Noise, Filtering and Smoothing

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Noise Sources

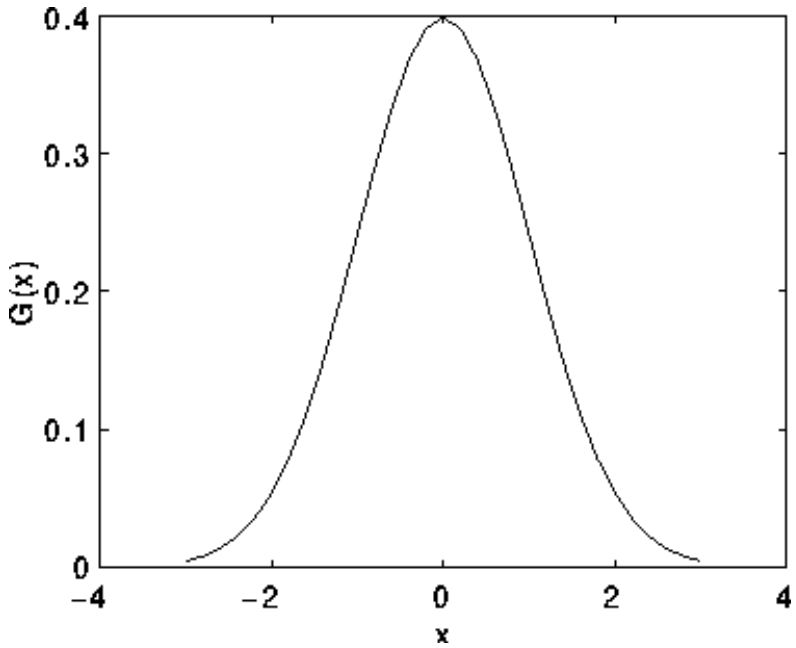
- Photon noise: variation in the #photons falling on a pixel per time interval T.
- Saturation: each pixel can only generate a limited amount of charge.
- Blooming: saturated pixel can overflow to neighboring pixels.

Noise Sources - continued

- Thermal noise: heat can free electrons and generate a response when there is no real signal.
- Electronic noise.
- Burned pixels.
- Black is not black.
- Keep in mind: Camera response may not be linear over the number of photons falling on a surface (camera gamma)

Detector Noise

- Source of noise: the discrete nature of radiation, i.e. the fact that each imaging system is recording an image by counting photons.
- Can be modeled as an independent additive noise which can be described by a zero-mean Gaussian.



Salt and Pepper Noise

- A common form of noise is caused by *data drop-out noise.*
- It is also known as commonly referred to as intensity spikes, speckle, or salt and pepper noise.
- Sources of error:
  - Errors in the data transmission.
  - Burned pixels: the corrupted pixels are either set to the maximum value (which looks like snow in the image) or are set to zero ("peppered" appearance), or a combination of the two.
  - Single bits are flipped over.
- Isolated/localized noise. It only affects individual pixels.

Filtering

- Most of the images we capture are noisy.
- Goal:

  Noisy Image$_{in}$ $\rightarrow$ Filter $\rightarrow$ Clean Image$_{out}$

- This notion of filtering is more general and can be used in a wide range of transformations that we may want to apply to images.

  Image$_{in}$ $\rightarrow$ Filter $\rightarrow$ Image$_{out}$

- Mathematically, a filter H can be treated as a function on an input image I:

$$H(I) = R$$

- Note: We use the terms *filter* and *transformation* interchangeably.

Linear Transformation

- A transformation H is **linear** if, for any inputs $I_1(x,y)$ and $I_2(x,y)$ (in our case input images), and for any constant scalar α we have:

$$H(\alpha I_1(x,y)) = \alpha H(I_1(x,y))$$

and

$$H(I_1(x,y) + I_2(x,y)) = H(I_1(x,y)) + H(I_2(x,y))$$

- This means:
  - Scaling of the input corresponds to scaling of the output.
  - Filtering an additive image is equivalent to filtering each image separately and then adding the results.

Shift-Invariant Transformation

- A transformation *H* is **shift-invariant** if for every pair (x_0, y_0) and for every input image $I(x,y)$, such that

$$H(I(x,y)) = R(x,y)$$

we get

$$H(I(x-x_0, y-y_0)) = R(x-x_0, y-y_0)$$

- This means that the filter *H* does not change as we shift it in the image (as we move it from one position to the next).

Convolution

- If a transformation (or filter) is linear shift-invariant (LSI) then one can apply it in a systematic manner over every pixel in the image.
- **Convolution** is the process through which we apply **linear shift-invariant filters** on an image.

$$I \longrightarrow \boxed{\text{LSI Filter } H} \longrightarrow R$$

- Convolution is defined as:

$$R(x,y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} H(x-i, y-j) I(i,j)$$

and is denoted as:

$$R = H * I$$

Another Look at Convolution

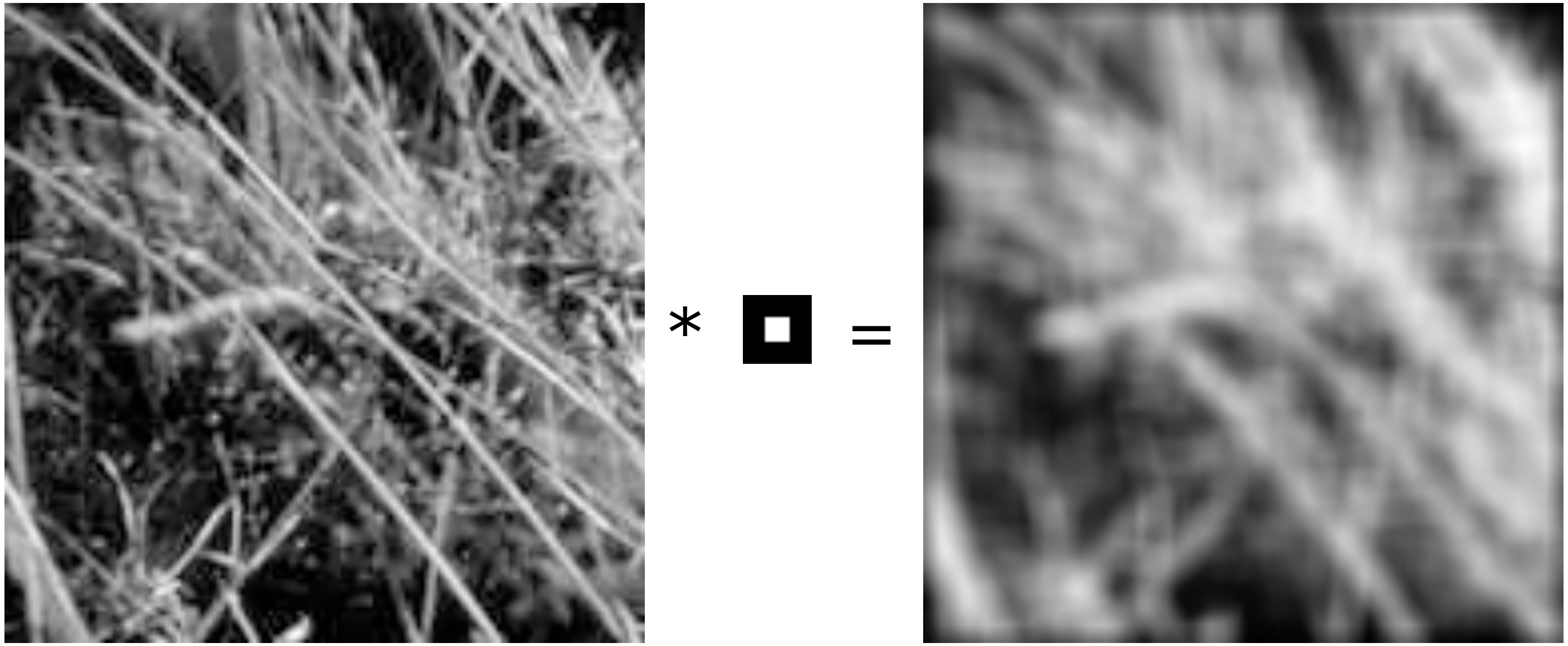
- Filtering often involves replacing the value of a pixel in the input image *F* with the weighted sum of its neighbors.
- Represent these weights as an image, ***H***
- ***H*** is usually called the **kernel**
- The operation for computing this weighted sum is called **convolution.**

$$R = H * I$$

- Convolution is:
  - commutative, $H * I = I * H$
  - associative, $H_1 * (H_2 * I) = (H_1 * H_2) * I$
  - distributive, $(H_1 + H_2) * I = (H_1 * I) + (H_2 * I)$

Smoothing via Simple Averaging

- One of the simplest filters is the mean filter: $H = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$
- In this case, $R(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} I(x-i, y-j) H(i,j)$
- It is used for removing image noise, i.e. for smoothing.



Original image

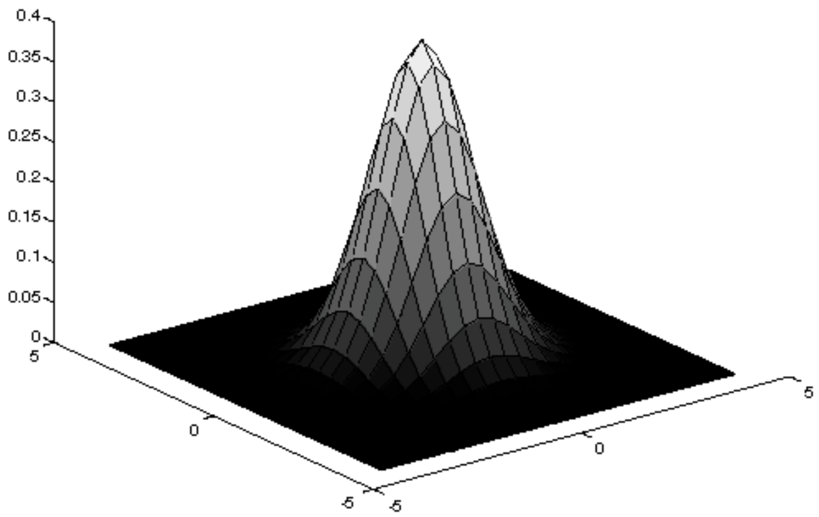
Image after mean filtering (25x25 kernel)

Gaussian Smoothing

- Idea: Use a weighted average. Pixels closest to the central pixel are more heavily weighted.

- The Gaussian function has exactly that profile.

- Gaussian also better approximates the behavior of a defocused lens.



Isotropic Gaussian Filter

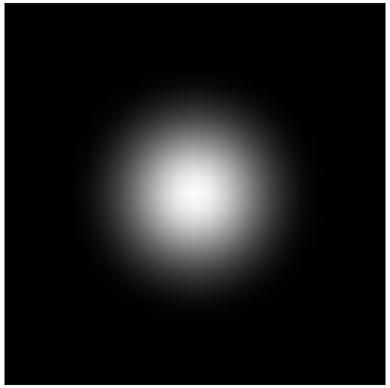
- To build a filter *H*, whose weights resemble the Gaussian distribution, assign the weight values on the matrix *H* according to the Gaussian function:

$$H(i,j) = \mathrm{e}^{-(i^2 + j^2)/2\sigma^2}$$

$$H_{Gauss} = \begin{bmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{bmatrix}$$

- Small σ, almost no effect, weights at neighboring points are negligible.
- Large σ, blurring, neighbors have almost the same weight as the central pixel.
- Commonly used σ values: Let w be the size of the kernel *H*. Then σ=w/5.

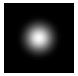
  For example for a 3x3 kernel, σ=3/5=0.6



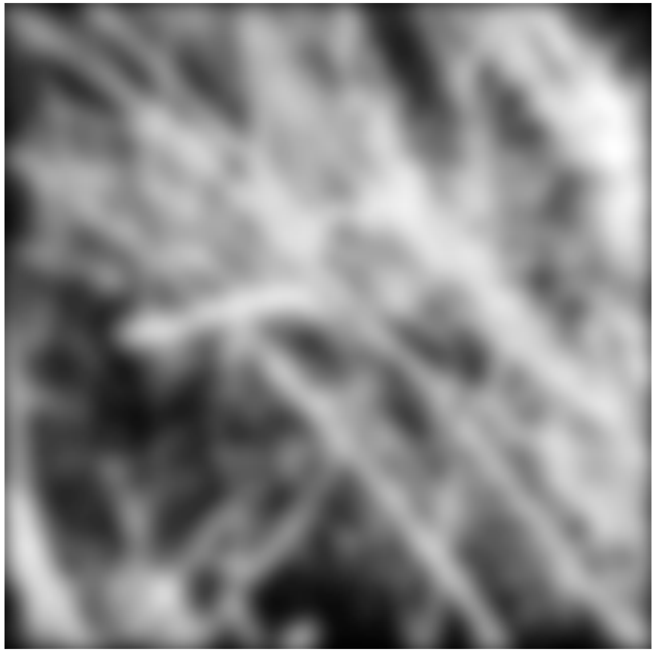
Gaussian Smoothing Example

- Compared to mean filtering, Gaussian filtering exhibits no “ringing” effect.

$*$



$=$



Original image

Image after Gaussian filtering (25x25 kernel)

"Ringing" effect

Original image

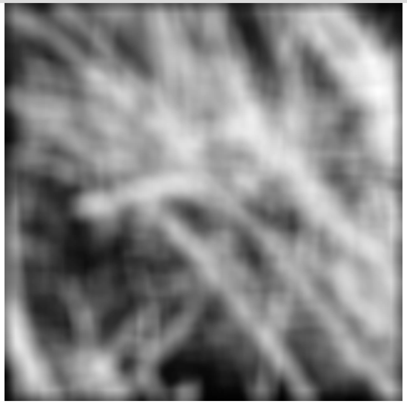


Image after Mean filtering (25x25 kernel)

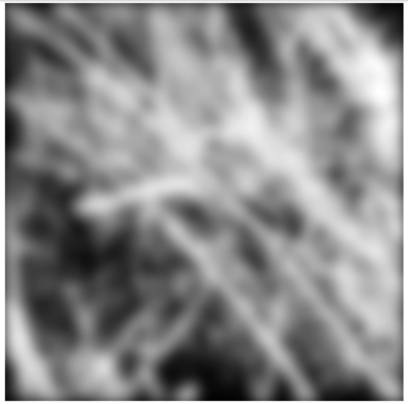
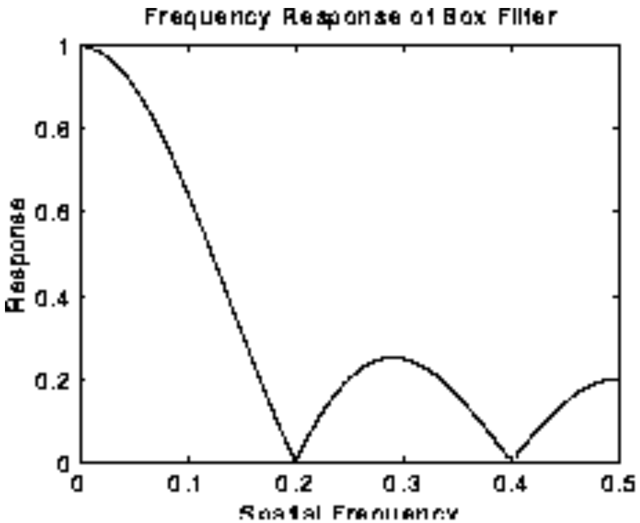
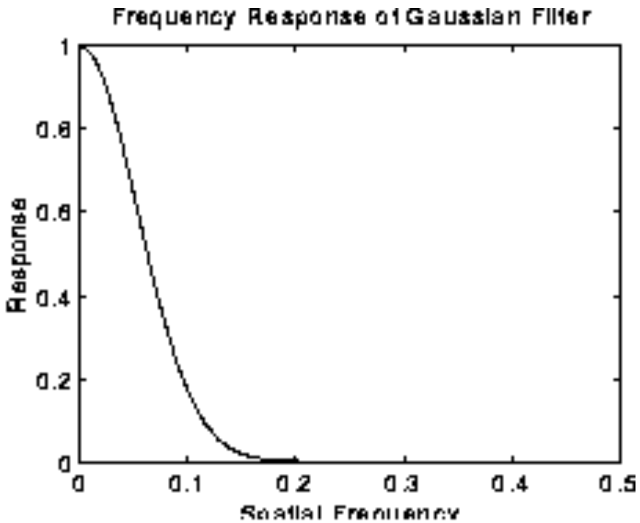


Image after Gaussian filtering (25x25 kernel)

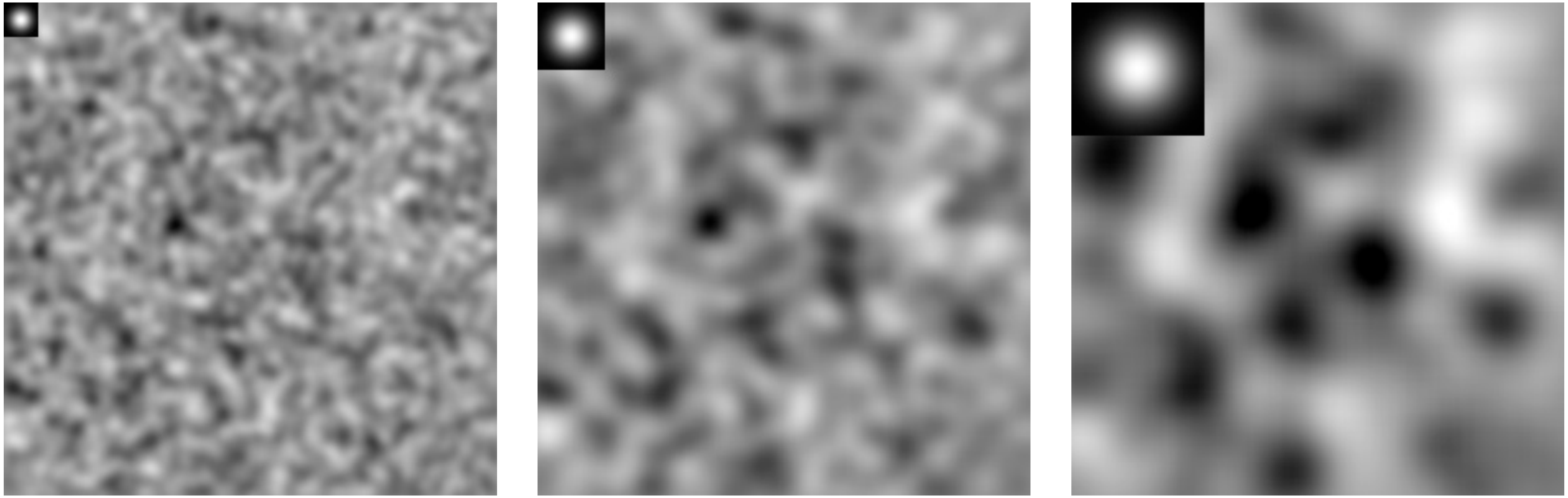




A close look at the frequency response of the two filters show that: compared to Gaussian filtering, mean filtering exhibits oscillations

The Effect of σ

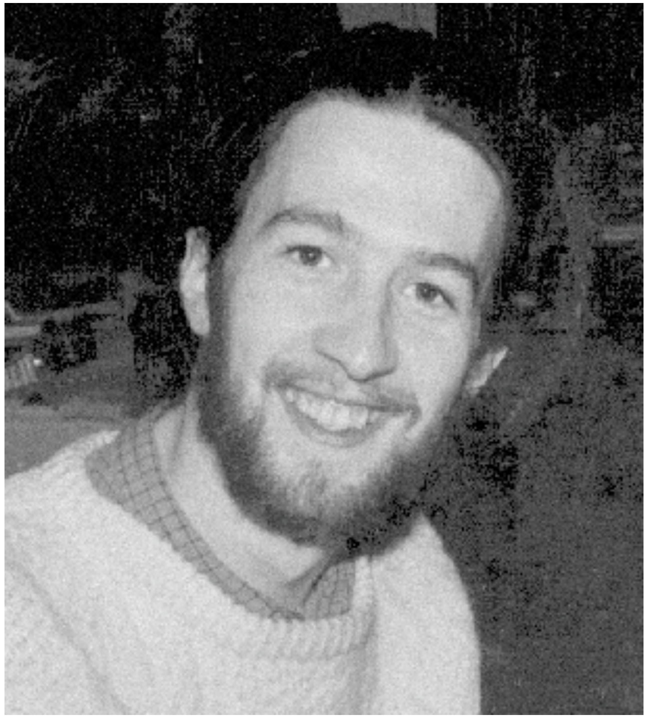
- Different σ values affect the amount of blurring, but also emphasize different characteristics of the image.



Non-Linear Smoothing

- The **median** filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings.
- It replaces a pixel value with the median of all pixel values in the neighborhood.
- It is a relatively slow filter, because it involves sorting.
- Can **not** be implemented via convolution.

Smoothing Examples



Original image corrupted by a zero mean Gaussian noise with σ=8.

Image after 5x5 Mean filtering

Image after 5x5 Gaussian filtering

Mean Filter

Original image

Image after 3x3 Mean filtering

Image after 7x7 Mean filtering

Image after applying 3 times 3x3 Mean filtering

- Mean filtering is sensitive to outliers.
- It typically blurs edges.
- It often causes a ringing effect.

Gaussian Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (1% prob. that a bit is flipped)

Image after 5x5 Gaussian filtering, σ=1.0

Image after 9x9 Gaussian filtering, σ=2.0

- Gaussian filtering works very well for images affected by Gaussian noise.
- It is not very effective in removing Salt and Pepper noise. Small σ values do not remove the Salt & Pepper noise, while large σ values blur the image too much.

Gaussian Filtering and Salt & Pepper Noise


Original image


Image with salt-pepper noise (1% prob. that a bit is flipped)


After 3x3 mean filtering


After 7x7 mean filtering


After 5x5 Gaussian filter, σ=1.0


After 9x9 Gaussian filter, σ=2.0

Median Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (5% prob. that a bit is flipped)

Image after 3x3 Median filtering

Image after 7x7 Median filtering

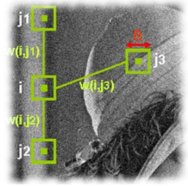
Image after applying 3 times 3x3 Median filtering

- Median filtering preserves high spatial frequency details.
- It works well when less than half of the pixels in the smoothing window have been affected by noise.
- It is not very effective in removing Gaussian noise.

Non-Local Means

- The output pixel is a weighted average of all the image pixels.

$$R(\vec{p}) = \sum_{\vec{q}\in N} w(\vec{p},\vec{q}) I(\vec{q})$$



where *N* is the set of all pixel positions, $\vec{p} = (x, y)$ is a pixel position, $0 \leq w(\vec{p},\vec{q}) \leq 1$ and $\sum_{\vec{q}\in N} w(\vec{p},\vec{q}) = 1$.

- The weight assigned to each pair of pixels depends on the similarity of the *grey values* in the neighborhood $\mathscr{N}(\vec{p})$ centered around each of the two pixels:

$$w(\vec{p},\vec{q}) = \frac{1}{Z(\vec{p})} e^{-\frac{\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2}{h^2}} \qquad Z(\vec{p}) = \sum_{\vec{q}} e^{-\frac{\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2}{h^2}}$$

where $\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2$ is the Euclidean distance weighted by a Gaussian function of standard deviation σ.

Examples of Weight Values

- In this figure, the original image is on the left and the weights for the central pixel (white dot) are shown on the right.

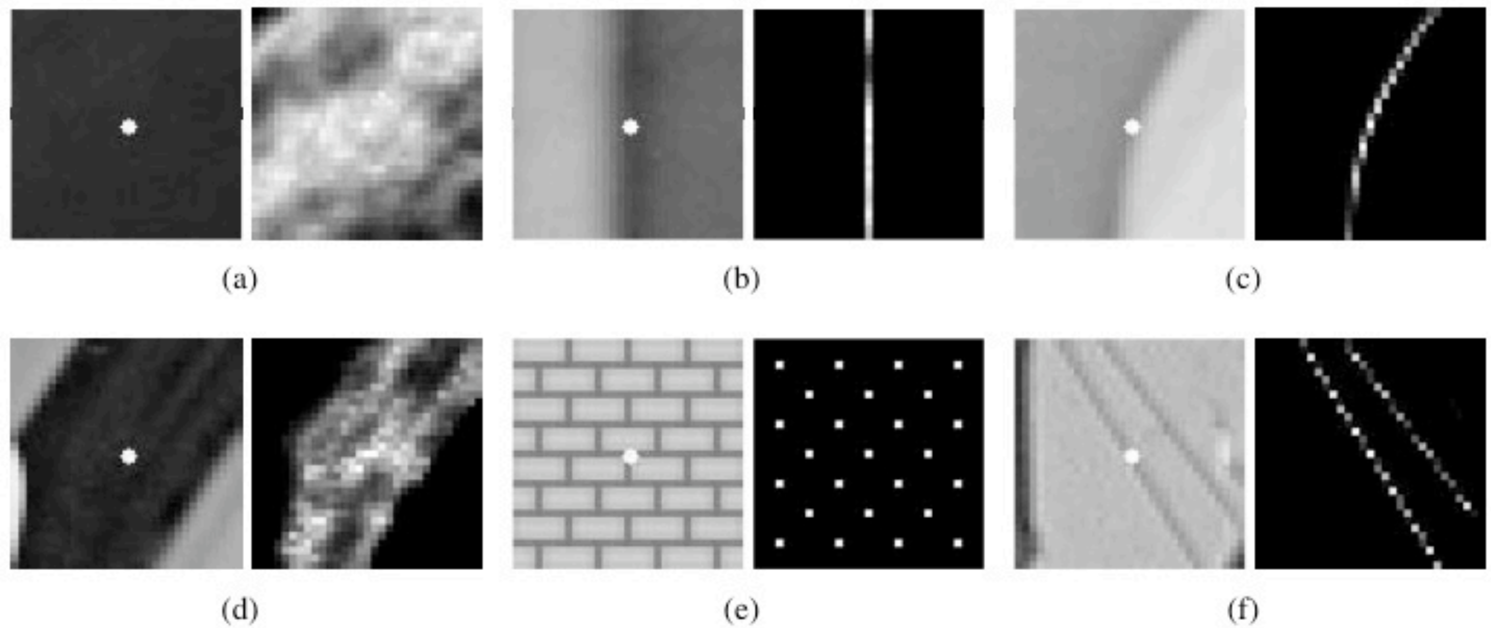
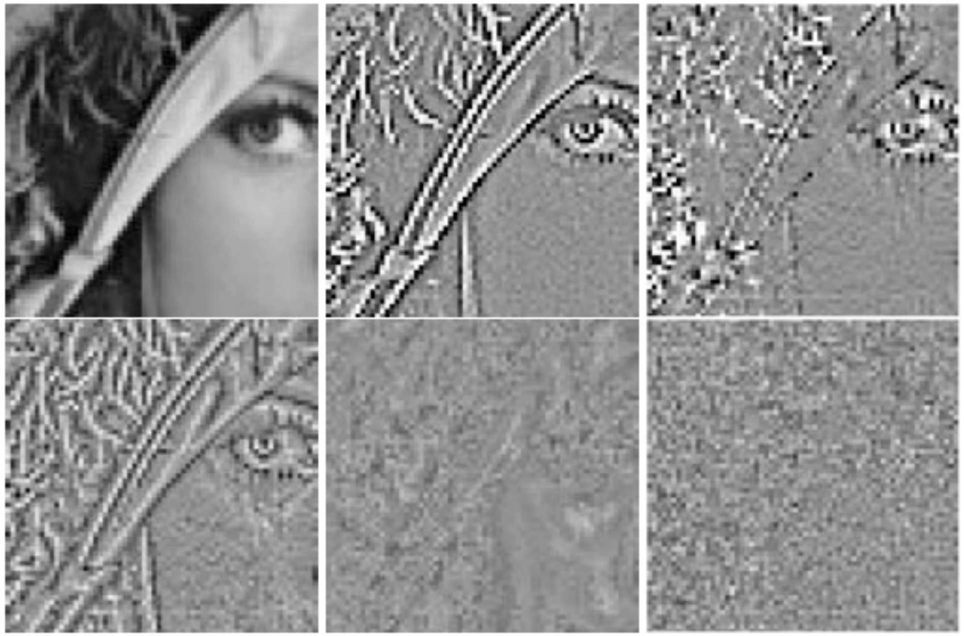


Figure 2. Display of the NL-means weight distribution used to estimate the central pixel of every image. The weights go from 1(white) to zero(black).

Example of Non-Local Means



Left figure: Denoising using from left to right and from top to bottom Gaussian filter, anisotropic filter, total variation denoising, neighborhood filtering and non-local means.

Right figure: The original picture an the differences between the denoised and the original image for each of the methods.

Non-Local Means Example 2

noisy

denoised

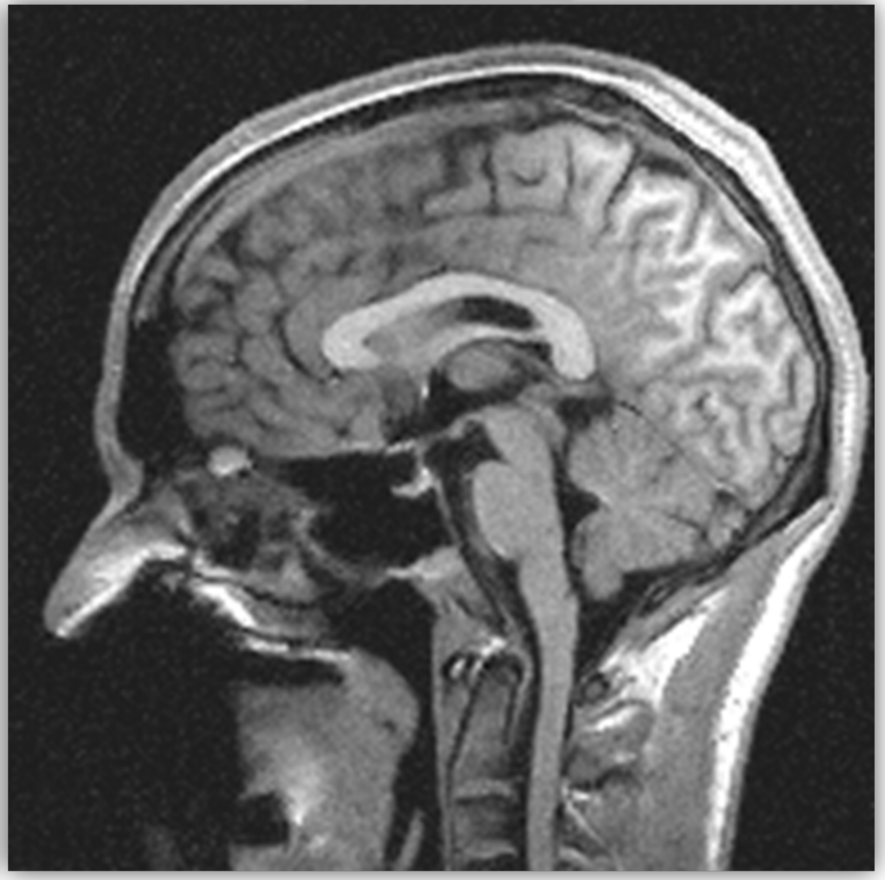


Image Sources

1. “Image with salt & pepper noise”, Marko Meza.
2. Many of the smoothing and edge detection images are from the slides by D.A. Forsyth, University of Illinois at Urbana-Champaign.
3. The examples in slides 18-21 are courtesy of R. Fisher, S. Perkins, A. Walker and E. Wolfart
4. Non-Local Means figures are from the paper A. Buades, B. Coll and J.-M. Morel, “A Non-Local Algorithm for Image Denoising,” *Computer Vision and Pattern Recognition*, 2005.