

Approximation of the scaling and wavelet functions using the cascade algorithm

```
in[26]= cascadehipw[h_, n_] := Module[{degrees, min, max, right, hpol, hpols, clist, tbl, z},
  degrees = Map[First[#] &, h];
  min = Min[degrees];
  max = Max[degrees];
  hpol := Apply[Plus, Map[Last[#] z^First[#] &, h]];
  hpols := Expand[Product[hpol /. z -> z^(2^t), {t, 0, n-1}]];
  clist = 2^n CoefficientList[z^(-min (2^n - 1)) hpols, z];
  right = (1 - 2^(-n)) (max - min) + 2^(-n);
  tbl = Table[{clist[[k]], (k-1) 2^(-n) ≤ t < k 2^(-n)}, {k, 1, Length[clist]}];
  Plot[Piecewise[tbl], {t, 0, right}, PlotRange -> All, Filling -> Axis]
]
```

```
in[27]= cascadehipw::usage
```

```
"level-n iteration for approximating the scaling function belonging
to the filter h using the cascade algorithm.
Display as a step function(piecewise)";
```

```
in[28]= cascadehidis[h_, n_] := Module[{degrees, min, max, hpol, hpols, clist, z},
  degrees = Map[First[#] &, h];
  min = Min[degrees];
  max = Max[degrees];
  hpol := Apply[Plus, Map[Last[#] z^First[#] &, h]];
  hpols := Expand[Product[hpol /. z -> z^(2^t), {t, 0, n-1}]];
  clist = 2^n CoefficientList[z^(-min (2^n - 1)) hpols, z];
  tbl = Table[{(k-1) 2^(-n), clist[[k]]}, {k, 1, Length[clist]}];
  ListPlot[tbl, PlotRange -> All, Filling -> Axis]
]
```

```
in[29]= cascadehidis::usage "level-n iteration for approximating
the scaling function belonging to the filter h using the
cascade algorithm.
Display as discrete values";
```

```
in[30]= cascadepsipw[h_, n_] := Module[{degrees, min, max, right, hpol, clist, tbl, z},
  degrees = Map[First[#] &, h];
  min = Min[degrees];
  max = Max[degrees];
  hpol := Apply[Plus, Map[Last[#] z^First[#] &, h]];
  gpol := hpol /. (z -> -1/z);
  hpols :=
  Expand[Product[(hpol /. z -> z^(2^t)), {t, 0, n-2}] (gpol /. z -> z^(2^(n-1)))];
  clist = 2^n CoefficientList[z^(-min (2^(n-1) - 1) + max 2^(n-1)) hpols, z];
  right = (1 - 2^(-n)) (max - min) + 2^(-n);
  tbl = Table[{clist[[k]], (k-1) 2^(-n) ≤ t < k 2^(-n)}, {k, 1, Length[clist]}];
  Plot[Piecewise[tbl], {t, 0, right}, PlotRange -> All, Filling -> Axis]
]
```

```

In[31]= cascadepsipw::usage
  "level-n iteration for approximating the wavelet function belonging
    to the filter h using the cascade algorithm.
    Display as a step function(piecewise)";
In[32]= cascadepsidis[h_, n_] := Module[{degrees, min, max, hpol, clist, z},
  degrees = Map[First[#] &, h];
  min = Min[degrees];
  max = Max[degrees];
  hpol := Apply[Plus, Map[Last[#] z^First[#] &, h]];
  gpol := hpol /. (z -> -1/z);
  hpols :=
    Expand[Product[hpol /. z -> z^(2^t), {t, 0, n-2}] (gpol /. z -> z^(2^(n-1)))]];
  clist = 2^n CoefficientList[z^(-min (2^(n-1) - 1) + 2^(n-1) max) hpols, z];
  tbl = Table[{(k-1) 2^(-n), clist[[k]]}, {k, 1, Length[clist]}];
  ListPlot[tbl, PlotRange -> All, Filling -> Axis]
]
In[33]= cascadepsidis::usage
  "level-n iteration for approximating the wavelet function belonging
    to the filter h using the cascade algorithm.
    Display as discrete values";

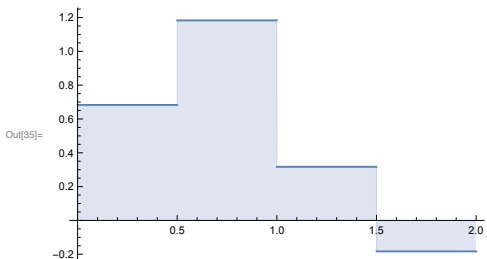
```

Example Daubechies-4

```

In[34]= db4 = WaveletFilterCoefficients[DaubechiesWavelet[2]]
Out[34]= {{0, 0.341506}, {1, 0.591506}, {2, 0.158494}, {3, -0.0915064}}
In[35]= cascadepsipw[db4, 1]

```



```

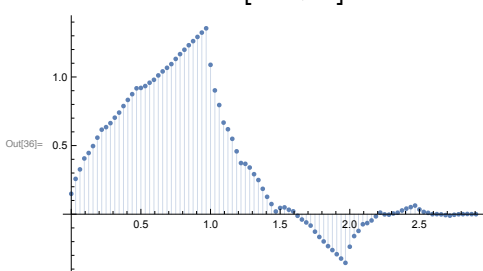
Out[35]=

```

```

In[36]= cascadepsidis[db4, 5]

```

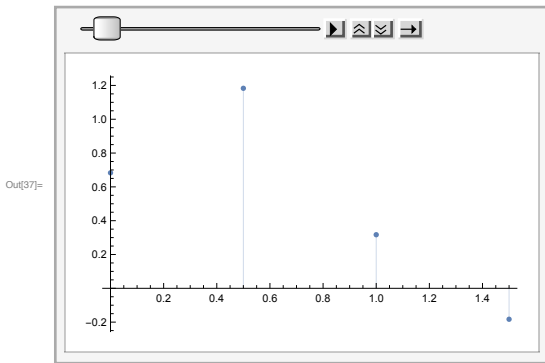


```

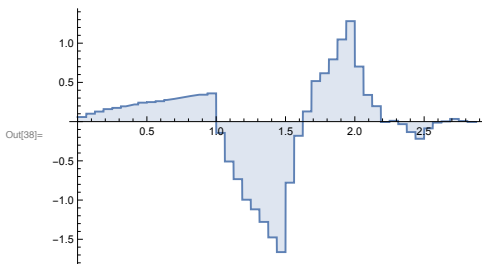
Out[36]=

```

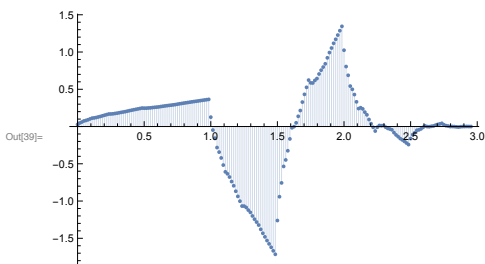
```
In[37]= ListAnimate[Table[cascadephidis[db4, t], {t, 1, 7}], AnimationRunning -> False]
```



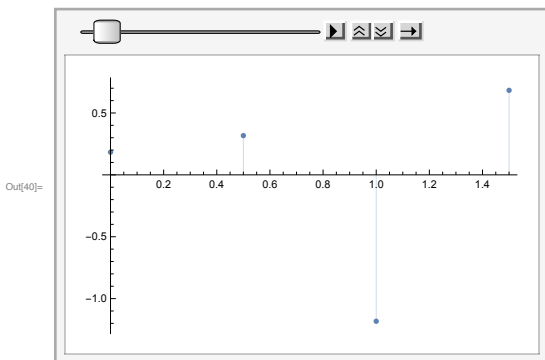
```
In[38]= cascadepsipw[db4, 4]
```



```
In[39]= cascadepsidis[db4, 6]
```



```
In[40]= ListAnimate[Table[cascadepsidis[db4, t], {t, 1, 7}], AnimationRunning -> False]
```

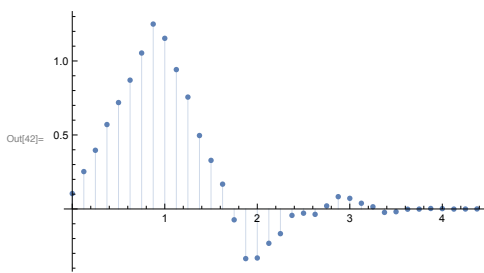


Example Daubechies-6

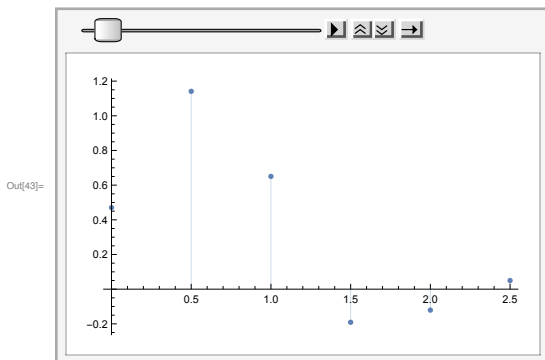
```
In[41]= db6 = WaveletFilterCoefficients[DaubechiesWavelet[3]]
```

```
Out[41]= {{0, 0.235234}, {1, 0.570558}, {2, 0.325183}, {3, -0.0954672}, {4, -0.0604161}, {5, 0.0249087}}
```

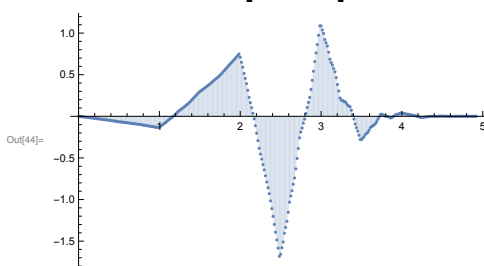
```
In[42]= cascadephidis[db6, 3]
```



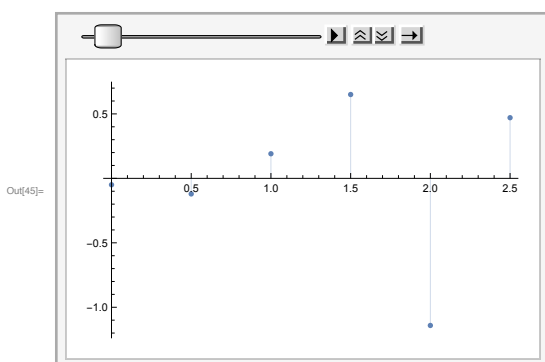
```
In[43]= ListAnimate[Table[cascadephidis[db6, t], {t, 1, 7}], AnimationRunning -> False]
```



```
In[44]= cascadepsidis[db6, 6]
```



```
In[45]= ListAnimate[Table[cascadepsidis[db6, t], {t, 1, 7}], AnimationRunning -> False]
```

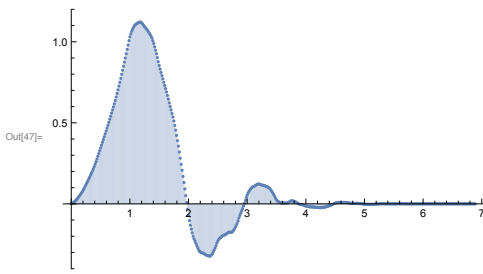


Example Daubechies-8

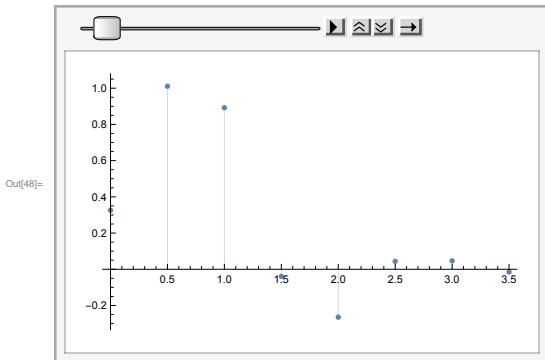
```
In[46]= db8 = WaveletFilterCoefficients[DaubechiesWavelet[4]]
```

```
Out[46]= {{0, 0.162902}, {1, 0.505473}, {2, 0.4461}, {3, -0.0197875}, {4, -0.132254},
          {5, 0.0218082}, {6, 0.0232518}, {7, -0.00749349}}
```

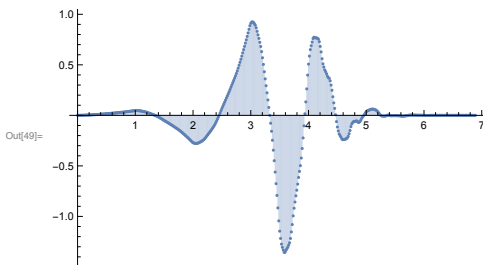
In[47]= cascadephidis[db8, 6]



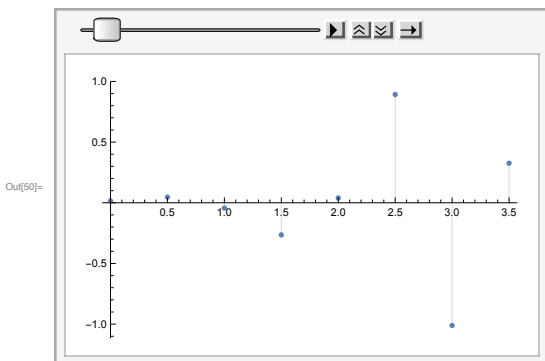
In[48]= ListAnimate[Table[cascadephidis[db8, t], {t, 1, 7}], AnimationRunning -> False]



In[49]= cascadepsidis[db8, 6]



In[50]= ListAnimate[Table[cascadepsidis[db8, t], {t, 1, 7}], AnimationRunning -> False]

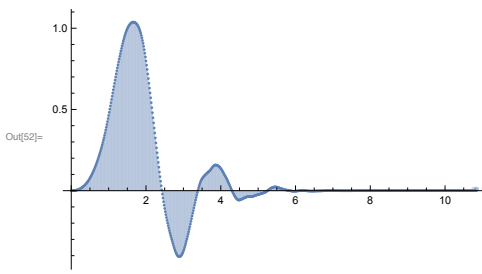


Example Daubechies-12

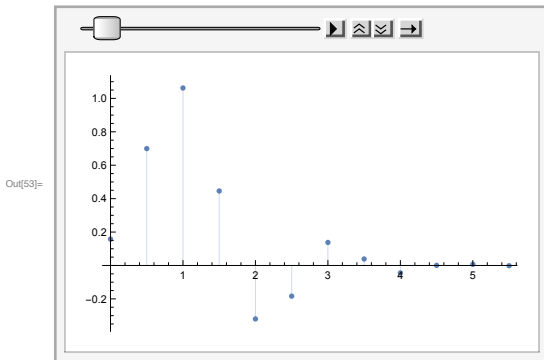
In[51]= db12 = WaveletFilterCoefficients[DaubechiesWavelet[6]]

Out[51]= {{0, 0.0788712}, {1, 0.349752}, {2, 0.531132}, {3, 0.222916}, {4, -0.159993},
 {5, -0.091759}, {6, 0.068944}, {7, 0.0194616}, {8, -0.0223319}, {9, 0.000391626},
 {10, 0.00337803}, {11, -0.000761767}}

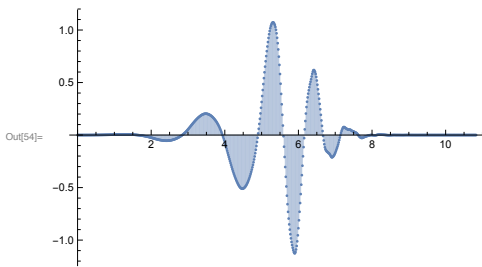
```
In[52]= cascadephidis[db12, 6]
```



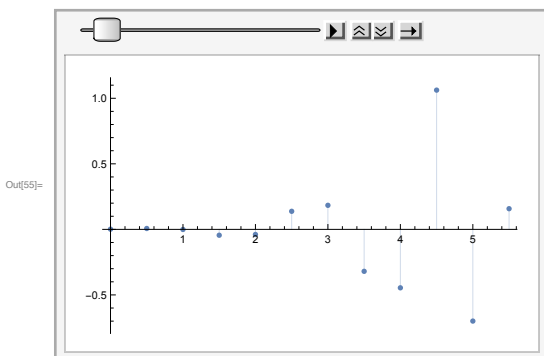
```
In[53]= ListAnimate[Table[cascadephidis[db12, t], {t, 1, 7}], AnimationRunning -> False]
```



```
In[54]= cascadepsidis[db12, 6]
```



```
In[55]= ListAnimate[Table[cascadepsidis[db12, t], {t, 1, 7}], AnimationRunning -> False]
```

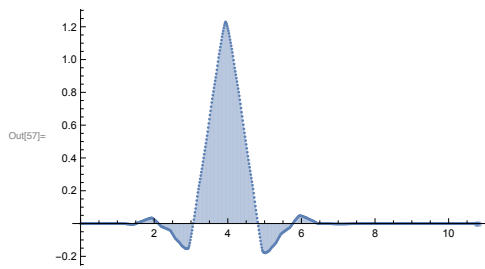


Example Coiflet-12

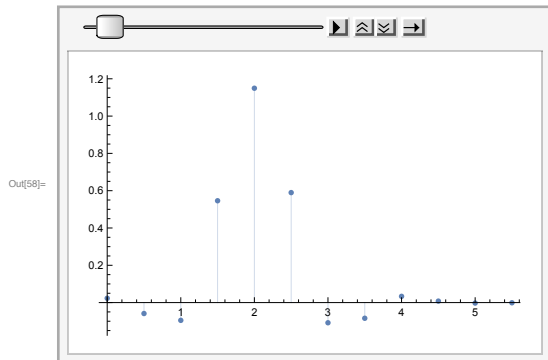
```
In[56]= coif12 = WaveletFilterCoefficients[CoifletWavelet[2]]
```

```
Out[56]= {{-4, 0.0115876}, {-3, -0.0293201}, {-2, -0.0476396}, {-1, 0.273021}, {0, 0.574682},
{1, 0.294867}, {2, -0.0540856}, {3, -0.0420265}, {4, 0.0167444}, {5, 0.00396788},
{6, -0.0012892}, {7, -0.000509505}}
```

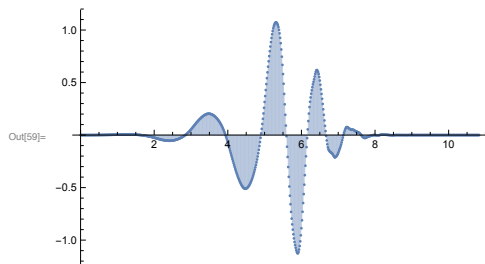
In[57]= cascadephidis[coif12, 6]



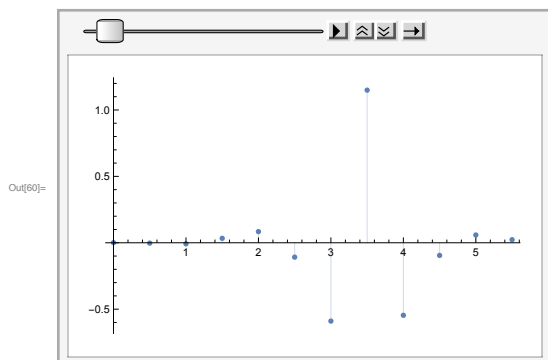
In[58]= ListAnimate[Table[cascadephidis[coif12, t], {t, 1, 7}], AnimationRunning -> False]



In[59]= cascadepsidis[db12, 6]



In[60]= ListAnimate[Table[cascadepsidis[coif12, t], {t, 1, 7}], AnimationRunning -> False]

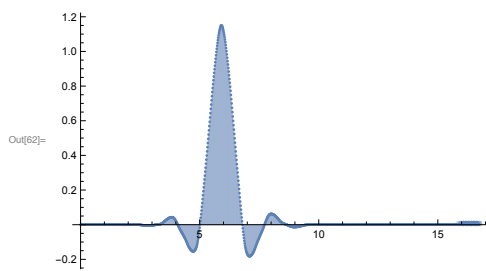


Example Coiflet-24

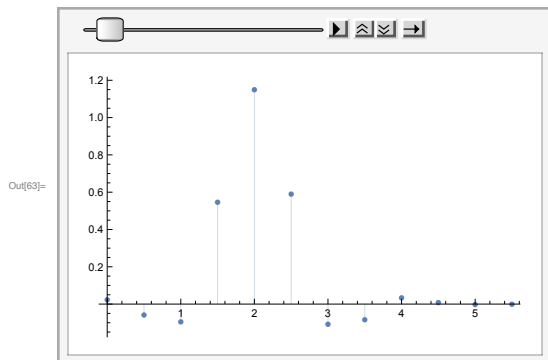
In[61]= coif24 = WaveletFilterCoefficients[CoifletWavelet[3]]

Out[61]= {{-6, -0.00268242}, {-5, 0.00550313}, {-4, 0.0165836}, {-3, -0.0465078},
 {-2, -0.0432208}, {-1, 0.286503}, {0, 0.561285}, {1, 0.302984}, {2, -0.0507701},
 {3, -0.0581963}, {4, 0.0244341}, {5, 0.0112292}, {6, -0.0063696}, {7, -0.00182046},
 {8, 0.000790205}, {9, 0.000329665}, {10, -0.0000501928}, {11, -0.0000244657}}

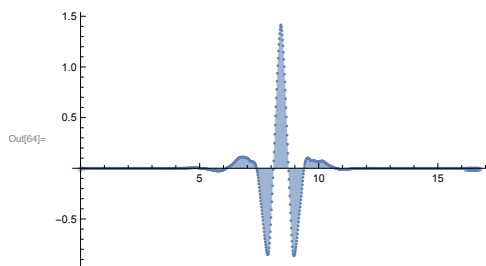
In[62]= `cascadephidis[coif24, 6]`



In[63]= `ListAnimate[Table[cascadephidis[coif12, t], {t, 1, 7}], AnimationRunning -> False]`



In[64]= `cascadepsidis[coif24, 6]`



In[65]= `ListAnimate[Table[cascadepsidis[coif12, t], {t, 1, 7}], AnimationRunning -> False]`

