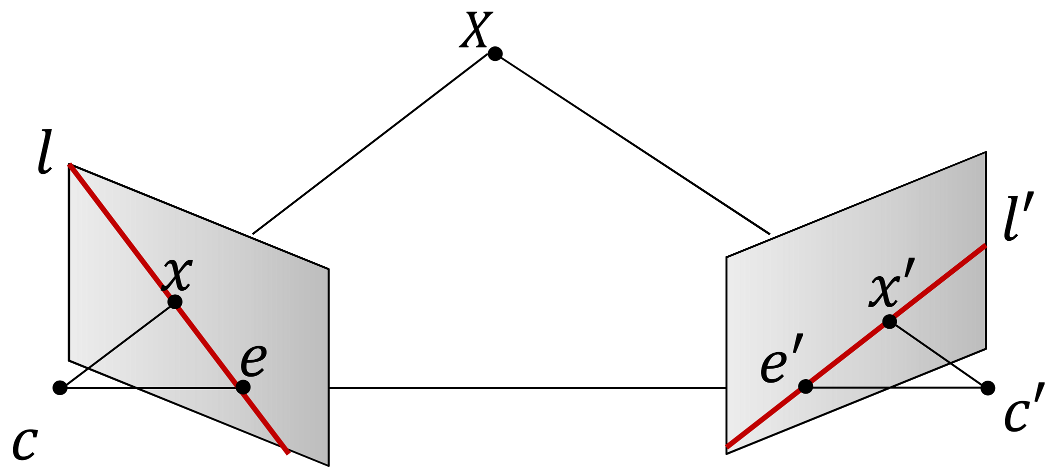
dc divided by the number of tracked points, resulting in the average distance to the origin.

Then, we apply a scale factor *s,* to satisfy the criteria that the average distance of a point p from the origin is $\sqrt{2}$.

Finally, the normalized data can be computed by apply the transformation

$$T = \begin{pmatrix} s & 0 & -st_x \\ 0 & s & -st_y \\ 0 & 0 & 1 \end{pmatrix}$$

Compute Epipolar line and Epipole



Epipolar lines:

- $\boldsymbol{l}' = \boldsymbol{F}\boldsymbol{x}$ is the epipolar line corresponding to $\boldsymbol{x}$
- Assume $\boldsymbol{l}'$=(a,b,c), then line equation is

$$ax + by + c = 0$$

- $\boldsymbol{x}' = (x', y', 1)$ lies on the line $\boldsymbol{l}'$

$\rightarrow 0 = ax' + by' + c = \boldsymbol{x}'^T \boldsymbol{l}'$

$\rightarrow 0 = \boldsymbol{x}'^T \boldsymbol{F}\boldsymbol{x}$

Epipole: $\boldsymbol{F}\boldsymbol{e} = \boldsymbol{0}$ (slide 27) $\rightarrow$ *e is eigenvector with smallest eigenvalue*