Some examples of wavelet filter coefficients

> *Please note: Mathematica has a convention for normalizing filter coefficients which is different from the one used in the lecture. The coefficients given in this notebook have to be multiplied by $\sqrt{2}$ und order to obtain the usual values. Note also that the positions of the filter coefficients are shifted.*

In[1]:=
```
PS = {PlotStyle →
      {Directive[Red, PointSize[Large]], Directive[Blue, PointSize[Large]]},
    PlotRange → All, Filling → Axis};
```

The Haar Filter (D2)

In[2]:=
```
D2 = WaveletFilterCoefficients[
      DaubechiesWavelet[1], {"PrimalLowpass", "PrimalHighpass"}];
D2 // MatrixForm
```

Out[3]//MatrixForm=

$$\left(\begin{array}{cc} \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} & \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.5 \end{pmatrix} \end{array} \right)$$
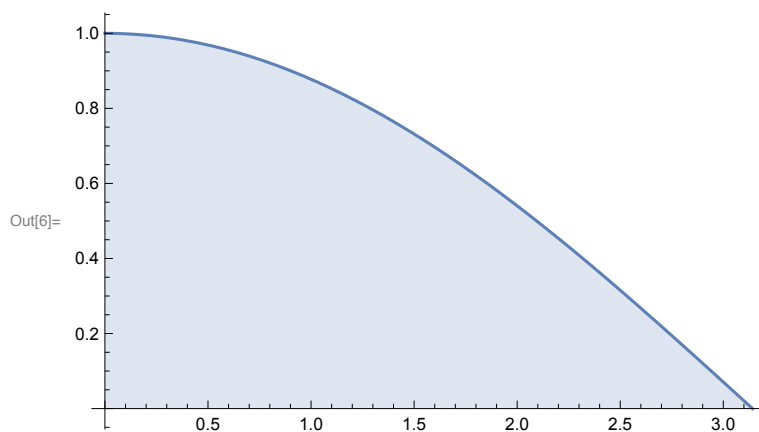
z-Transform, Frequency response

In[4]:=
```
p2[z_] = Sum[D2[[1, k + 1, 2]] z^k, {k, 0, 1}]
```

Out[4]= $0.5 + 0.5\, z$

In[5]:=
```
P2[ω_] = p2[Exp[I ω]]
```

Out[5]= $0.5 + 0.5\, e^{i\,\omega}$

In[6]:=
```
Plot[Abs[P2[ω]], {ω, 0, Pi}, Filling → Axis]
```

Out[6]=

Checking orthogonality

In[7]:=
```
Abs[P2[ω]]^2 + Abs[P2[ω + Pi]]^2
```

Out[7]= $\mathrm{Abs}\left[0.5 + 0.5\, e^{i\,\omega}\right]^2 + \mathrm{Abs}\left[0.5 + 0.5\, e^{i\,(\pi+\omega)}\right]^2$
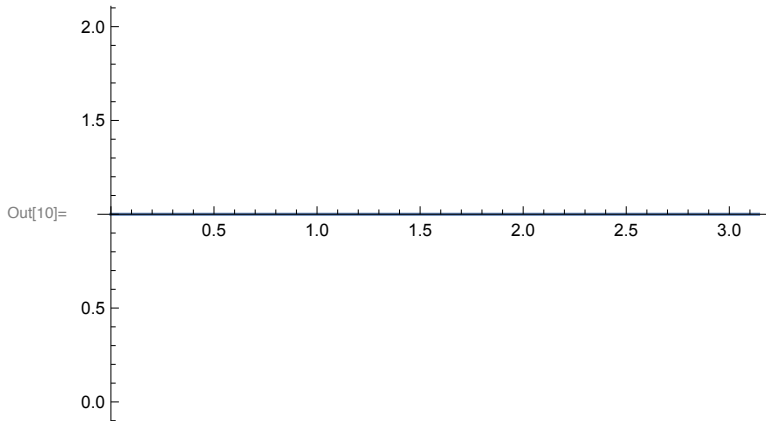
In[8]:= `ComplexExpand[%]`

Out[8]= $0.5 + 0.5\,Cos[\omega]^2 + 0.5\,Sin[\omega]^2$

In[9]:= `Simplify[%]`

Out[9]= $1.$

In[10]:= $\text{Plot}\left[Abs[P2[\omega]]^2 + Abs[P2[\omega + Pi]]^2, \{\omega, 0, Pi\}\right]$

Out[10]=

The D4 Filter

In[11]:= `D4 = WaveletFilterCoefficients[`
     `  DaubechiesWavelet[2], {"PrimalLowpass", "PrimalHighpass"}];`
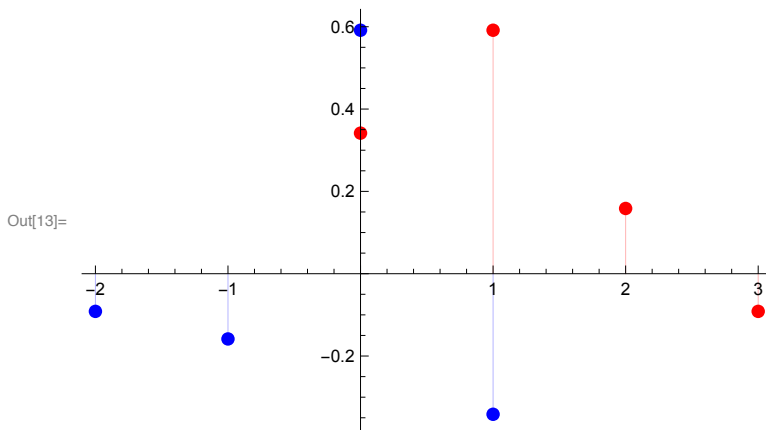   `D4 // MatrixForm`

Out[11]//MatrixForm=
$$\left(\begin{array}{cccc} \binom{0}{0.341506} & \binom{1}{0.591506} & \binom{2}{0.158494} & \binom{3}{-0.0915064} \\ \binom{-2}{-0.0915064} & \binom{-1}{-0.158494} & \binom{0}{0.591506} & \binom{1}{-0.341506} \end{array}\right)$$

In[12]:= `Transpose[Map[Total[#] &, D4]][[2]]`

Out[12]= $\left\{1., -5.55112 \times 10^{-17}\right\}$

Scaling filter coefficients in red, wavelet filter coefficients in blue

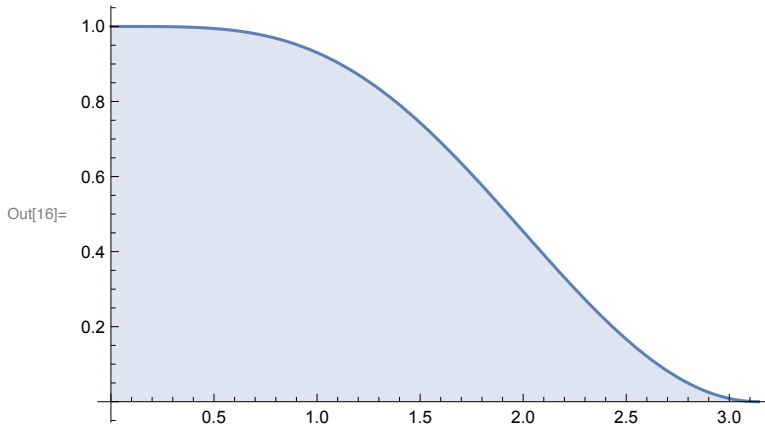In[13]:= `ListPlot[D4, PS]`

Out[13]=

z-Transform, Frequency response

In[14]:= `p4[z_] = Sum[D4[[1, k + 1, 2]] z^k, {k, 0, 3}]`

Out[14]= $0.341506 + 0.591506\, z + 0.158494\, z^2 - 0.0915064\, z^3$

In[15]:= `P4[ω_] = p4[Exp[I ω]]`

Out[15]= $0.341506 + 0.591506\, \mathbb{e}^{\mathbb{i}\,\omega} + 0.158494\, \mathbb{e}^{2\,\mathbb{i}\,\omega} - 0.0915064\, \mathbb{e}^{3\,\mathbb{i}\,\omega}$

In[16]:= `Plot[Abs[P4[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[16]=

Checking orthogonality

In[17]:= `ComplexExpand[Abs[P4[ω]]^2 + Abs[P4[ω + Pi]]^2]`

Out[17]= $0.233253 + 0.69976\, \text{Cos}[\omega]^2 + 0.108253\, \text{Cos}[2\,\omega] + 0.1875\, \text{Cos}[\omega]\, \text{Cos}[2\,\omega] +$
$0.0251202\, \text{Cos}[2\,\omega]^2 - 0.0625\, \text{Cos}[3\,\omega] - 0.108253\, \text{Cos}[\omega]\, \text{Cos}[3\,\omega] -$
$0.0290064\, \text{Cos}[2\,\omega]\, \text{Cos}[3\,\omega] + 0.00837341\, \text{Cos}[3\,\omega]^2 + 0.108253\, \text{Cos}[2\,(\pi+\omega)] -$
$0.1875\, \text{Cos}[\omega]\, \text{Cos}[2\,(\pi+\omega)] + 0.0251202\, \text{Cos}[2\,(\pi+\omega)]^2 - 0.0625\, \text{Cos}[3\,(\pi+\omega)] +$
$0.108253\, \text{Cos}[\omega]\, \text{Cos}[3\,(\pi+\omega)] - 0.0290064\, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[3\,(\pi+\omega)] +$
$0.00837341\, \text{Cos}[3\,(\pi+\omega)]^2 + 0.69976\, \text{Sin}[\omega]^2 + 0.1875\, \text{Sin}[\omega]\, \text{Sin}[2\,\omega] +$
$0.0251202\, \text{Sin}[2\,\omega]^2 - 0.108253\, \text{Sin}[\omega]\, \text{Sin}[3\,\omega] - 0.0290064\, \text{Sin}[2\,\omega]\, \text{Sin}[3\,\omega] +$
$0.00837341\, \text{Sin}[3\,\omega]^2 - 0.1875\, \text{Sin}[\omega]\, \text{Sin}[2\,(\pi+\omega)] +$
$0.0251202\, \text{Sin}[2\,(\pi+\omega)]^2 + 0.108253\, \text{Sin}[\omega]\, \text{Sin}[3\,(\pi+\omega)] -$
$0.0290064\, \text{Sin}[2\,(\pi+\omega)]\, \text{Sin}[3\,(\pi+\omega)] + 0.00837341\, \text{Sin}[3\,(\pi+\omega)]^2$

In[18]:= `Simplify[%]`

Out[18]= $1. + 8.32667 \times 10^{-17}\, \text{Cos}[\omega]^2 - 8.32667 \times 10^{-17}\, \text{Sin}[\omega]^2$

In[19]:= `Chop[%]`

Out[19]= $1.$

The D6 Filter

In[21]:= `D6 = WaveletFilterCoefficients[`
    `DaubechiesWavelet[3], {"PrimalLowpass", "PrimalHighpass"}];`

In[22]:= `Transpose[D6] // MatrixForm`

Out[22]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0.235234 \end{pmatrix} & \begin{pmatrix} -4 \\ 0.0249087 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0.570558 \end{pmatrix} & \begin{pmatrix} -3 \\ 0.0604161 \end{pmatrix} \\ \begin{pmatrix} 2 \\ 0.325183 \end{pmatrix} & \begin{pmatrix} -2 \\ -0.0954672 \end{pmatrix} \\ \begin{pmatrix} 3 \\ -0.0954672 \end{pmatrix} & \begin{pmatrix} -1 \\ -0.325183 \end{pmatrix} \\ \begin{pmatrix} 4 \\ -0.0604161 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.570558 \end{pmatrix} \\ \begin{pmatrix} 5 \\ 0.0249087 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.235234 \end{pmatrix} \end{pmatrix}$$

In[23]:= `Transpose[Map[Total[#] &, D6]][[2]]`

Out[23]= $\left\{ 1., 8.32667 \times 10^{-17} \right\}$

In[24]:= `ListPlot[D6, PS]`

Out[24]=
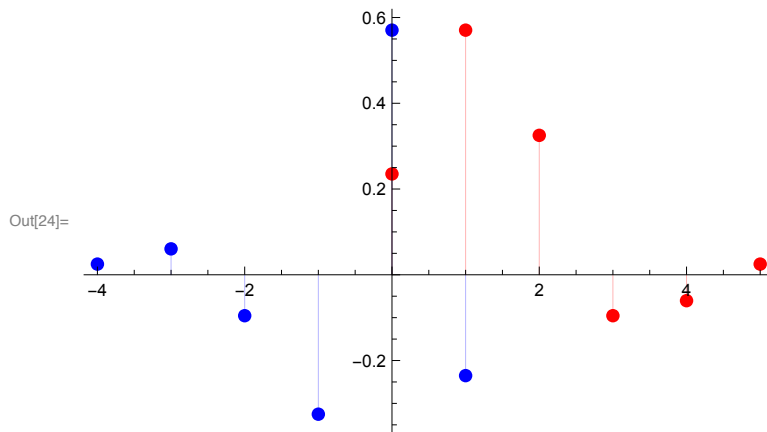
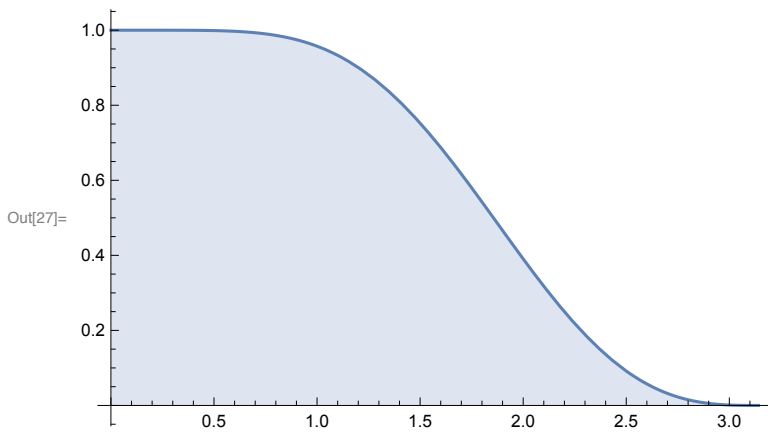## z-Transform, Frequency response

In[25]:= `p6[z_] = Sum[D6[[1, k + 1, 2]] z^k, {k, 0, 5}]`

Out[25]= $0.235234 + 0.570558\, z + 0.325183\, z^2 - 0.0954672\, z^3 - 0.0604161\, z^4 + 0.0249087\, z^5$

In[26]:= `P6[ω_] = p6[Exp[I ω]]`

Out[26]= $0.235234 + 0.570558\, e^{i\,\omega} + 0.325183\, e^{2\,i\,\omega} - 0.0954672\, e^{3\,i\,\omega} - 0.0604161\, e^{4\,i\,\omega} + 0.0249087\, e^{5\,i\,\omega}$

In[27]:= `Plot[Abs[P6[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[27]=

Checking orthogonality

In[28]:= `ComplexExpand[Abs[P6[ω]]² + Abs[P6[ω + Pi]]²]`

Out[28]= $0.11067 + 0.651074 \, \text{Cos}[\omega]^2 + 0.152988 \, \text{Cos}[2\,\omega] + 0.371071 \, \text{Cos}[\omega] \, \text{Cos}[2\,\omega] +$
$0.105744 \, \text{Cos}[2\,\omega]^2 - 0.0449142 \, \text{Cos}[3\,\omega] - 0.108939 \, \text{Cos}[\omega] \, \text{Cos}[3\,\omega] -$
$0.0620885 \, \text{Cos}[2\,\omega] \, \text{Cos}[3\,\omega] + 0.00911399 \, \text{Cos}[3\,\omega]^2 - 0.0284238 \, \text{Cos}[4\,\omega] -$
$0.0689418 \, \text{Cos}[\omega] \, \text{Cos}[4\,\omega] - 0.0392925 \, \text{Cos}[2\,\omega] \, \text{Cos}[4\,\omega] + 0.0115355 \, \text{Cos}[3\,\omega] \, \text{Cos}[4\,\omega] +$
$0.00365011 \, \text{Cos}[4\,\omega]^2 + 0.0117188 \, \text{Cos}[5\,\omega] + 0.0284238 \, \text{Cos}[\omega] \, \text{Cos}[5\,\omega] +$
$0.0161998 \, \text{Cos}[2\,\omega] \, \text{Cos}[5\,\omega] - 0.00475594 \, \text{Cos}[3\,\omega] \, \text{Cos}[5\,\omega] -$
$0.00300978 \, \text{Cos}[4\,\omega] \, \text{Cos}[5\,\omega] + 0.000620446 \, \text{Cos}[5\,\omega]^2 + 0.152988 \, \text{Cos}[2\,(\pi+\omega)] -$
$0.371071 \, \text{Cos}[\omega] \, \text{Cos}[2\,(\pi+\omega)] + 0.105744 \, \text{Cos}[2\,(\pi+\omega)]^2 - 0.0449142 \, \text{Cos}[3\,(\pi+\omega)] +$
$0.108939 \, \text{Cos}[\omega] \, \text{Cos}[3\,(\pi+\omega)] - 0.0620885 \, \text{Cos}[2\,(\pi+\omega)] \, \text{Cos}[3\,(\pi+\omega)] +$
$0.00911399 \, \text{Cos}[3\,(\pi+\omega)]^2 - 0.0284238 \, \text{Cos}[4\,(\pi+\omega)] + 0.0689418 \, \text{Cos}[\omega] \, \text{Cos}[4\,(\pi+\omega)] -$
$0.0392925 \, \text{Cos}[2\,(\pi+\omega)] \, \text{Cos}[4\,(\pi+\omega)] + 0.0115355 \, \text{Cos}[3\,(\pi+\omega)] \, \text{Cos}[4\,(\pi+\omega)] +$
$0.00365011 \, \text{Cos}[4\,(\pi+\omega)]^2 + 0.0117188 \, \text{Cos}[5\,(\pi+\omega)] - 0.0284238 \, \text{Cos}[\omega] \, \text{Cos}[5\,(\pi+\omega)] +$
$0.0161998 \, \text{Cos}[2\,(\pi+\omega)] \, \text{Cos}[5\,(\pi+\omega)] - 0.00475594 \, \text{Cos}[3\,(\pi+\omega)] \, \text{Cos}[5\,(\pi+\omega)] -$
$0.00300978 \, \text{Cos}[4\,(\pi+\omega)] \, \text{Cos}[5\,(\pi+\omega)] + 0.000620446 \, \text{Cos}[5\,(\pi+\omega)]^2 +$
$0.651074 \, \text{Sin}[\omega]^2 + 0.371071 \, \text{Sin}[\omega] \, \text{Sin}[2\,\omega] + 0.105744 \, \text{Sin}[2\,\omega]^2 -$
$0.108939 \, \text{Sin}[\omega] \, \text{Sin}[3\,\omega] - 0.0620885 \, \text{Sin}[2\,\omega] \, \text{Sin}[3\,\omega] + 0.00911399 \, \text{Sin}[3\,\omega]^2 -$
$0.0689418 \, \text{Sin}[\omega] \, \text{Sin}[4\,\omega] - 0.0392925 \, \text{Sin}[2\,\omega] \, \text{Sin}[4\,\omega] + 0.0115355 \, \text{Sin}[3\,\omega] \, \text{Sin}[4\,\omega] +$
$0.00365011 \, \text{Sin}[4\,\omega]^2 + 0.0284238 \, \text{Sin}[\omega] \, \text{Sin}[5\,\omega] + 0.0161998 \, \text{Sin}[2\,\omega] \, \text{Sin}[5\,\omega] -$
$0.00475594 \, \text{Sin}[3\,\omega] \, \text{Sin}[5\,\omega] - 0.00300978 \, \text{Sin}[4\,\omega] \, \text{Sin}[5\,\omega] +$
$0.000620446 \, \text{Sin}[5\,\omega]^2 - 0.371071 \, \text{Sin}[\omega] \, \text{Sin}[2\,(\pi+\omega)] + 0.105744 \, \text{Sin}[2\,(\pi+\omega)]^2 +$
$0.108939 \, \text{Sin}[\omega] \, \text{Sin}[3\,(\pi+\omega)] - 0.0620885 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[3\,(\pi+\omega)] +$
$0.00911399 \, \text{Sin}[3\,(\pi+\omega)]^2 + 0.0689418 \, \text{Sin}[\omega] \, \text{Sin}[4\,(\pi+\omega)] -$
$0.0392925 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[4\,(\pi+\omega)] + 0.0115355 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[4\,(\pi+\omega)] +$
$0.00365011 \, \text{Sin}[4\,(\pi+\omega)]^2 - 0.0284238 \, \text{Sin}[\omega] \, \text{Sin}[5\,(\pi+\omega)] +$
$0.0161998 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] - 0.00475594 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] -$
$0.00300978 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] + 0.000620446 \, \text{Sin}[5\,(\pi+\omega)]^2$

In[29]:= `Simplify[%]`

Out[29]= $1. - 5.55112 \times 10^{-17} \, \text{Cos}[\omega]^3 + 1.38778 \times 10^{-17} \, \text{Cos}[\omega]^4 +$
$1.66533 \times 10^{-16} \, \text{Cos}[\omega] \, \text{Sin}[\omega]^2 + 1.38778 \times 10^{-17} \, \text{Sin}[\omega]^4 - 2.08167 \times 10^{-17} \, \text{Sin}[2\,\omega]^2$

In[30]:= **Chop[%]**

Out[30]= 1.

The D8 Filter

In[31]:= **D8 = WaveletFilterCoefficients[**
DaubechiesWavelet[4], {"PrimalLowpass", "PrimalHighpass"}];

In[32]:= **Transpose[D8] // MatrixForm**

Out[32]//MatrixForm=

$$
\begin{pmatrix}
\begin{pmatrix} 0 \\ 0.162902 \end{pmatrix} & \begin{pmatrix} -6 \\ -0.00749349 \end{pmatrix} \\
\begin{pmatrix} 1 \\ 0.505473 \end{pmatrix} & \begin{pmatrix} -5 \\ -0.0232518 \end{pmatrix} \\
\begin{pmatrix} 2 \\ 0.4461 \end{pmatrix} & \begin{pmatrix} -4 \\ 0.0218082 \end{pmatrix} \\
\begin{pmatrix} 3 \\ -0.0197875 \end{pmatrix} & \begin{pmatrix} -3 \\ 0.132254 \end{pmatrix} \\
\begin{pmatrix} 4 \\ -0.132254 \end{pmatrix} & \begin{pmatrix} -2 \\ -0.0197875 \end{pmatrix} \\
\begin{pmatrix} 5 \\ 0.0218082 \end{pmatrix} & \begin{pmatrix} -1 \\ -0.4461 \end{pmatrix} \\
\begin{pmatrix} 6 \\ 0.0232518 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.505473 \end{pmatrix} \\
\begin{pmatrix} 7 \\ -0.00749349 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.162902 \end{pmatrix}
\end{pmatrix}
$$

In[33]:= **Transpose[Map[Total[#] &, D8]][[2]]**

Out[33]= $\{1., 8.60423 \times 10^{-16}\}$

In[34]:= **ListPlot[D8, PS]**

Out[34]=
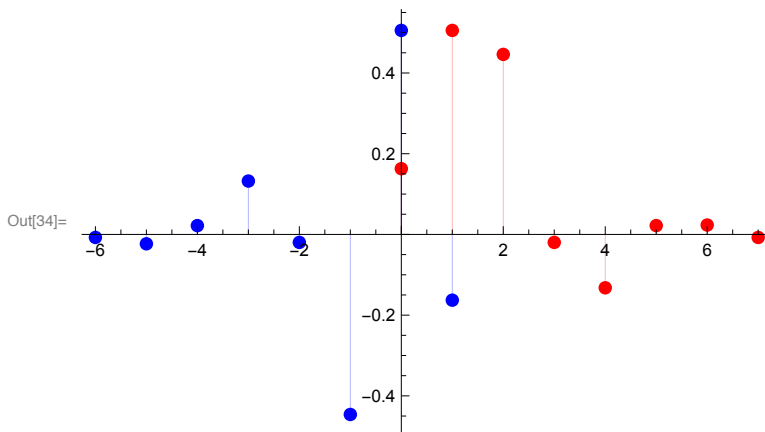
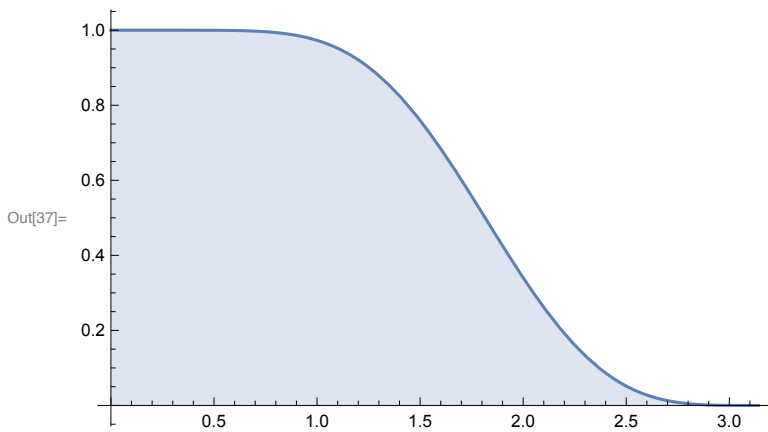
z-Transform, Frequency response

In[35]:= **p8[z_] = Sum[D8[[1, k + 1, 2]] z^k, {k, 0, 7}]**

Out[35]= $0.162902 + 0.505473\, z + 0.4461\, z^2 - 0.0197875\, z^3 - 0.132254\, z^4 + 0.0218082\, z^5 + 0.0232518\, z^6 - 0.00749349\, z^7$

In[36]:= **P8[ω_] = p8[Exp[I ω]]**

Out[36]= $0.162902 + 0.505473\, e^{i\,\omega} + 0.4461\, e^{2\,i\,\omega} - 0.0197875\, e^{3\,i\,\omega} - 0.132254\, e^{4\,i\,\omega} + 0.0218082\, e^{5\,i\,\omega} + 0.0232518\, e^{6\,i\,\omega} - 0.00749349\, e^{7\,i\,\omega}$

In[37]:= `Plot[Abs[P8[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[37]=

Checking orthogonality

In[38]:= $\text{ComplexExpand}\left[\text{Abs}[\text{P8}[\omega]]^2 + \text{Abs}[\text{P8}[\omega + \text{Pi}]]^2\right]$

Out[38]= $0.0530739 + 0.511006 \, \text{Cos}[\omega]^2 + 0.145341 \, \text{Cos}[2\,\omega] + 0.450983 \, \text{Cos}[\omega]\, \text{Cos}[2\,\omega] +$
$0.199005 \, \text{Cos}[2\,\omega]^2 - 0.00644684 \, \text{Cos}[3\,\omega] - 0.0200041 \, \text{Cos}[\omega]\, \text{Cos}[3\,\omega] -$
$0.0176544 \, \text{Cos}[2\,\omega]\, \text{Cos}[3\,\omega] + 0.000391546 \, \text{Cos}[3\,\omega]^2 - 0.0430887 \, \text{Cos}[4\,\omega] -$
$0.133701 \, \text{Cos}[\omega]\, \text{Cos}[4\,\omega] - 0.117997 \, \text{Cos}[2\,\omega]\, \text{Cos}[4\,\omega] + 0.00523394 \, \text{Cos}[3\,\omega]\, \text{Cos}[4\,\omega] +$
$0.017491 \, \text{Cos}[4\,\omega]^2 + 0.00710517 \, \text{Cos}[5\,\omega] + 0.0220469 \, \text{Cos}[\omega]\, \text{Cos}[5\,\omega] +$
$0.0194572 \, \text{Cos}[2\,\omega]\, \text{Cos}[5\,\omega] - 0.000863058 \, \text{Cos}[3\,\omega]\, \text{Cos}[5\,\omega] -$
$0.00576841 \, \text{Cos}[4\,\omega]\, \text{Cos}[5\,\omega] + 0.000475595 \, \text{Cos}[5\,\omega]^2 + 0.00757552 \, \text{Cos}[6\,\omega] +$
$0.0235063 \, \text{Cos}[\omega]\, \text{Cos}[6\,\omega] + 0.0207453 \, \text{Cos}[2\,\omega]\, \text{Cos}[6\,\omega] -$
$0.000920191 \, \text{Cos}[3\,\omega]\, \text{Cos}[6\,\omega] - 0.00615027 \, \text{Cos}[4\,\omega]\, \text{Cos}[6\,\omega] +$
$0.00101416 \, \text{Cos}[5\,\omega]\, \text{Cos}[6\,\omega] + 0.000540646 \, \text{Cos}[6\,\omega]^2 - 0.00244141 \, \text{Cos}[7\,\omega] -$
$0.00757552 \, \text{Cos}[\omega]\, \text{Cos}[7\,\omega] - 0.0066857 \, \text{Cos}[2\,\omega]\, \text{Cos}[7\,\omega] +$
$0.000296555 \, \text{Cos}[3\,\omega]\, \text{Cos}[7\,\omega] + 0.00198208 \, \text{Cos}[4\,\omega]\, \text{Cos}[7\,\omega] -$
$0.000326839 \, \text{Cos}[5\,\omega]\, \text{Cos}[7\,\omega] - 0.000348474 \, \text{Cos}[6\,\omega]\, \text{Cos}[7\,\omega] +$
$0.0000561525 \, \text{Cos}[7\,\omega]^2 + 0.145341 \, \text{Cos}[2\,(\pi+\omega)] - 0.450983 \, \text{Cos}[\omega]\, \text{Cos}[2\,(\pi+\omega)] +$
$0.199005 \, \text{Cos}[2\,(\pi+\omega)]^2 - 0.00644684 \, \text{Cos}[3\,(\pi+\omega)] + 0.0200041 \, \text{Cos}[\omega]\, \text{Cos}[3\,(\pi+\omega)] -$
$0.0176544 \, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[3\,(\pi+\omega)] + 0.000391546 \, \text{Cos}[3\,(\pi+\omega)]^2 -$
$0.0430887 \, \text{Cos}[4\,(\pi+\omega)] + 0.133701 \, \text{Cos}[\omega]\, \text{Cos}[4\,(\pi+\omega)] -$
$0.117997 \, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[4\,(\pi+\omega)] + 0.00523394 \, \text{Cos}[3\,(\pi+\omega)]\, \text{Cos}[4\,(\pi+\omega)] +$
$0.017491 \, \text{Cos}[4\,(\pi+\omega)]^2 + 0.00710517 \, \text{Cos}[5\,(\pi+\omega)] - 0.0220469 \, \text{Cos}[\omega]\, \text{Cos}[5\,(\pi+\omega)] +$
$0.0194572 \, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[5\,(\pi+\omega)] - 0.000863058 \, \text{Cos}[3\,(\pi+\omega)]\, \text{Cos}[5\,(\pi+\omega)] -$
$0.00576841 \, \text{Cos}[4\,(\pi+\omega)]\, \text{Cos}[5\,(\pi+\omega)] + 0.000475595 \, \text{Cos}[5\,(\pi+\omega)]^2 +$
$0.00757552 \, \text{Cos}[6\,(\pi+\omega)] - 0.0235063 \, \text{Cos}[\omega]\, \text{Cos}[6\,(\pi+\omega)] +$
$0.0207453 \, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[6\,(\pi+\omega)] - 0.000920191 \, \text{Cos}[3\,(\pi+\omega)]\, \text{Cos}[6\,(\pi+\omega)] -$
$0.00615027 \, \text{Cos}[4\,(\pi+\omega)]\, \text{Cos}[6\,(\pi+\omega)] + 0.00101416 \, \text{Cos}[5\,(\pi+\omega)]\, \text{Cos}[6\,(\pi+\omega)] +$
$0.000540646 \, \text{Cos}[6\,(\pi+\omega)]^2 - 0.00244141 \, \text{Cos}[7\,(\pi+\omega)] +$
$0.00757552 \, \text{Cos}[\omega]\, \text{Cos}[7\,(\pi+\omega)] - 0.0066857 \, \text{Cos}[2\,(\pi+\omega)]\, \text{Cos}[7\,(\pi+\omega)] +$
$0.000296555 \, \text{Cos}[3\,(\pi+\omega)]\, \text{Cos}[7\,(\pi+\omega)] + 0.00198208 \, \text{Cos}[4\,(\pi+\omega)]\, \text{Cos}[7\,(\pi+\omega)] -$
$0.000326839 \, \text{Cos}[5\,(\pi+\omega)]\, \text{Cos}[7\,(\pi+\omega)] - 0.000348474 \, \text{Cos}[6\,(\pi+\omega)]\, \text{Cos}[7\,(\pi+\omega)] +$
$0.0000561525 \, \text{Cos}[7\,(\pi+\omega)]^2 + 0.511006 \, \text{Sin}[\omega]^2 + 0.450983 \, \text{Sin}[\omega]\, \text{Sin}[2\,\omega] +$
$0.199005 \, \text{Sin}[2\,\omega]^2 - 0.0200041 \, \text{Sin}[\omega]\, \text{Sin}[3\,\omega] - 0.0176544 \, \text{Sin}[2\,\omega]\, \text{Sin}[3\,\omega] +$
$0.000391546 \, \text{Sin}[3\,\omega]^2 - 0.133701 \, \text{Sin}[\omega]\, \text{Sin}[4\,\omega] - 0.117997 \, \text{Sin}[2\,\omega]\, \text{Sin}[4\,\omega] +$

$0.00523394\,\mathrm{Sin}[3\,\omega]\,\mathrm{Sin}[4\,\omega] + 0.017491\,\mathrm{Sin}[4\,\omega]^2 + 0.0220469\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[5\,\omega] +$
$0.0194572\,\mathrm{Sin}[2\,\omega]\,\mathrm{Sin}[5\,\omega] - 0.000863058\,\mathrm{Sin}[3\,\omega]\,\mathrm{Sin}[5\,\omega] -$
$0.00576841\,\mathrm{Sin}[4\,\omega]\,\mathrm{Sin}[5\,\omega] + 0.000475595\,\mathrm{Sin}[5\,\omega]^2 + 0.0235063\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[6\,\omega] +$
$0.0207453\,\mathrm{Sin}[2\,\omega]\,\mathrm{Sin}[6\,\omega] - 0.000920191\,\mathrm{Sin}[3\,\omega]\,\mathrm{Sin}[6\,\omega] -$
$0.00615027\,\mathrm{Sin}[4\,\omega]\,\mathrm{Sin}[6\,\omega] + 0.00101416\,\mathrm{Sin}[5\,\omega]\,\mathrm{Sin}[6\,\omega] +$
$0.000540646\,\mathrm{Sin}[6\,\omega]^2 - 0.00757552\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[7\,\omega] - 0.0066857\,\mathrm{Sin}[2\,\omega]\,\mathrm{Sin}[7\,\omega] +$
$0.000296555\,\mathrm{Sin}[3\,\omega]\,\mathrm{Sin}[7\,\omega] + 0.00198208\,\mathrm{Sin}[4\,\omega]\,\mathrm{Sin}[7\,\omega] -$
$0.000326839\,\mathrm{Sin}[5\,\omega]\,\mathrm{Sin}[7\,\omega] - 0.000348474\,\mathrm{Sin}[6\,\omega]\,\mathrm{Sin}[7\,\omega] +$
$0.0000561525\,\mathrm{Sin}[7\,\omega]^2 - 0.450983\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[2\,(\pi+\omega)] +$
$0.199005\,\mathrm{Sin}[2\,(\pi+\omega)]^2 + 0.0200041\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[3\,(\pi+\omega)] -$
$0.0176544\,\mathrm{Sin}[2\,(\pi+\omega)]\,\mathrm{Sin}[3\,(\pi+\omega)] + 0.000391546\,\mathrm{Sin}[3\,(\pi+\omega)]^2 +$
$0.133701\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[4\,(\pi+\omega)] - 0.117997\,\mathrm{Sin}[2\,(\pi+\omega)]\,\mathrm{Sin}[4\,(\pi+\omega)] +$
$0.00523394\,\mathrm{Sin}[3\,(\pi+\omega)]\,\mathrm{Sin}[4\,(\pi+\omega)] + 0.017491\,\mathrm{Sin}[4\,(\pi+\omega)]^2 -$
$0.0220469\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[5\,(\pi+\omega)] + 0.0194572\,\mathrm{Sin}[2\,(\pi+\omega)]\,\mathrm{Sin}[5\,(\pi+\omega)] -$
$0.000863058\,\mathrm{Sin}[3\,(\pi+\omega)]\,\mathrm{Sin}[5\,(\pi+\omega)] - 0.00576841\,\mathrm{Sin}[4\,(\pi+\omega)]\,\mathrm{Sin}[5\,(\pi+\omega)] +$
$0.000475595\,\mathrm{Sin}[5\,(\pi+\omega)]^2 - 0.0235063\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[6\,(\pi+\omega)] +$
$0.0207453\,\mathrm{Sin}[2\,(\pi+\omega)]\,\mathrm{Sin}[6\,(\pi+\omega)] - 0.000920191\,\mathrm{Sin}[3\,(\pi+\omega)]\,\mathrm{Sin}[6\,(\pi+\omega)] -$
$0.00615027\,\mathrm{Sin}[4\,(\pi+\omega)]\,\mathrm{Sin}[6\,(\pi+\omega)] + 0.00101416\,\mathrm{Sin}[5\,(\pi+\omega)]\,\mathrm{Sin}[6\,(\pi+\omega)] +$
$0.000540646\,\mathrm{Sin}[6\,(\pi+\omega)]^2 + 0.00757552\,\mathrm{Sin}[\omega]\,\mathrm{Sin}[7\,(\pi+\omega)] -$
$0.0066857\,\mathrm{Sin}[2\,(\pi+\omega)]\,\mathrm{Sin}[7\,(\pi+\omega)] + 0.000296555\,\mathrm{Sin}[3\,(\pi+\omega)]\,\mathrm{Sin}[7\,(\pi+\omega)] +$
$0.00198208\,\mathrm{Sin}[4\,(\pi+\omega)]\,\mathrm{Sin}[7\,(\pi+\omega)] - 0.000326839\,\mathrm{Sin}[5\,(\pi+\omega)]\,\mathrm{Sin}[7\,(\pi+\omega)] -$
$0.000348474\,\mathrm{Sin}[6\,(\pi+\omega)]\,\mathrm{Sin}[7\,(\pi+\omega)] + 0.0000561525\,\mathrm{Sin}[7\,(\pi+\omega)]^2$

In[39]:= `Simplify[%]`

Out[39]= $1. - 1.11022 \times 10^{-16}\,\mathrm{Cos}[2\,\omega] - 6.59195 \times 10^{-17}\,\mathrm{Cos}[4\,\omega] +$
$1.38778 \times 10^{-17}\,\mathrm{Cos}[6\,\omega] + 6.93889 \times 10^{-18}\,\mathrm{Cos}[7\,\omega] - 4.33681 \times 10^{-19}\,\mathrm{Cos}[11\,\omega]$

In[40]:= `Chop[%]`

Out[40]= $1.$

The D20 Filter

In[41]:= ```
D20 = WaveletFilterCoefficients[
    DaubechiesWavelet[10], {"PrimalLowpass", "PrimalHighpass"}];
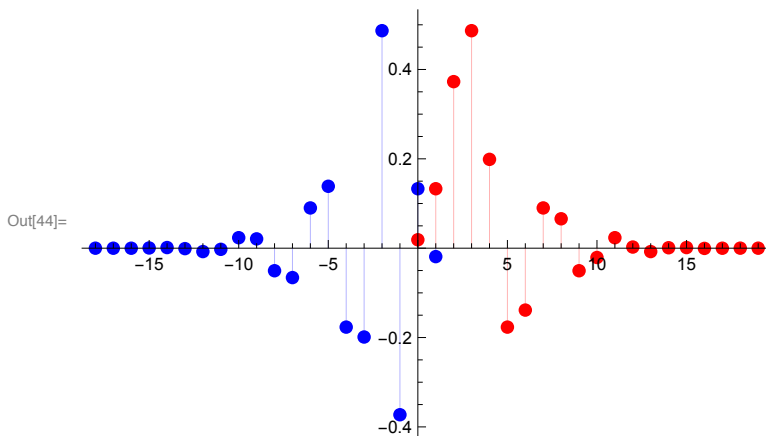```

In[42]:= **Transpose[D20] // MatrixForm**

Out[42]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} 0 \\ 0.0188586 \end{pmatrix} & \begin{pmatrix} -18 \\ -9.37921 \times 10^{-6} \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0.133061 \end{pmatrix} & \begin{pmatrix} -17 \\ -0.0000661772 \end{pmatrix} \\ \begin{pmatrix} 2 \\ 0.372788 \end{pmatrix} & \begin{pmatrix} -16 \\ -0.0000823545 \end{pmatrix} \\ \begin{pmatrix} 3 \\ 0.486814 \end{pmatrix} & \begin{pmatrix} -15 \\ 0.000484974 \end{pmatrix} \\ \begin{pmatrix} 4 \\ 0.198819 \end{pmatrix} & \begin{pmatrix} -14 \\ 0.00140884 \end{pmatrix} \\ \begin{pmatrix} 5 \\ -0.176668 \end{pmatrix} & \begin{pmatrix} -13 \\ -0.000986663 \end{pmatrix} \\ \begin{pmatrix} 6 \\ -0.138555 \end{pmatrix} & \begin{pmatrix} -12 \\ -0.0075895 \end{pmatrix} \\ \begin{pmatrix} 7 \\ 0.0900637 \end{pmatrix} & \begin{pmatrix} -11 \\ -0.00255022 \end{pmatrix} \\ \begin{pmatrix} 8 \\ 0.0658015 \end{pmatrix} & \begin{pmatrix} -10 \\ 0.0234849 \end{pmatrix} \\ \begin{pmatrix} 9 \\ -0.0504833 \end{pmatrix} & \begin{pmatrix} -9 \\ 0.0208296 \end{pmatrix} \\ \begin{pmatrix} 10 \\ -0.0208296 \end{pmatrix} & \begin{pmatrix} -8 \\ -0.0504833 \end{pmatrix} \\ \begin{pmatrix} 11 \\ 0.0234849 \end{pmatrix} & \begin{pmatrix} -7 \\ -0.0658015 \end{pmatrix} \\ \begin{pmatrix} 12 \\ 0.00255022 \end{pmatrix} & \begin{pmatrix} -6 \\ 0.0900637 \end{pmatrix} \\ \begin{pmatrix} 13 \\ -0.0075895 \end{pmatrix} & \begin{pmatrix} -5 \\ 0.138555 \end{pmatrix} \\ \begin{pmatrix} 14 \\ 0.000986663 \end{pmatrix} & \begin{pmatrix} -4 \\ -0.176668 \end{pmatrix} \\ \begin{pmatrix} 15 \\ 0.00140884 \end{pmatrix} & \begin{pmatrix} -3 \\ -0.198819 \end{pmatrix} \\ \begin{pmatrix} 16 \\ -0.000484974 \end{pmatrix} & \begin{pmatrix} -2 \\ 0.486814 \end{pmatrix} \\ \begin{pmatrix} 17 \\ -0.0000823545 \end{pmatrix} & \begin{pmatrix} -1 \\ -0.372788 \end{pmatrix} \\ \begin{pmatrix} 18 \\ 0.0000661772 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.133061 \end{pmatrix} \\ \begin{pmatrix} 19 \\ -9.37921 \times 10^{-6} \end{pmatrix} & \begin{pmatrix} 1 \\ -0.0188586 \end{pmatrix} \end{pmatrix}$$

In[43]:= **Transpose[Map[Total[#] &, D20]][[2]]**

Out[43]= $\{1., 2.04697 \times 10^{-15}\}$

In[44]:= `ListPlot[D20, PS]`

Out[44]=

z-Transform, Frequency response

In[45]:= `p20[z_] = Sum[D20[[1, k + 1, 2]] z^k, {k, 0, 19}]`

Out[45]= $0.0188586 + 0.133061\, z + 0.372788\, z^2 + 0.486814\, z^3 + 0.198819\, z^4 - 0.176668\, z^5 -$
$0.138555\, z^6 + 0.0900637\, z^7 + 0.0658015\, z^8 - 0.0504833\, z^9 - 0.0208296\, z^{10} +$
$0.0234849\, z^{11} + 0.00255022\, z^{12} - 0.0075895\, z^{13} + 0.000986663\, z^{14} + 0.00140884\, z^{15} -$
$0.000484974\, z^{16} - 0.0000823545\, z^{17} + 0.0000661772\, z^{18} - 9.37921 \times 10^{-6}\, z^{19}$

In[46]:= `P20[ω_] = p20[Exp[I ω]]`

Out[46]= $0.0188586 + 0.133061\, e^{i\,\omega} + 0.372788\, e^{2\,i\,\omega} + 0.486814\, e^{3\,i\,\omega} + 0.198819\, e^{4\,i\,\omega} -$
$0.176668\, e^{5\,i\,\omega} - 0.138555\, e^{6\,i\,\omega} + 0.0900637\, e^{7\,i\,\omega} + 0.0658015\, e^{8\,i\,\omega} -$
$0.0504833\, e^{9\,i\,\omega} - 0.0208296\, e^{10\,i\,\omega} + 0.0234849\, e^{11\,i\,\omega} + 0.00255022\, e^{12\,i\,\omega} -$
$0.0075895\, e^{13\,i\,\omega} + 0.000986663\, e^{14\,i\,\omega} + 0.00140884\, e^{15\,i\,\omega} - 0.000484974\, e^{16\,i\,\omega} -$
$0.0000823545\, e^{17\,i\,\omega} + 0.0000661772\, e^{18\,i\,\omega} - 9.37921 \times 10^{-6}\, e^{19\,i\,\omega}$

In[47]:= `Plot[Abs[P20[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[47]=

Checking orthogonality

In[48]:= $\text{Plot}\left[\text{Abs}[\text{P20}[\omega]]^2 + \text{Abs}[\text{P20}[\omega + \text{Pi}]]^2, \{\omega, 0, \text{Pi}\}\right]$

Out[48]=

The C6 Filter

In[49]:= ```
C6 = WaveletFilterCoefficients[
    CoifletWavelet[1], {"PrimalLowpass", "PrimalHighpass"}];
```
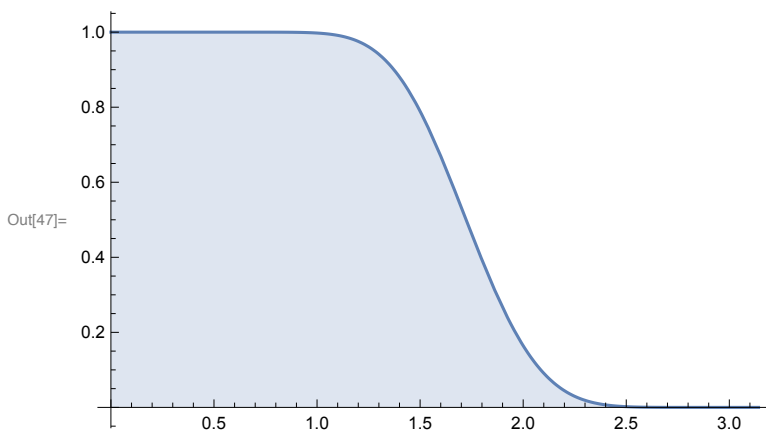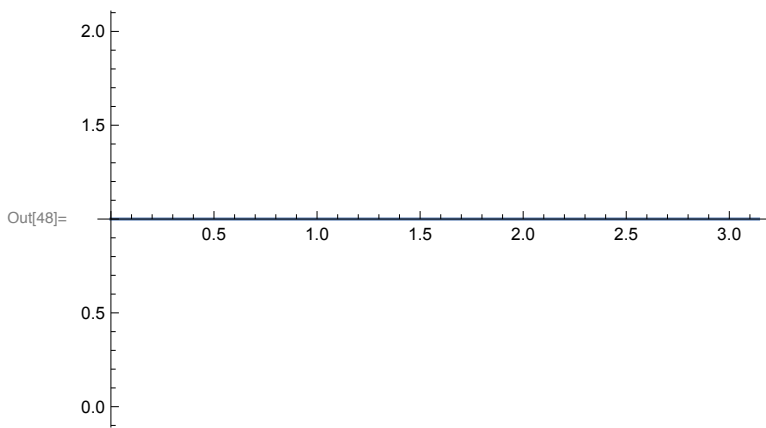
In[50]:= `Transpose[C6] // MatrixForm`

Out[50]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -2 \\ -0.0514297 \end{pmatrix} & \begin{pmatrix} -2 \\ -0.0110703 \end{pmatrix} \\ \begin{pmatrix} -1 \\ 0.23893 \end{pmatrix} & \begin{pmatrix} -1 \\ 0.0514297 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0.602859 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.272141 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0.272141 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.602859 \end{pmatrix} \\ \begin{pmatrix} 2 \\ -0.0514297 \end{pmatrix} & \begin{pmatrix} 2 \\ 0.23893 \end{pmatrix} \\ \begin{pmatrix} 3 \\ -0.0110703 \end{pmatrix} & \begin{pmatrix} 3 \\ 0.0514297 \end{pmatrix} \end{pmatrix}$$

In[51]:= `Transpose[Map[Total[#] &, C6]][[2]]`

Out[51]= $\left\{1., 5.55112 \times 10^{-17}\right\}$

In[52]:= `ListPlot[C6, PS]`

Out[52]=
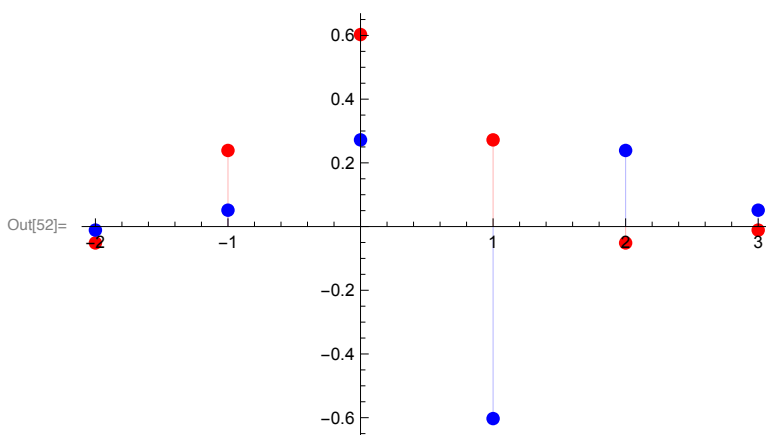


## z-Transform, Frequency response

In[53]:= `coif6[z_] = Sum[C6[[1, k + 1, 2]] z^k, {k, 0, 5}]`

Out[53]= $-0.0514297 + 0.23893 z + 0.602859 z^2 + 0.272141 z^3 - 0.0514297 z^4 - 0.0110703 z^5$

In[54]:= `Coif6[ω_] = c6[Exp[I ω]]`

Out[54]= $c6\left[e^{i\,\omega}\right]$

In[55]:= `Plot[Abs[Coif6[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[55]=
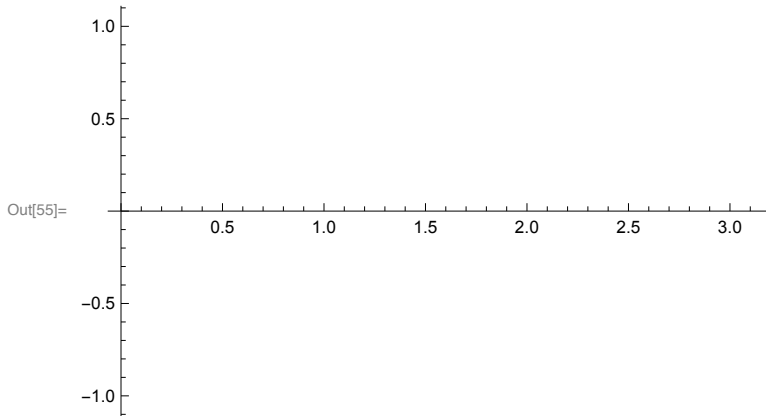


## Checking orthogonality

In[56]:= `ComplexExpand[Abs[Coif6[ω]]^2 + Abs[Coif6[ω + Pi]]^2]`

Out[56]= $c6\left[e^{i\,\omega}\right]^2 + c6\left[e^{i\,(\pi+\omega)}\right]^2$

In[57]:= `Simplify[%]`

Out[57]= $c6\left[e^{i\,\omega}\right]^2 + c6\left[e^{i\,(\pi+\omega)}\right]^2$

In[58]:= `Chop[%]`

Out[58]= $c6\left[e^{i\,\omega}\right]^2 + c6\left[e^{i\,(\pi+\omega)}\right]^2$

## The C12 Filter

In[59]:= `C12 = WaveletFilterCoefficients[`
`    CoifletWavelet[2], {"PrimalLowpass", "PrimalHighpass"}];`

In[60]:= **Transpose[C12] // MatrixForm**

Out[60]//MatrixForm=

$$
\begin{pmatrix}
\begin{pmatrix} -4 \\ 0.0115876 \end{pmatrix} & \begin{pmatrix} -6 \\ -0.000509505 \end{pmatrix} \\
\begin{pmatrix} -3 \\ -0.0293201 \end{pmatrix} & \begin{pmatrix} -5 \\ 0.0012892 \end{pmatrix} \\
\begin{pmatrix} -2 \\ -0.0476396 \end{pmatrix} & \begin{pmatrix} -4 \\ 0.00396788 \end{pmatrix} \\
\begin{pmatrix} -1 \\ 0.273021 \end{pmatrix} & \begin{pmatrix} -3 \\ -0.0167444 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0.574682 \end{pmatrix} & \begin{pmatrix} -2 \\ -0.0420265 \end{pmatrix} \\
\begin{pmatrix} 1 \\ 0.294867 \end{pmatrix} & \begin{pmatrix} -1 \\ 0.0540856 \end{pmatrix} \\
\begin{pmatrix} 2 \\ -0.0540856 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.294867 \end{pmatrix} \\
\begin{pmatrix} 3 \\ -0.0420265 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.574682 \end{pmatrix} \\
\begin{pmatrix} 4 \\ 0.0167444 \end{pmatrix} & \begin{pmatrix} 2 \\ 0.273021 \end{pmatrix} \\
\begin{pmatrix} 5 \\ 0.00396788 \end{pmatrix} & \begin{pmatrix} 3 \\ 0.0476396 \end{pmatrix} \\
\begin{pmatrix} 6 \\ -0.0012892 \end{pmatrix} & \begin{pmatrix} 4 \\ -0.0293201 \end{pmatrix} \\
\begin{pmatrix} 7 \\ -0.000509505 \end{pmatrix} & \begin{pmatrix} 5 \\ -0.0115876 \end{pmatrix}
\end{pmatrix}
$$

In[61]:= **Transpose[Map[Total[#] &, C12]][[2]]**

Out[61]= $\{1., -7.80626 \times 10^{-18}\}$

In[62]:= **ListPlot[C12, PS]**

Out[62]=
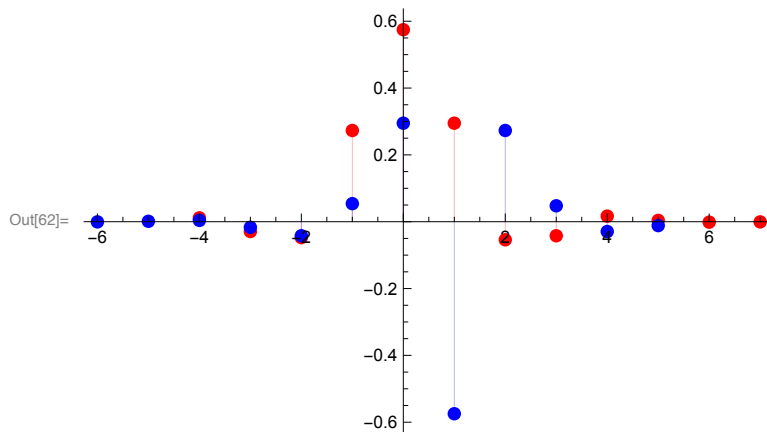


## z-Transform, Frequency response

In[63]:= **coif12[z_] = Sum[C12[[1, k + 1, 2]] z^k, {k, 0, 11}]**

Out[63]= $0.0115876 - 0.0293201\, z - 0.0476396\, z^2 + 0.273021\, z^3 +$
$0.574682\, z^4 + 0.294867\, z^5 - 0.0540856\, z^6 - 0.0420265\, z^7 +$
$0.0167444\, z^8 + 0.00396788\, z^9 - 0.0012892\, z^{10} - 0.000509505\, z^{11}$

In[64]:= `Coif12[ω_] = coif12[Exp[I ω]]`

Out[64]= $0.0115876 - 0.0293201\, e^{i\omega} - 0.0476396\, e^{2i\omega} + 0.273021\, e^{3i\omega} +$
$0.574682\, e^{4i\omega} + 0.294867\, e^{5i\omega} - 0.0540856\, e^{6i\omega} - 0.0420265\, e^{7i\omega} +$
$0.0167444\, e^{8i\omega} + 0.00396788\, e^{9i\omega} - 0.0012892\, e^{10i\omega} - 0.000509505\, e^{11i\omega}$

In[65]:= `Plot[Abs[Coif12[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[65]=
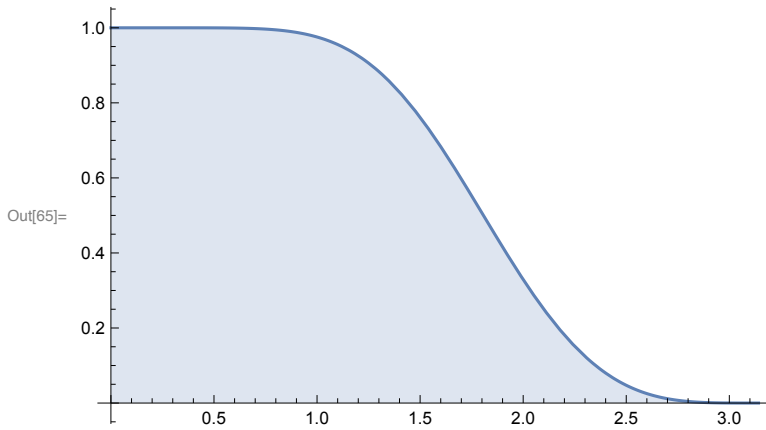


## Checking orthogonality

In[66]:= `ComplexExpand[Abs[Coif12[ω]]^2 + Abs[Coif12[ω + Pi]]^2]`

Out[66]= $0.000268545 + 0.00171934\, \cos[\omega]^2 - 0.00110406\, \cos[2\omega] + 0.0027936\, \cos[\omega]\cos[2\omega] +$
$0.00226953\, \cos[2\omega]^2 + 0.00632732\, \cos[3\omega] - 0.01601\, \cos[\omega]\cos[3\omega] -$
$0.0260132\, \cos[2\omega]\cos[3\omega] + 0.0745405\, \cos[3\omega]^2 + 0.0133184\, \cos[4\omega] -$
$0.0336995\, \cos[\omega]\cos[4\omega] - 0.0547553\, \cos[2\omega]\cos[4\omega] + 0.313801\, \cos[3\omega]\cos[4\omega] +$
$0.33026\, \cos[4\omega]^2 + 0.0068336\, \cos[5\omega] - 0.0172911\, \cos[\omega]\cos[5\omega] -$
$0.0280947\, \cos[2\omega]\cos[5\omega] + 0.16101\, \cos[3\omega]\cos[5\omega] + 0.33891\, \cos[4\omega]\cos[5\omega] +$
$0.0869467\, \cos[5\omega]^2 - 0.00125344\, \cos[6\omega] + 0.00317159\, \cos[\omega]\cos[6\omega] +$
$0.00515323\, \cos[2\omega]\cos[6\omega] - 0.029533\, \cos[3\omega]\cos[6\omega] -$
$0.0621641\, \cos[4\omega]\cos[6\omega] - 0.0318961\, \cos[5\omega]\cos[6\omega] + 0.00292525\, \cos[6\omega]^2 -$
$0.000973972\, \cos[7\omega] + 0.00246444\, \cos[\omega]\cos[7\omega] + 0.00400425\, \cos[2\omega]\cos[7\omega] -$
$0.0229482\, \cos[3\omega]\cos[7\omega] - 0.0483038\, \cos[4\omega]\cos[7\omega] -$
$0.0247845\, \cos[5\omega]\cos[7\omega] + 0.00454606\, \cos[6\omega]\cos[7\omega] +$
$0.00176623\, \cos[7\omega]^2 + 0.000388055\, \cos[8\omega] - 0.000981897\, \cos[\omega]\cos[8\omega] -$
$0.00159539\, \cos[2\omega]\cos[8\omega] + 0.00914315\, \cos[3\omega]\cos[8\omega] +$
$0.0192454\, \cos[4\omega]\cos[8\omega] + 0.00987475\, \cos[5\omega]\cos[8\omega] -$
$0.00181126\, \cos[6\omega]\cos[8\omega] - 0.00140742\, \cos[7\omega]\cos[8\omega] +$
$0.000280375\, \cos[8\omega]^2 + 0.0000919565\, \cos[9\omega] - 0.000232678\, \cos[\omega]\cos[9\omega] -$
$0.000378057\, \cos[2\omega]\cos[9\omega] + 0.00216663\, \cos[3\omega]\cos[9\omega] +$
$0.00456055\, \cos[4\omega]\cos[9\omega] + 0.00234\, \cos[5\omega]\cos[9\omega] -$
$0.000429211\, \cos[6\omega]\cos[9\omega] - 0.000333512\, \cos[7\omega]\cos[9\omega] +$
$0.00013288\, \cos[8\omega]\cos[9\omega] + 0.0000157441\, \cos[9\omega]^2 - 0.0000298775\, \cos[10\omega] +$
$0.0000755992\, \cos[\omega]\cos[10\omega] + 0.000122834\, \cos[2\omega]\cos[10\omega] -$
$0.000703959\, \cos[3\omega]\cos[10\omega] - 0.00148176\, \cos[4\omega]\cos[10\omega] -$
$0.000760288\, \cos[5\omega]\cos[10\omega] + 0.000139455\, \cos[6\omega]\cos[10\omega] +$
$0.000108361\, \cos[7\omega]\cos[10\omega] - 0.0000431739\, \cos[8\omega]\cos[10\omega] -$
$0.0000102308\, \cos[9\omega]\cos[10\omega] + 1.66205 \times 10^{-6}\, \cos[10\omega]^2 - 0.0000118079\, \cos[11\omega] +$

$0.0000298775\,\text{Cos}[\omega]\,\text{Cos}[11\,\omega]+0.0000485453\,\text{Cos}[2\,\omega]\,\text{Cos}[11\,\omega]-$

$0.000278211\,\text{Cos}[3\,\omega]\,\text{Cos}[11\,\omega]-0.000585608\,\text{Cos}[4\,\omega]\,\text{Cos}[11\,\omega]-$

$0.000300473\,\text{Cos}[5\,\omega]\,\text{Cos}[11\,\omega]+0.0000551138\,\text{Cos}[6\,\omega]\,\text{Cos}[11\,\omega]+$

$0.0000428254\,\text{Cos}[7\,\omega]\,\text{Cos}[11\,\omega]-0.0000170627\,\text{Cos}[8\,\omega]\,\text{Cos}[11\,\omega]-$

$4.04332\times10^{-6}\,\text{Cos}[9\,\omega]\,\text{Cos}[11\,\omega]+1.31371\times10^{-6}\,\text{Cos}[10\,\omega]\,\text{Cos}[11\,\omega]+$

$2.59596\times10^{-7}\,\text{Cos}[11\,\omega]^2-0.00110406\,\text{Cos}[2\,(\pi+\omega)]-0.0027936\,\text{Cos}[\omega]\,\text{Cos}[2\,(\pi+\omega)]+$

$0.00226953\,\text{Cos}[2\,(\pi+\omega)]^2+0.00632732\,\text{Cos}[3\,(\pi+\omega)]+0.01601\,\text{Cos}[\omega]\,\text{Cos}[3\,(\pi+\omega)]-$

$0.0260132\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[3\,(\pi+\omega)]+0.0745405\,\text{Cos}[3\,(\pi+\omega)]^2+$

$0.0133184\,\text{Cos}[4\,(\pi+\omega)]+0.0336995\,\text{Cos}[\omega]\,\text{Cos}[4\,(\pi+\omega)]-$

$0.0547553\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[4\,(\pi+\omega)]+0.313801\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[4\,(\pi+\omega)]+$

$0.33026\,\text{Cos}[4\,(\pi+\omega)]^2+0.0068336\,\text{Cos}[5\,(\pi+\omega)]+0.0172911\,\text{Cos}[\omega]\,\text{Cos}[5\,(\pi+\omega)]-$

$0.0280947\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[5\,(\pi+\omega)]+0.16101\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[5\,(\pi+\omega)]+$

$0.33891\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[5\,(\pi+\omega)]+0.0869467\,\text{Cos}[5\,(\pi+\omega)]^2-$

$0.00125344\,\text{Cos}[6\,(\pi+\omega)]-0.00317159\,\text{Cos}[\omega]\,\text{Cos}[6\,(\pi+\omega)]+$

$0.00515323\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[6\,(\pi+\omega)]-0.029533\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[6\,(\pi+\omega)]-$

$0.0621641\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[6\,(\pi+\omega)]-0.0318961\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[6\,(\pi+\omega)]+$

$0.00292525\,\text{Cos}[6\,(\pi+\omega)]^2-0.000973972\,\text{Cos}[7\,(\pi+\omega)]-$

$0.00246444\,\text{Cos}[\omega]\,\text{Cos}[7\,(\pi+\omega)]+0.00400425\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[7\,(\pi+\omega)]-$

$0.0229482\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[7\,(\pi+\omega)]-0.0483038\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[7\,(\pi+\omega)]-$

$0.0247845\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[7\,(\pi+\omega)]+0.00454606\,\text{Cos}[6\,(\pi+\omega)]\,\text{Cos}[7\,(\pi+\omega)]+$

$0.00176623\,\text{Cos}[7\,(\pi+\omega)]^2+0.000388055\,\text{Cos}[8\,(\pi+\omega)]+$

$0.000981897\,\text{Cos}[\omega]\,\text{Cos}[8\,(\pi+\omega)]-0.00159539\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]+$

$0.00914315\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]+0.0192454\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]+$

$0.00987475\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]-0.00181126\,\text{Cos}[6\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]-$

$0.00140742\,\text{Cos}[7\,(\pi+\omega)]\,\text{Cos}[8\,(\pi+\omega)]+0.000280375\,\text{Cos}[8\,(\pi+\omega)]^2+$

$0.0000919565\,\text{Cos}[9\,(\pi+\omega)]+0.000232678\,\text{Cos}[\omega]\,\text{Cos}[9\,(\pi+\omega)]-$

$0.000378057\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]+0.00216663\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]+$

$0.00456055\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]+0.00234\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]-$

$0.000429211\,\text{Cos}[6\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]-0.000333512\,\text{Cos}[7\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]+$

$0.00013288\,\text{Cos}[8\,(\pi+\omega)]\,\text{Cos}[9\,(\pi+\omega)]+0.0000157441\,\text{Cos}[9\,(\pi+\omega)]^2-$

$0.0000298775\,\text{Cos}[10\,(\pi+\omega)]-0.0000755992\,\text{Cos}[\omega]\,\text{Cos}[10\,(\pi+\omega)]+$

$0.000122834\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]-0.000703959\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]-$

$0.00148176\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]-0.000760288\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]+$

$0.000139455\,\text{Cos}[6\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]+0.000108361\,\text{Cos}[7\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]-$

$0.0000431739\,\text{Cos}[8\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]-$

$0.0000102308\,\text{Cos}[9\,(\pi+\omega)]\,\text{Cos}[10\,(\pi+\omega)]+1.66205\times10^{-6}\,\text{Cos}[10\,(\pi+\omega)]^2-$

$0.0000118079\,\text{Cos}[11\,(\pi+\omega)]-0.0000298775\,\text{Cos}[\omega]\,\text{Cos}[11\,(\pi+\omega)]+$

$0.0000485453\,\text{Cos}[2\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]-$

$0.000278211\,\text{Cos}[3\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]-$

$0.000585608\,\text{Cos}[4\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]-0.000300473\,\text{Cos}[5\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]+$

$0.0000551138\,\text{Cos}[6\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]+$

$0.0000428254\,\text{Cos}[7\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]-0.0000170627\,\text{Cos}[8\,(\pi+\omega)]$

$\text{Cos}[11\,(\pi+\omega)]-4.04332\times10^{-6}\,\text{Cos}[9\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]+$

$1.31371\times10^{-6}\,\text{Cos}[10\,(\pi+\omega)]\,\text{Cos}[11\,(\pi+\omega)]+2.59596\times10^{-7}\,\text{Cos}[11\,(\pi+\omega)]^2+$

$0.00171934\,\text{Sin}[\omega]^2+0.0027936\,\text{Sin}[\omega]\,\text{Sin}[2\,\omega]+0.00226953\,\text{Sin}[2\,\omega]^2-$

$0.01601\,\text{Sin}[\omega]\,\text{Sin}[3\,\omega]-0.0260132\,\text{Sin}[2\,\omega]\,\text{Sin}[3\,\omega]+0.0745405\,\text{Sin}[3\,\omega]^2-$

$0.0336995\,\text{Sin}[\omega]\,\text{Sin}[4\,\omega]-0.0547553\,\text{Sin}[2\,\omega]\,\text{Sin}[4\,\omega]+0.313801\,\text{Sin}[3\,\omega]\,\text{Sin}[4\,\omega]+$

$0.33026 \sin[4\omega]^2 - 0.0172911 \sin[\omega] \sin[5\omega] - 0.0280947 \sin[2\omega] \sin[5\omega] +$
$0.16101 \sin[3\omega] \sin[5\omega] + 0.33891 \sin[4\omega] \sin[5\omega] + 0.0869467 \sin[5\omega]^2 +$
$0.00317159 \sin[\omega] \sin[6\omega] + 0.00515323 \sin[2\omega] \sin[6\omega] -$
$0.029533 \sin[3\omega] \sin[6\omega] - 0.0621641 \sin[4\omega] \sin[6\omega] -$
$0.0318961 \sin[5\omega] \sin[6\omega] + 0.00292525 \sin[6\omega]^2 + 0.00246444 \sin[\omega] \sin[7\omega] +$
$0.00400425 \sin[2\omega] \sin[7\omega] - 0.0229482 \sin[3\omega] \sin[7\omega] -$
$0.0483038 \sin[4\omega] \sin[7\omega] - 0.0247845 \sin[5\omega] \sin[7\omega] +$
$0.00454606 \sin[6\omega] \sin[7\omega] + 0.00176623 \sin[7\omega]^2 - 0.000981897 \sin[\omega] \sin[8\omega] -$
$0.00159539 \sin[2\omega] \sin[8\omega] + 0.00914315 \sin[3\omega] \sin[8\omega] +$
$0.0192454 \sin[4\omega] \sin[8\omega] + 0.00987475 \sin[5\omega] \sin[8\omega] -$
$0.00181126 \sin[6\omega] \sin[8\omega] - 0.00140742 \sin[7\omega] \sin[8\omega] + 0.000280375 \sin[8\omega]^2 -$
$0.000232678 \sin[\omega] \sin[9\omega] - 0.000378057 \sin[2\omega] \sin[9\omega] +$
$0.00216663 \sin[3\omega] \sin[9\omega] + 0.00456055 \sin[4\omega] \sin[9\omega] +$
$0.00234 \sin[5\omega] \sin[9\omega] - 0.000429211 \sin[6\omega] \sin[9\omega] -$
$0.000333512 \sin[7\omega] \sin[9\omega] + 0.00013288 \sin[8\omega] \sin[9\omega] + 0.0000157441 \sin[9\omega]^2 +$
$0.0000755992 \sin[\omega] \sin[10\omega] + 0.000122834 \sin[2\omega] \sin[10\omega] -$
$0.000703959 \sin[3\omega] \sin[10\omega] - 0.00148176 \sin[4\omega] \sin[10\omega] -$
$0.000760288 \sin[5\omega] \sin[10\omega] + 0.000139455 \sin[6\omega] \sin[10\omega] +$
$0.000108361 \sin[7\omega] \sin[10\omega] - 0.0000431739 \sin[8\omega] \sin[10\omega] -$
$0.0000102308 \sin[9\omega] \sin[10\omega] + 1.66205 \times 10^{-6} \sin[10\omega]^2 +$
$0.0000298775 \sin[\omega] \sin[11\omega] + 0.0000485453 \sin[2\omega] \sin[11\omega] -$
$0.000278211 \sin[3\omega] \sin[11\omega] - 0.000585608 \sin[4\omega] \sin[11\omega] -$
$0.000300473 \sin[5\omega] \sin[11\omega] + 0.0000551138 \sin[6\omega] \sin[11\omega] +$
$0.0000428254 \sin[7\omega] \sin[11\omega] - 0.0000170627 \sin[8\omega] \sin[11\omega] -$
$4.04332 \times 10^{-6} \sin[9\omega] \sin[11\omega] + 1.31371 \times 10^{-6} \sin[10\omega] \sin[11\omega] +$
$2.59596 \times 10^{-7} \sin[11\omega]^2 - 0.0027936 \sin[\omega] \sin[2(\pi+\omega)] +$
$0.00226953 \sin[2(\pi+\omega)]^2 + 0.01601 \sin[\omega] \sin[3(\pi+\omega)] -$
$0.0260132 \sin[2(\pi+\omega)] \sin[3(\pi+\omega)] + 0.0745405 \sin[3(\pi+\omega)]^2 +$
$0.0336995 \sin[\omega] \sin[4(\pi+\omega)] - 0.0547553 \sin[2(\pi+\omega)] \sin[4(\pi+\omega)] +$
$0.313801 \sin[3(\pi+\omega)] \sin[4(\pi+\omega)] + 0.33026 \sin[4(\pi+\omega)]^2 +$
$0.0172911 \sin[\omega] \sin[5(\pi+\omega)] - 0.0280947 \sin[2(\pi+\omega)] \sin[5(\pi+\omega)] +$
$0.16101 \sin[3(\pi+\omega)] \sin[5(\pi+\omega)] + 0.33891 \sin[4(\pi+\omega)] \sin[5(\pi+\omega)] +$
$0.0869467 \sin[5(\pi+\omega)]^2 - 0.00317159 \sin[\omega] \sin[6(\pi+\omega)] +$
$0.00515323 \sin[2(\pi+\omega)] \sin[6(\pi+\omega)] - 0.029533 \sin[3(\pi+\omega)] \sin[6(\pi+\omega)] -$
$0.0621641 \sin[4(\pi+\omega)] \sin[6(\pi+\omega)] - 0.0318961 \sin[5(\pi+\omega)] \sin[6(\pi+\omega)] +$
$0.00292525 \sin[6(\pi+\omega)]^2 - 0.00246444 \sin[\omega] \sin[7(\pi+\omega)] +$
$0.00400425 \sin[2(\pi+\omega)] \sin[7(\pi+\omega)] - 0.0229482 \sin[3(\pi+\omega)] \sin[7(\pi+\omega)] -$
$0.0483038 \sin[4(\pi+\omega)] \sin[7(\pi+\omega)] - 0.0247845 \sin[5(\pi+\omega)] \sin[7(\pi+\omega)] +$
$0.00454606 \sin[6(\pi+\omega)] \sin[7(\pi+\omega)] + 0.00176623 \sin[7(\pi+\omega)]^2 +$
$0.000981897 \sin[\omega] \sin[8(\pi+\omega)] - 0.00159539 \sin[2(\pi+\omega)] \sin[8(\pi+\omega)] +$
$0.00914315 \sin[3(\pi+\omega)] \sin[8(\pi+\omega)] + 0.0192454 \sin[4(\pi+\omega)] \sin[8(\pi+\omega)] +$
$0.00987475 \sin[5(\pi+\omega)] \sin[8(\pi+\omega)] - 0.00181126 \sin[6(\pi+\omega)] \sin[8(\pi+\omega)] -$
$0.00140742 \sin[7(\pi+\omega)] \sin[8(\pi+\omega)] + 0.000280375 \sin[8(\pi+\omega)]^2 +$
$0.000232678 \sin[\omega] \sin[9(\pi+\omega)] - 0.000378057 \sin[2(\pi+\omega)] \sin[9(\pi+\omega)] +$
$0.00216663 \sin[3(\pi+\omega)] \sin[9(\pi+\omega)] + 0.00456055 \sin[4(\pi+\omega)] \sin[9(\pi+\omega)] +$
$0.00234 \sin[5(\pi+\omega)] \sin[9(\pi+\omega)] - 0.000429211 \sin[6(\pi+\omega)] \sin[9(\pi+\omega)] -$
$0.000333512 \sin[7(\pi+\omega)] \sin[9(\pi+\omega)] + 0.00013288 \sin[8(\pi+\omega)] \sin[9(\pi+\omega)] +$

$0.0000157441 \, \text{Sin}[9 \, (\pi + \omega)]^2 - 0.0000755992 \, \text{Sin}[\omega] \, \text{Sin}[10 \, (\pi + \omega)] +$

$0.000122834 \, \text{Sin}[2 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] - 0.000703959 \, \text{Sin}[3 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] -$

$0.00148176 \, \text{Sin}[4 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] - 0.000760288 \, \text{Sin}[5 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] +$

$0.000139455 \, \text{Sin}[6 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] + 0.000108361 \, \text{Sin}[7 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] -$

$0.0000431739 \, \text{Sin}[8 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] -$

$0.0000102308 \, \text{Sin}[9 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] + 1.66205 \times 10^{-6} \, \text{Sin}[10 \, (\pi + \omega)]^2 -$

$0.0000298775 \, \text{Sin}[\omega] \, \text{Sin}[11 \, (\pi + \omega)] + 0.0000485453 \, \text{Sin}[2 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] -$

$0.000278211 \, \text{Sin}[3 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] -$

$0.000585608 \, \text{Sin}[4 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] - 0.000300473 \, \text{Sin}[5 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$

$0.0000551138 \, \text{Sin}[6 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$

$0.0000428254 \, \text{Sin}[7 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] -$

$0.0000170627 \, \text{Sin}[8 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] -$

$4.04332 \times 10^{-6} \, \text{Sin}[9 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$

$1.31371 \times 10^{-6} \, \text{Sin}[10 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] + 2.59596 \times 10^{-7} \, \text{Sin}[11 \, (\pi + \omega)]^2$

In[67]:= `Simplify[%]`

Out[67]= $1. + 1.11022 \times 10^{-16} \, \text{Cos}[\omega] - 5.55112 \times 10^{-17} \, \text{Cos}[2 \, \omega] -$

$1.38778 \times 10^{-17} \, \text{Cos}[3 \, \omega] - 7.97973 \times 10^{-17} \, \text{Cos}[4 \, \omega] - 6.93889 \times 10^{-18} \, \text{Cos}[5 \, \omega] -$

$1.11022 \times 10^{-16} \, \text{Cos}[8 \, \omega] + 1.73472 \times 10^{-18} \, \text{Cos}[13 \, \omega] + 4.33681 \times 10^{-19} \, \text{Cos}[14 \, \omega]$

In[68]:= `Chop[%]`

Out[68]= $1.$

## The Sym8 Filter

In[69]:= 
```
Sym8 = WaveletFilterCoefficients[
    SymletWavelet[4], {"PrimalLowpass", "PrimalHighpass"}];
```

In[70]:= `Transpose[Sym8] // MatrixForm`

Out[70]//MatrixForm=

$$
\begin{pmatrix}
\begin{pmatrix} 0 \\ -0.0535745 \end{pmatrix} & \begin{pmatrix} -6 \\ 0.0227852 \end{pmatrix} \\
\begin{pmatrix} 1 \\ -0.0209555 \end{pmatrix} & \begin{pmatrix} -5 \\ 0.00891235 \end{pmatrix} \\
\begin{pmatrix} 2 \\ 0.35187 \end{pmatrix} & \begin{pmatrix} -4 \\ -0.0701588 \end{pmatrix} \\
\begin{pmatrix} 3 \\ 0.568329 \end{pmatrix} & \begin{pmatrix} -3 \\ -0.210617 \end{pmatrix} \\
\begin{pmatrix} 4 \\ 0.210617 \end{pmatrix} & \begin{pmatrix} -2 \\ 0.568329 \end{pmatrix} \\
\begin{pmatrix} 5 \\ -0.0701588 \end{pmatrix} & \begin{pmatrix} -1 \\ -0.35187 \end{pmatrix} \\
\begin{pmatrix} 6 \\ -0.00891235 \end{pmatrix} & \begin{pmatrix} 0 \\ -0.0209555 \end{pmatrix} \\
\begin{pmatrix} 7 \\ 0.0227852 \end{pmatrix} & \begin{pmatrix} 1 \\ 0.0535745 \end{pmatrix}
\end{pmatrix}
$$

In[71]:= `Transpose[Map[Total[#] &, Sym8]][[2]]`

Out[71]= $\{1., -1.11022 \times 10^{-16}\}$

In[72]:= `ListPlot[Sym8, PS]`

Out[72]=
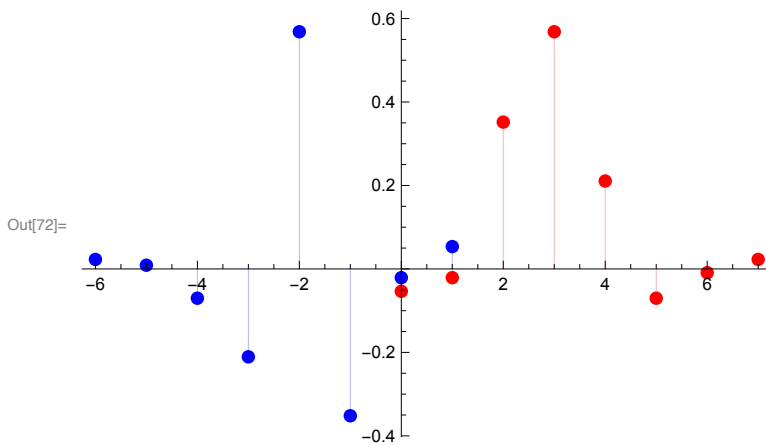


## z-Transform, Frequency response

In[73]:= `sym8[z_] = Sum[Sym8[[1, k + 1, 2]] z^k, {k, 0, 7}]`

Out[73]= $-0.0535745 - 0.0209555\,z + 0.35187\,z^2 + 0.568329\,z^3 +$
$0.210617\,z^4 - 0.0701588\,z^5 - 0.00891235\,z^6 + 0.0227852\,z^7$

In[74]:= `Syml8[ω_] = sym8[Exp[I ω]]`

Out[74]= $-0.0535745 - 0.0209555\,e^{i\,\omega} + 0.35187\,e^{2\,i\,\omega} + 0.568329\,e^{3\,i\,\omega} +$
$0.210617\,e^{4\,i\,\omega} - 0.0701588\,e^{5\,i\,\omega} - 0.00891235\,e^{6\,i\,\omega} + 0.0227852\,e^{7\,i\,\omega}$

In[75]:= `Plot[Abs[Syml8[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[75]=
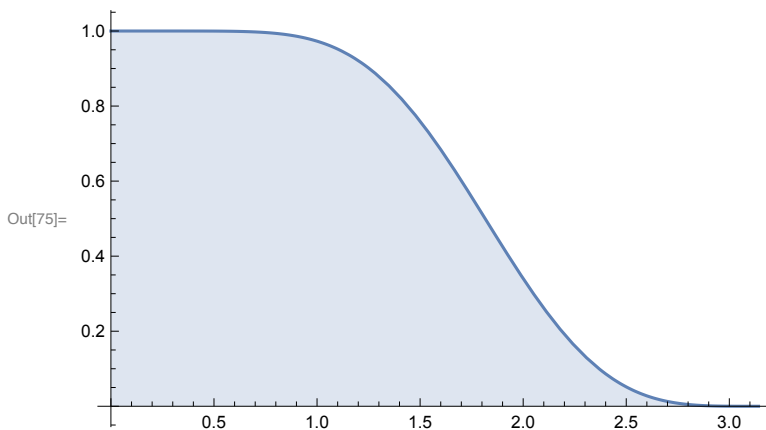


## Checking orthogonality

In[76]:= `ComplexExpand[Abs[Syml8[ω]]^2 + Abs[Syml8[ω + Pi]]^2]`

Out[76]= $0.00574044 + 0.000878264\,\mathrm{Cos}[\omega]^2 - 0.0377024\,\mathrm{Cos}[2\,\omega] -$
$0.0147472\,\mathrm{Cos}[\omega]\,\mathrm{Cos}[2\,\omega] + 0.123812\,\mathrm{Cos}[2\,\omega]^2 - 0.0608958\,\mathrm{Cos}[3\,\omega] -$
$0.0238192\,\mathrm{Cos}[\omega]\,\mathrm{Cos}[3\,\omega] + 0.399955\,\mathrm{Cos}[2\,\omega]\,\mathrm{Cos}[3\,\omega] + 0.322998\,\mathrm{Cos}[3\,\omega]^2 -$
$0.0225674\,\mathrm{Cos}[4\,\omega] - 0.00882717\,\mathrm{Cos}[\omega]\,\mathrm{Cos}[4\,\omega] + 0.14822\,\mathrm{Cos}[2\,\omega]\,\mathrm{Cos}[4\,\omega] +$
$0.2394\,\mathrm{Cos}[3\,\omega]\,\mathrm{Cos}[4\,\omega] + 0.0443596\,\mathrm{Cos}[4\,\omega]^2 + 0.00751744\,\mathrm{Cos}[5\,\omega] +$
$0.00294042\,\mathrm{Cos}[\omega]\,\mathrm{Cos}[5\,\omega] - 0.0493735\,\mathrm{Cos}[2\,\omega]\,\mathrm{Cos}[5\,\omega] -$
$0.0797466\,\mathrm{Cos}[3\,\omega]\,\mathrm{Cos}[5\,\omega] - 0.0295533\,\mathrm{Cos}[4\,\omega]\,\mathrm{Cos}[5\,\omega] +$
$0.00492226\,\mathrm{Cos}[5\,\omega]^2 + 0.000954949\,\mathrm{Cos}[6\,\omega] + 0.000373525\,\mathrm{Cos}[\omega]\,\mathrm{Cos}[6\,\omega] -$

0.00627197 Cos[2 ω] Cos[6 ω] − 0.0101303 Cos[3 ω] Cos[6 ω] −
0.00375419 Cos[4 ω] Cos[6 ω] + 0.00125056 Cos[5 ω] Cos[6 ω] +
0.00007943 Cos[6 ω]² − 0.00244141 Cos[7 ω] − 0.000954949 Cos[ω] Cos[7 ω] +
0.0160348 Cos[2 ω] Cos[7 ω] + 0.025899 Cos[3 ω] Cos[7 ω] +
0.0095979 Cos[4 ω] Cos[7 ω] − 0.00319716 Cos[5 ω] Cos[7 ω] −
0.000406139 Cos[6 ω] Cos[7 ω] + 0.000519164 Cos[7 ω]² − 0.0377024 Cos[2 (π + ω)] +
0.0147472 Cos[ω] Cos[2 (π + ω)] + 0.123812 Cos[2 (π + ω)]² − 0.0608958 Cos[3 (π + ω)] +
0.0238192 Cos[ω] Cos[3 (π + ω)] + 0.399955 Cos[2 (π + ω)] Cos[3 (π + ω)] +
0.322998 Cos[3 (π + ω)]² − 0.0225674 Cos[4 (π + ω)] + 0.00882717 Cos[ω] Cos[4 (π + ω)] +
0.14822 Cos[2 (π + ω)] Cos[4 (π + ω)] + 0.2394 Cos[3 (π + ω)] Cos[4 (π + ω)] +
0.0443596 Cos[4 (π + ω)]² + 0.00751744 Cos[5 (π + ω)] −
0.00294042 Cos[ω] Cos[5 (π + ω)] − 0.0493735 Cos[2 (π + ω)] Cos[5 (π + ω)] −
0.0797466 Cos[3 (π + ω)] Cos[5 (π + ω)] − 0.0295533 Cos[4 (π + ω)] Cos[5 (π + ω)] +
0.00492226 Cos[5 (π + ω)]² + 0.000954949 Cos[6 (π + ω)] −
0.000373525 Cos[ω] Cos[6 (π + ω)] − 0.00627197 Cos[2 (π + ω)] Cos[6 (π + ω)] −
0.0101303 Cos[3 (π + ω)] Cos[6 (π + ω)] − 0.00375419 Cos[4 (π + ω)] Cos[6 (π + ω)] +
0.00125056 Cos[5 (π + ω)] Cos[6 (π + ω)] + 0.00007943 Cos[6 (π + ω)]² −
0.00244141 Cos[7 (π + ω)] + 0.000954949 Cos[ω] Cos[7 (π + ω)] +
0.0160348 Cos[2 (π + ω)] Cos[7 (π + ω)] + 0.025899 Cos[3 (π + ω)] Cos[7 (π + ω)] +
0.0095979 Cos[4 (π + ω)] Cos[7 (π + ω)] − 0.00319716 Cos[5 (π + ω)] Cos[7 (π + ω)] −
0.000406139 Cos[6 (π + ω)] Cos[7 (π + ω)] + 0.000519164 Cos[7 (π + ω)]² +
0.000878264 Sin[ω]² − 0.0147472 Sin[ω] Sin[2 ω] + 0.123812 Sin[2 ω]² −
0.0238192 Sin[ω] Sin[3 ω] + 0.399955 Sin[2 ω] Sin[3 ω] + 0.322998 Sin[3 ω]² −
0.00882717 Sin[ω] Sin[4 ω] + 0.14822 Sin[2 ω] Sin[4 ω] + 0.2394 Sin[3 ω] Sin[4 ω] +
0.0443596 Sin[4 ω]² + 0.00294042 Sin[ω] Sin[5 ω] − 0.0493735 Sin[2 ω] Sin[5 ω] −
0.0797466 Sin[3 ω] Sin[5 ω] − 0.0295533 Sin[4 ω] Sin[5 ω] + 0.00492226 Sin[5 ω]² +
0.000373525 Sin[ω] Sin[6 ω] − 0.00627197 Sin[2 ω] Sin[6 ω] −
0.0101303 Sin[3 ω] Sin[6 ω] − 0.00375419 Sin[4 ω] Sin[6 ω] +
0.00125056 Sin[5 ω] Sin[6 ω] + 0.00007943 Sin[6 ω]² − 0.000954949 Sin[ω] Sin[7 ω] +
0.0160348 Sin[2 ω] Sin[7 ω] + 0.025899 Sin[3 ω] Sin[7 ω] + 0.0095979 Sin[4 ω] Sin[7 ω] −
0.00319716 Sin[5 ω] Sin[7 ω] − 0.000406139 Sin[6 ω] Sin[7 ω] +
0.000519164 Sin[7 ω]² + 0.0147472 Sin[ω] Sin[2 (π + ω)] +
0.123812 Sin[2 (π + ω)]² + 0.0238192 Sin[ω] Sin[3 (π + ω)] +
0.399955 Sin[2 (π + ω)] Sin[3 (π + ω)] + 0.322998 Sin[3 (π + ω)]² +
0.00882717 Sin[ω] Sin[4 (π + ω)] + 0.14822 Sin[2 (π + ω)] Sin[4 (π + ω)] +
0.2394 Sin[3 (π + ω)] Sin[4 (π + ω)] + 0.0443596 Sin[4 (π + ω)]² −
0.00294042 Sin[ω] Sin[5 (π + ω)] − 0.0493735 Sin[2 (π + ω)] Sin[5 (π + ω)] −
0.0797466 Sin[3 (π + ω)] Sin[5 (π + ω)] − 0.0295533 Sin[4 (π + ω)] Sin[5 (π + ω)] +
0.00492226 Sin[5 (π + ω)]² − 0.000373525 Sin[ω] Sin[6 (π + ω)] −
0.00627197 Sin[2 (π + ω)] Sin[6 (π + ω)] − 0.0101303 Sin[3 (π + ω)] Sin[6 (π + ω)] −
0.00375419 Sin[4 (π + ω)] Sin[6 (π + ω)] + 0.00125056 Sin[5 (π + ω)] Sin[6 (π + ω)] +
0.00007943 Sin[6 (π + ω)]² + 0.000954949 Sin[ω] Sin[7 (π + ω)] +
0.0160348 Sin[2 (π + ω)] Sin[7 (π + ω)] + 0.025899 Sin[3 (π + ω)] Sin[7 (π + ω)] +
0.0095979 Sin[4 (π + ω)] Sin[7 (π + ω)] − 0.00319716 Sin[5 (π + ω)] Sin[7 (π + ω)] −
0.000406139 Sin[6 (π + ω)] Sin[7 (π + ω)] + 0.000519164 Sin[7 (π + ω)]²

In[77]:= **Simplify[%]**

Out[77]= $1. - 5.55112 \times 10^{-17} \operatorname{Cos}[\omega] + 6.66134 \times 10^{-16} \operatorname{Cos}[2\,\omega] - 1.38778 \times 10^{-17} \operatorname{Cos}[3\,\omega] +$
$4.51028 \times 10^{-17} \operatorname{Cos}[4\,\omega] - 5.55112 \times 10^{-17} \operatorname{Cos}[6\,\omega] - 3.46945 \times 10^{-18} \operatorname{Cos}[10\,\omega]$

In[78]:= **Chop[%]**

Out[78]= $1.$

## The BL25 Filter

In[79]:= **BL25 = WaveletFilterCoefficients[**
**BattleLemarieWavelet[2, 5], {"PrimalLowpass", "PrimalHighpass"}];**

In[80]:= **Transpose[BL25] // MatrixForm**

Out[80]//MatrixForm=

$$\begin{pmatrix}
\begin{pmatrix} -5 \\ 0.00299683 \end{pmatrix} & \begin{pmatrix} -5 \\ -0.00299683 \end{pmatrix} \\
\begin{pmatrix} -4 \\ 0.0296858 \end{pmatrix} & \begin{pmatrix} -4 \\ 0.0296858 \end{pmatrix} \\
\begin{pmatrix} -3 \\ -0.0146705 \end{pmatrix} & \begin{pmatrix} -3 \\ 0.0146705 \end{pmatrix} \\
\begin{pmatrix} -2 \\ -0.0882717 \end{pmatrix} & \begin{pmatrix} -2 \\ -0.0882717 \end{pmatrix} \\
\begin{pmatrix} -1 \\ 0.0975546 \end{pmatrix} & \begin{pmatrix} -1 \\ -0.0975546 \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0.481094 \end{pmatrix} & \begin{pmatrix} 0 \\ 0.481094 \end{pmatrix} \\
\begin{pmatrix} 1 \\ 0.481094 \end{pmatrix} & \begin{pmatrix} 1 \\ -0.481094 \end{pmatrix} \\
\begin{pmatrix} 2 \\ 0.0975546 \end{pmatrix} & \begin{pmatrix} 2 \\ 0.0975546 \end{pmatrix} \\
\begin{pmatrix} 3 \\ -0.0882717 \end{pmatrix} & \begin{pmatrix} 3 \\ 0.0882717 \end{pmatrix} \\
\begin{pmatrix} 4 \\ -0.0146705 \end{pmatrix} & \begin{pmatrix} 4 \\ -0.0146705 \end{pmatrix} \\
\begin{pmatrix} 5 \\ 0.0296858 \end{pmatrix} & \begin{pmatrix} 5 \\ -0.0296858 \end{pmatrix} \\
\begin{pmatrix} 6 \\ 0.00299683 \end{pmatrix} & \begin{pmatrix} 6 \\ 0.00299683 \end{pmatrix}
\end{pmatrix}$$

In[81]:= **Transpose[Map[Total[#] &, BL25]][[2]]**

Out[81]= $\{1.01678, 0.\}$

In[82]:= **ListPlot[BL25, PS]**

Out[82]=
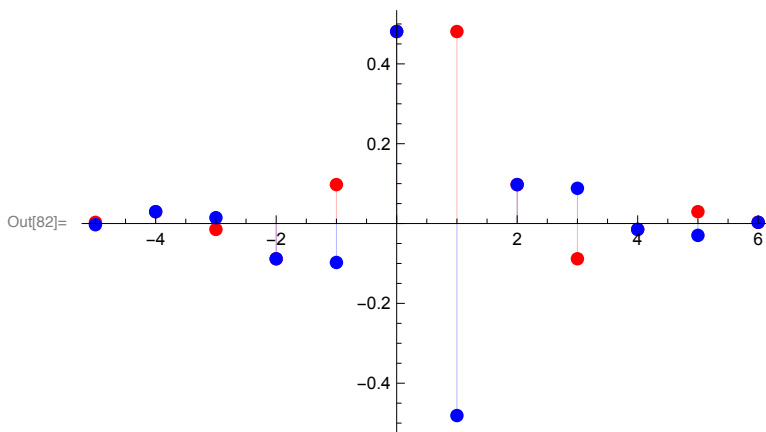


## z-Transform, Frequency response

In[83]:= `blm[z_] = Sum[BL25[[1, k + 1, 2]] z^k, {k, 0, 11}]`

Out[83]= $0.00299683 + 0.0296858\, z - 0.0146705\, z^2 - 0.0882717\, z^3 + 0.0975546\, z^4 + 0.481094\, z^5 +$
$0.481094\, z^6 + 0.0975546\, z^7 - 0.0882717\, z^8 - 0.0146705\, z^9 + 0.0296858\, z^{10} + 0.00299683\, z^{11}$

In[84]:= `BLM[ω_] = blm[Exp[I ω]]`

Out[84]= $0.00299683 + 0.0296858\, e^{i\,\omega} - 0.0146705\, e^{2\,i\,\omega} - 0.0882717\, e^{3\,i\,\omega} +$
$0.0975546\, e^{4\,i\,\omega} + 0.481094\, e^{5\,i\,\omega} + 0.481094\, e^{6\,i\,\omega} + 0.0975546\, e^{7\,i\,\omega} -$
$0.0882717\, e^{8\,i\,\omega} - 0.0146705\, e^{9\,i\,\omega} + 0.0296858\, e^{10\,i\,\omega} + 0.00299683\, e^{11\,i\,\omega}$

In[85]:= `Plot[Abs[BLM[ω]], {ω, 0, Pi}, Filling → Axis]`

Out[85]=
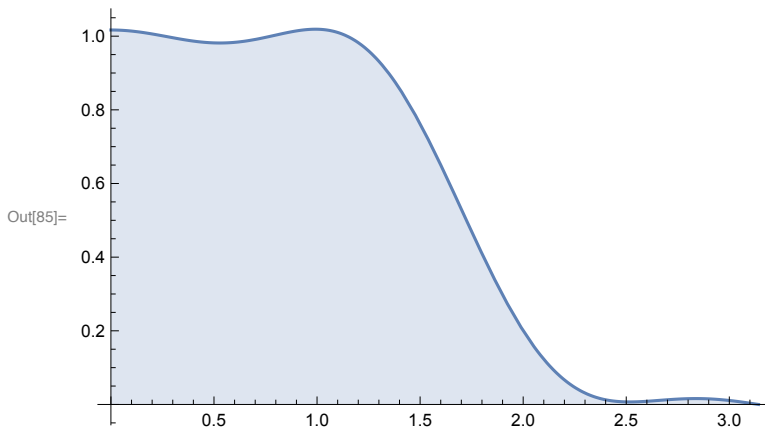


## Checking orthogonality

In[86]:= `ComplexExpand[Abs[BLM[ω]]^2 + Abs[BLM[ω + Pi]]^2]`

Out[86]= $0.000017962 + 0.0017625\, \text{Cos}[\omega]^2 - 0.00008793\, \text{Cos}[2\,\omega] - 0.000871011\, \text{Cos}[\omega]\, \text{Cos}[2\,\omega] +$
$0.000215223\, \text{Cos}[2\,\omega]^2 - 0.000529071\, \text{Cos}[3\,\omega] - 0.00524084\, \text{Cos}[\omega]\, \text{Cos}[3\,\omega] +$
$0.00258998\, \text{Cos}[2\,\omega]\, \text{Cos}[3\,\omega] + 0.0077919\, \text{Cos}[3\,\omega]^2 + 0.00058471\, \text{Cos}[4\,\omega] +$
$0.00579198\, \text{Cos}[\omega]\, \text{Cos}[4\,\omega] - 0.00286235\, \text{Cos}[2\,\omega]\, \text{Cos}[4\,\omega] -$
$0.0172226\, \text{Cos}[3\,\omega]\, \text{Cos}[4\,\omega] + 0.0095169\, \text{Cos}[4\,\omega]^2 + 0.00288352\, \text{Cos}[5\,\omega] +$
$0.0285634\, \text{Cos}[\omega]\, \text{Cos}[5\,\omega] - 0.0141158\, \text{Cos}[2\,\omega]\, \text{Cos}[5\,\omega] - 0.084934\, \text{Cos}[3\,\omega]\, \text{Cos}[5\,\omega] +$
$0.0938659\, \text{Cos}[4\,\omega]\, \text{Cos}[5\,\omega] + 0.231452\, \text{Cos}[5\,\omega]^2 + 0.00288352\, \text{Cos}[6\,\omega] +$
$0.0285634\, \text{Cos}[\omega]\, \text{Cos}[6\,\omega] - 0.0141158\, \text{Cos}[2\,\omega]\, \text{Cos}[6\,\omega] - 0.084934\, \text{Cos}[3\,\omega]\, \text{Cos}[6\,\omega] +$
$0.0938659\, \text{Cos}[4\,\omega]\, \text{Cos}[6\,\omega] + 0.462903\, \text{Cos}[5\,\omega]\, \text{Cos}[6\,\omega] + 0.231452\, \text{Cos}[6\,\omega]^2 +$
$0.00058471\, \text{Cos}[7\,\omega] + 0.00579198\, \text{Cos}[\omega]\, \text{Cos}[7\,\omega] - 0.00286235\, \text{Cos}[2\,\omega]\, \text{Cos}[7\,\omega] -$
$0.0172226\, \text{Cos}[3\,\omega]\, \text{Cos}[7\,\omega] + 0.0190338\, \text{Cos}[4\,\omega]\, \text{Cos}[7\,\omega] +$
$0.0938659\, \text{Cos}[5\,\omega]\, \text{Cos}[7\,\omega] + 0.0938659\, \text{Cos}[6\,\omega]\, \text{Cos}[7\,\omega] + 0.0095169\, \text{Cos}[7\,\omega]^2 -$
$0.000529071\, \text{Cos}[8\,\omega] - 0.00524084\, \text{Cos}[\omega]\, \text{Cos}[8\,\omega] + 0.00258998\, \text{Cos}[2\,\omega]\, \text{Cos}[8\,\omega] +$
$0.0155838\, \text{Cos}[3\,\omega]\, \text{Cos}[8\,\omega] - 0.0172226\, \text{Cos}[4\,\omega]\, \text{Cos}[8\,\omega] - 0.084934\, \text{Cos}[5\,\omega]\, \text{Cos}[8\,\omega] -$
$0.084934\, \text{Cos}[6\,\omega]\, \text{Cos}[8\,\omega] - 0.0172226\, \text{Cos}[7\,\omega]\, \text{Cos}[8\,\omega] + 0.0077919\, \text{Cos}[8\,\omega]^2 -$
$0.00008793\, \text{Cos}[9\,\omega] - 0.000871011\, \text{Cos}[\omega]\, \text{Cos}[9\,\omega] + 0.000430446\, \text{Cos}[2\,\omega]\, \text{Cos}[9\,\omega] +$
$0.00258998\, \text{Cos}[3\,\omega]\, \text{Cos}[9\,\omega] - 0.00286235\, \text{Cos}[4\,\omega]\, \text{Cos}[9\,\omega] -$
$0.0141158\, \text{Cos}[5\,\omega]\, \text{Cos}[9\,\omega] - 0.0141158\, \text{Cos}[6\,\omega]\, \text{Cos}[9\,\omega] -$
$0.00286235\, \text{Cos}[7\,\omega]\, \text{Cos}[9\,\omega] + 0.00258998\, \text{Cos}[8\,\omega]\, \text{Cos}[9\,\omega] +$
$0.000215223\, \text{Cos}[9\,\omega]^2 + 0.000177927\, \text{Cos}[10\,\omega] + 0.0017625\, \text{Cos}[\omega]\, \text{Cos}[10\,\omega] -$
$0.000871011\, \text{Cos}[2\,\omega]\, \text{Cos}[10\,\omega] - 0.00524084\, \text{Cos}[3\,\omega]\, \text{Cos}[10\,\omega] +$

```
0.00579198 Cos[4 ω] Cos[10 ω] + 0.0285634 Cos[5 ω] Cos[10 ω] +
0.0285634 Cos[6 ω] Cos[10 ω] + 0.00579198 Cos[7 ω] Cos[10 ω] -
0.00524084 Cos[8 ω] Cos[10 ω] - 0.000871011 Cos[9 ω] Cos[10 ω] +
0.000881248 Cos[10 ω]² + 0.000017962 Cos[11 ω] + 0.000177927 Cos[ω] Cos[11 ω] -
0.00008793 Cos[2 ω] Cos[11 ω] - 0.000529071 Cos[3 ω] Cos[11 ω] +
0.00058471 Cos[4 ω] Cos[11 ω] + 0.00288352 Cos[5 ω] Cos[11 ω] +
0.00288352 Cos[6 ω] Cos[11 ω] + 0.00058471 Cos[7 ω] Cos[11 ω] -
0.000529071 Cos[8 ω] Cos[11 ω] - 0.00008793 Cos[9 ω] Cos[11 ω] +
0.000177927 Cos[10 ω] Cos[11 ω] + 8.98101 × 10⁻⁶ Cos[11 ω]² - 0.00008793 Cos[2 (π + ω)] +
0.000871011 Cos[ω] Cos[2 (π + ω)] + 0.000215223 Cos[2 (π + ω)]² -
0.000529071 Cos[3 (π + ω)] + 0.00524084 Cos[ω] Cos[3 (π + ω)] +
0.00258998 Cos[2 (π + ω)] Cos[3 (π + ω)] + 0.0077919 Cos[3 (π + ω)]² +
0.00058471 Cos[4 (π + ω)] - 0.00579198 Cos[ω] Cos[4 (π + ω)] -
0.00286235 Cos[2 (π + ω)] Cos[4 (π + ω)] - 0.0172226 Cos[3 (π + ω)] Cos[4 (π + ω)] +
0.0095169 Cos[4 (π + ω)]² + 0.00288352 Cos[5 (π + ω)] - 0.0285634 Cos[ω] Cos[5 (π + ω)] -
0.0141158 Cos[2 (π + ω)] Cos[5 (π + ω)] - 0.084934 Cos[3 (π + ω)] Cos[5 (π + ω)] +
0.0938659 Cos[4 (π + ω)] Cos[5 (π + ω)] + 0.231452 Cos[5 (π + ω)]² +
0.00288352 Cos[6 (π + ω)] - 0.0285634 Cos[ω] Cos[6 (π + ω)] -
0.0141158 Cos[2 (π + ω)] Cos[6 (π + ω)] - 0.084934 Cos[3 (π + ω)] Cos[6 (π + ω)] +
0.0938659 Cos[4 (π + ω)] Cos[6 (π + ω)] + 0.462903 Cos[5 (π + ω)] Cos[6 (π + ω)] +
0.231452 Cos[6 (π + ω)]² + 0.00058471 Cos[7 (π + ω)] - 0.00579198 Cos[ω] Cos[7 (π + ω)] -
0.00286235 Cos[2 (π + ω)] Cos[7 (π + ω)] - 0.0172226 Cos[3 (π + ω)] Cos[7 (π + ω)] +
0.0190338 Cos[4 (π + ω)] Cos[7 (π + ω)] + 0.0938659 Cos[5 (π + ω)] Cos[7 (π + ω)] +
0.0938659 Cos[6 (π + ω)] Cos[7 (π + ω)] + 0.0095169 Cos[7 (π + ω)]² -
0.000529071 Cos[8 (π + ω)] + 0.00524084 Cos[ω] Cos[8 (π + ω)] +
0.00258998 Cos[2 (π + ω)] Cos[8 (π + ω)] + 0.0155838 Cos[3 (π + ω)] Cos[8 (π + ω)] -
0.0172226 Cos[4 (π + ω)] Cos[8 (π + ω)] - 0.084934 Cos[5 (π + ω)] Cos[8 (π + ω)] -
0.084934 Cos[6 (π + ω)] Cos[8 (π + ω)] - 0.0172226 Cos[7 (π + ω)] Cos[8 (π + ω)] +
0.0077919 Cos[8 (π + ω)]² - 0.00008793 Cos[9 (π + ω)] +
0.000871011 Cos[ω] Cos[9 (π + ω)] + 0.000430446 Cos[2 (π + ω)] Cos[9 (π + ω)] +
0.00258998 Cos[3 (π + ω)] Cos[9 (π + ω)] - 0.00286235 Cos[4 (π + ω)] Cos[9 (π + ω)] -
0.0141158 Cos[5 (π + ω)] Cos[9 (π + ω)] - 0.0141158 Cos[6 (π + ω)] Cos[9 (π + ω)] -
0.00286235 Cos[7 (π + ω)] Cos[9 (π + ω)] + 0.00258998 Cos[8 (π + ω)] Cos[9 (π + ω)] +
0.000215223 Cos[9 (π + ω)]² + 0.000177927 Cos[10 (π + ω)] -
0.0017625 Cos[ω] Cos[10 (π + ω)] - 0.000871011 Cos[2 (π + ω)] Cos[10 (π + ω)] -
0.00524084 Cos[3 (π + ω)] Cos[10 (π + ω)] + 0.00579198 Cos[4 (π + ω)] Cos[10 (π + ω)] +
0.0285634 Cos[5 (π + ω)] Cos[10 (π + ω)] + 0.0285634 Cos[6 (π + ω)] Cos[10 (π + ω)] +
0.00579198 Cos[7 (π + ω)] Cos[10 (π + ω)] - 0.00524084 Cos[8 (π + ω)] Cos[10 (π + ω)] -
0.000871011 Cos[9 (π + ω)] Cos[10 (π + ω)] + 0.000881248 Cos[10 (π + ω)]² +
0.000017962 Cos[11 (π + ω)] - 0.000177927 Cos[ω] Cos[11 (π + ω)] -
0.00008793 Cos[2 (π + ω)] Cos[11 (π + ω)] - 0.000529071 Cos[3 (π + ω)] Cos[11 (π + ω)] +
0.00058471 Cos[4 (π + ω)] Cos[11 (π + ω)] + 0.00288352 Cos[5 (π + ω)] Cos[11 (π + ω)] +
0.00288352 Cos[6 (π + ω)] Cos[11 (π + ω)] + 0.00058471 Cos[7 (π + ω)] Cos[11 (π + ω)] -
0.000529071 Cos[8 (π + ω)] Cos[11 (π + ω)] - 0.00008793 Cos[9 (π + ω)] Cos[11 (π + ω)] +
0.000177927 Cos[10 (π + ω)] Cos[11 (π + ω)] + 8.98101 × 10⁻⁶ Cos[11 (π + ω)]² +
0.0017625 Sin[ω]² - 0.000871011 Sin[ω] Sin[2 ω] + 0.000215223 Sin[2 ω]² -
0.00524084 Sin[ω] Sin[3 ω] + 0.00258998 Sin[2 ω] Sin[3 ω] +
```

$0.0077919 \, \text{Sin}[3\,\omega]^2 + 0.00579198 \, \text{Sin}[\omega] \, \text{Sin}[4\,\omega] - 0.00286235 \, \text{Sin}[2\,\omega] \, \text{Sin}[4\,\omega] - 0.0172226 \, \text{Sin}[3\,\omega] \, \text{Sin}[4\,\omega] + 0.0095169 \, \text{Sin}[4\,\omega]^2 + 0.0285634 \, \text{Sin}[\omega] \, \text{Sin}[5\,\omega] - 0.0141158 \, \text{Sin}[2\,\omega] \, \text{Sin}[5\,\omega] - 0.084934 \, \text{Sin}[3\,\omega] \, \text{Sin}[5\,\omega] + 0.0938659 \, \text{Sin}[4\,\omega] \, \text{Sin}[5\,\omega] + 0.231452 \, \text{Sin}[5\,\omega]^2 + 0.0285634 \, \text{Sin}[\omega] \, \text{Sin}[6\,\omega] - 0.0141158 \, \text{Sin}[2\,\omega] \, \text{Sin}[6\,\omega] - 0.084934 \, \text{Sin}[3\,\omega] \, \text{Sin}[6\,\omega] + 0.0938659 \, \text{Sin}[4\,\omega] \, \text{Sin}[6\,\omega] + 0.462903 \, \text{Sin}[5\,\omega] \, \text{Sin}[6\,\omega] + 0.231452 \, \text{Sin}[6\,\omega]^2 + 0.00579198 \, \text{Sin}[\omega] \, \text{Sin}[7\,\omega] - 0.00286235 \, \text{Sin}[2\,\omega] \, \text{Sin}[7\,\omega] - 0.0172226 \, \text{Sin}[3\,\omega] \, \text{Sin}[7\,\omega] + 0.0190338 \, \text{Sin}[4\,\omega] \, \text{Sin}[7\,\omega] + 0.0938659 \, \text{Sin}[5\,\omega] \, \text{Sin}[7\,\omega] + 0.0938659 \, \text{Sin}[6\,\omega] \, \text{Sin}[7\,\omega] + 0.0095169 \, \text{Sin}[7\,\omega]^2 - 0.00524084 \, \text{Sin}[\omega] \, \text{Sin}[8\,\omega] + 0.00258998 \, \text{Sin}[2\,\omega] \, \text{Sin}[8\,\omega] + 0.0155838 \, \text{Sin}[3\,\omega] \, \text{Sin}[8\,\omega] - 0.0172226 \, \text{Sin}[4\,\omega] \, \text{Sin}[8\,\omega] - 0.084934 \, \text{Sin}[5\,\omega] \, \text{Sin}[8\,\omega] - 0.084934 \, \text{Sin}[6\,\omega] \, \text{Sin}[8\,\omega] - 0.0172226 \, \text{Sin}[7\,\omega] \, \text{Sin}[8\,\omega] + 0.0077919 \, \text{Sin}[8\,\omega]^2 - 0.000871011 \, \text{Sin}[\omega] \, \text{Sin}[9\,\omega] + 0.000430446 \, \text{Sin}[2\,\omega] \, \text{Sin}[9\,\omega] + 0.00258998 \, \text{Sin}[3\,\omega] \, \text{Sin}[9\,\omega] - 0.00286235 \, \text{Sin}[4\,\omega] \, \text{Sin}[9\,\omega] - 0.0141158 \, \text{Sin}[5\,\omega] \, \text{Sin}[9\,\omega] - 0.0141158 \, \text{Sin}[6\,\omega] \, \text{Sin}[9\,\omega] - 0.00286235 \, \text{Sin}[7\,\omega] \, \text{Sin}[9\,\omega] + 0.00258998 \, \text{Sin}[8\,\omega] \, \text{Sin}[9\,\omega] + 0.000215223 \, \text{Sin}[9\,\omega]^2 + 0.0017625 \, \text{Sin}[\omega] \, \text{Sin}[10\,\omega] - 0.000871011 \, \text{Sin}[2\,\omega] \, \text{Sin}[10\,\omega] - 0.00524084 \, \text{Sin}[3\,\omega] \, \text{Sin}[10\,\omega] + 0.00579198 \, \text{Sin}[4\,\omega] \, \text{Sin}[10\,\omega] + 0.0285634 \, \text{Sin}[5\,\omega] \, \text{Sin}[10\,\omega] + 0.0285634 \, \text{Sin}[6\,\omega] \, \text{Sin}[10\,\omega] + 0.00579198 \, \text{Sin}[7\,\omega] \, \text{Sin}[10\,\omega] - 0.00524084 \, \text{Sin}[8\,\omega] \, \text{Sin}[10\,\omega] - 0.000871011 \, \text{Sin}[9\,\omega] \, \text{Sin}[10\,\omega] + 0.000881248 \, \text{Sin}[10\,\omega]^2 + 0.000177927 \, \text{Sin}[\omega] \, \text{Sin}[11\,\omega] - 0.00008793 \, \text{Sin}[2\,\omega] \, \text{Sin}[11\,\omega] - 0.000529071 \, \text{Sin}[3\,\omega] \, \text{Sin}[11\,\omega] + 0.00058471 \, \text{Sin}[4\,\omega] \, \text{Sin}[11\,\omega] + 0.00288352 \, \text{Sin}[5\,\omega] \, \text{Sin}[11\,\omega] + 0.00288352 \, \text{Sin}[6\,\omega] \, \text{Sin}[11\,\omega] + 0.00058471 \, \text{Sin}[7\,\omega] \, \text{Sin}[11\,\omega] - 0.000529071 \, \text{Sin}[8\,\omega] \, \text{Sin}[11\,\omega] - 0.00008793 \, \text{Sin}[9\,\omega] \, \text{Sin}[11\,\omega] + 0.000177927 \, \text{Sin}[10\,\omega] \, \text{Sin}[11\,\omega] + 8.98101 \times 10^{-6} \, \text{Sin}[11\,\omega]^2 + 0.000871011 \, \text{Sin}[\omega] \, \text{Sin}[2\,(\pi+\omega)] + 0.000215223 \, \text{Sin}[2\,(\pi+\omega)]^2 + 0.00524084 \, \text{Sin}[\omega] \, \text{Sin}[3\,(\pi+\omega)] + 0.00258998 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[3\,(\pi+\omega)] + 0.0077919 \, \text{Sin}[3\,(\pi+\omega)]^2 - 0.00579198 \, \text{Sin}[\omega] \, \text{Sin}[4\,(\pi+\omega)] - 0.00286235 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[4\,(\pi+\omega)] - 0.0172226 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[4\,(\pi+\omega)] + 0.0095169 \, \text{Sin}[4\,(\pi+\omega)]^2 - 0.0285634 \, \text{Sin}[\omega] \, \text{Sin}[5\,(\pi+\omega)] - 0.0141158 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] - 0.084934 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] + 0.0938659 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[5\,(\pi+\omega)] + 0.231452 \, \text{Sin}[5\,(\pi+\omega)]^2 - 0.0285634 \, \text{Sin}[\omega] \, \text{Sin}[6\,(\pi+\omega)] - 0.0141158 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[6\,(\pi+\omega)] - 0.084934 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[6\,(\pi+\omega)] + 0.0938659 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[6\,(\pi+\omega)] + 0.462903 \, \text{Sin}[5\,(\pi+\omega)] \, \text{Sin}[6\,(\pi+\omega)] + 0.231452 \, \text{Sin}[6\,(\pi+\omega)]^2 - 0.00579198 \, \text{Sin}[\omega] \, \text{Sin}[7\,(\pi+\omega)] - 0.00286235 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[7\,(\pi+\omega)] - 0.0172226 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[7\,(\pi+\omega)] + 0.0190338 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[7\,(\pi+\omega)] + 0.0938659 \, \text{Sin}[5\,(\pi+\omega)] \, \text{Sin}[7\,(\pi+\omega)] + 0.0938659 \, \text{Sin}[6\,(\pi+\omega)] \, \text{Sin}[7\,(\pi+\omega)] + 0.0095169 \, \text{Sin}[7\,(\pi+\omega)]^2 + 0.00524084 \, \text{Sin}[\omega] \, \text{Sin}[8\,(\pi+\omega)] + 0.00258998 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] + 0.0155838 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] - 0.0172226 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] - 0.084934 \, \text{Sin}[5\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] - 0.084934 \, \text{Sin}[6\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] - 0.0172226 \, \text{Sin}[7\,(\pi+\omega)] \, \text{Sin}[8\,(\pi+\omega)] + 0.0077919 \, \text{Sin}[8\,(\pi+\omega)]^2 + 0.000871011 \, \text{Sin}[\omega] \, \text{Sin}[9\,(\pi+\omega)] + 0.000430446 \, \text{Sin}[2\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] + 0.00258998 \, \text{Sin}[3\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] - 0.00286235 \, \text{Sin}[4\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] - 0.0141158 \, \text{Sin}[5\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] - 0.0141158 \, \text{Sin}[6\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] - 0.00286235 \, \text{Sin}[7\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] + 0.00258998 \, \text{Sin}[8\,(\pi+\omega)] \, \text{Sin}[9\,(\pi+\omega)] +$

$0.000215223 \, \text{Sin}[9 \, (\pi + \omega)]^2 - 0.0017625 \, \text{Sin}[\omega] \, \text{Sin}[10 \, (\pi + \omega)] -$
$0.000871011 \, \text{Sin}[2 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] - 0.00524084 \, \text{Sin}[3 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] +$
$0.00579198 \, \text{Sin}[4 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] + 0.0285634 \, \text{Sin}[5 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] +$
$0.0285634 \, \text{Sin}[6 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] + 0.00579198 \, \text{Sin}[7 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] -$
$0.00524084 \, \text{Sin}[8 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] - 0.000871011 \, \text{Sin}[9 \, (\pi + \omega)] \, \text{Sin}[10 \, (\pi + \omega)] +$
$0.000881248 \, \text{Sin}[10 \, (\pi + \omega)]^2 - 0.000177927 \, \text{Sin}[\omega] \, \text{Sin}[11 \, (\pi + \omega)] -$
$0.00008793 \, \text{Sin}[2 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] - 0.000529071 \, \text{Sin}[3 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$
$0.00058471 \, \text{Sin}[4 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] + 0.00288352 \, \text{Sin}[5 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$
$0.00288352 \, \text{Sin}[6 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] + 0.00058471 \, \text{Sin}[7 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] -$
$0.000529071 \, \text{Sin}[8 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] - 0.00008793 \, \text{Sin}[9 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] +$
$0.000177927 \, \text{Sin}[10 \, (\pi + \omega)] \, \text{Sin}[11 \, (\pi + \omega)] + 8.98101 \times 10^{-6} \, \text{Sin}[11 \, (\pi + \omega)]^2$

In[87]:= **Simplify[%]**

Out[87]= $0.999464 - 5.55112 \times 10^{-17} \, \text{Cos}[\omega] + 0.00296316 \, \text{Cos}[2 \, \omega] - 1.38778 \times 10^{-17} \, \text{Cos}[3 \, \omega] -$
$0.00876136 \, \text{Cos}[4 \, \omega] - 3.46945 \times 10^{-18} \, \text{Cos}[5 \, \omega] + 0.0450619 \, \text{Cos}[6 \, \omega] -$
$3.46945 \times 10^{-18} \, \text{Cos}[7 \, \omega] - 0.00560033 \, \text{Cos}[8 \, \omega] - 1.38778 \times 10^{-17} \, \text{Cos}[9 \, \omega] +$
$0.000711708 \, \text{Cos}[10 \, \omega] - 1.38778 \times 10^{-17} \, \text{Cos}[13 \, \omega] - 3.46945 \times 10^{-18} \, \text{Cos}[15 \, \omega] -$
$8.67362 \times 10^{-19} \, \text{Cos}[17 \, \omega] + \left(0. + 1.38778 \times 10^{-17} \, \dot{\mathbb{i}}\right) \, \text{Cos}[\omega] \, \text{Sin}[\omega]$

In[88]:= **Plot$\left[\text{Abs}[\text{BLM}[\omega]]^2 + \text{Abs}[\text{BLM}[\omega + \text{Pi}]]^2, \{\omega, 0, \text{Pi}\}, \text{PlotRange} \rightarrow \{0, 1.2\}\right]$**

Out[88]=